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YOUVA

Unit 3

Introduction to Stats & Measurements of central Tendencies.

Finding mean of Inclusive type data.

CI	F
5-8	3
9-12	5
13-16	2
17-20	6
21-24	4
25-28	5

$$AF = \frac{9-8}{2} = \frac{1}{2} = 0.5$$

CI	F	x	Fx
4.5 - 8.5	3	6.5	19.5
8.5 - 12.5	5	10.5	52.5
12.5 - 16.5	2	14.5	29
16.5 - 20.5	6	18.5	111
20.5 - 24.5	4	22.5	90
24.5 - 28.5	5	26.5	132.5
	N=25		434.5

$$\bar{x} = \frac{434.5}{25} = \frac{4345 \times 4}{25 \times 4}$$

$$= 17.38$$

Combined Mean

Number	male	female
	60 N_1 =	N_2 = 40
wages (Avg)	$\bar{x}_1$ = 324	$\bar{x}_2$ = 350

$$\bar{x} = \frac{\Sigma x}{N}$$

$$\Sigma x = N \cdot \bar{x}$$

(male) = 60 × 344

(female) = 40 × 350 = 14000

$$\bar{x} = \frac{(60 \times 324) + (40 \times 350)}{60 + 40}$$

$$= \frac{N_1 \cdot \bar{x}_1 + N_2 \cdot \bar{x}_2}{N_1 + N_2}$$

$$= \frac{19440 + 14000}{100}$$

$$= \frac{33440}{100}$$

$$= 334.4$$

Q2 Avg daily wage of 100 workers (N) at a construct^n site is ₹ 334.40 ($\bar{x}$). The avg daily wage of 60 male workers (N_1) are ₹ 324 ($\bar{x}_1$). Then find the avg wages of the female workers.

$N_2 = 100 - 60$

$N_2 = 40$

	Male	Female	Total
Number	$N_1 = 60$	$N_2 = 100-60 = 40$	$N = 100$
Wages	$\bar{x}_1 = 324$	$\bar{x}_2 = ?$	$\bar{x} = 334.4$

$$\bar{x} = \frac{N_1 \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2}$$

$$334.4 = \frac{60 \times 324 + 40 \times \bar{x}_2}{100}$$

$$33440 = 19440 + 40 \times \bar{x}_2$$

$$14000 = 40 \times \bar{x}_2$$

$$\bar{x}_2 = 350$$

Q3

	Male	Female	Total
Number	$N_1 = ?$	$N_2 = 40$	100
Wages	$\bar{x}_1 = 324$	$\bar{x}_2 = ?$	$\bar{x} = 334.4$

$$\bar{x} = \frac{N_1 \cdot \bar{x}_1 + N_2 \cdot \bar{x}_2}{N_1 + N_2}$$

$$334.4 = \frac{N_1 \cdot 324 + 40 \cdot \bar{x}_2}{N_1 + 40}$$

$$334.4 N_1 + 13376 = 324 N_1 + 40 \cdot \bar{x}_2$$

$$10.4 N_1 + 13376 = 40 \bar{x}_2$$

$$N_1 = 60$$

$$40 \bar{x}_2 - 10.4 N_1 = 13376$$

$$40 \bar{x}_2 = 13376 + 10.4 N_1$$

$$\bar{x}_2 = \frac{13376 + 10.4 N_1}{40}$$

$$\bar{x}_2 = \frac{13376 + 624}{40}$$

$$\bar{x}_2 = \frac{14000}{40}$$

$$\bar{x}_2 = 350$$

The Avg daily wages of certain Male workers is ₹350, The Avg wages of ~~Female workers~~ is ₹324 & Avg wages of all the workers together is ₹334.4. Find

	Male	Female	Total
No	N_1	N_2	$N_1 + N_2$
Avg wages	350	324	334.4

Find :- 1) Ratio of Male to Female workers.
2) % of Female workers.

$$\bar{X} = \frac{N_1 \cdot \bar{x}_1 + N_2 \bar{x}_2}{N_1 + N_2}$$

$$334.4 = \frac{324N_1 + 350N_2}{N_1 + N_2}$$

$$N_1 + N_2 = \frac{324N_1 + 350N_2}{334.4}$$

$$334.4N_1 + 334.4N_2 = 324N_1 + 350N_2$$

$$10.4N_1 = 15.6N_2$$

$$\frac{N_1}{N_2} = \frac{15.6}{10.4} = \frac{156}{104} = \frac{39}{26} = \frac{3}{2}$$

$N_1 : N_2 = 3 : 2$

Assume no of male $= 3x$

Female $= 2x$

Total $= 3x + 2x = 5x$

% Female $= \frac{2x}{5x} \times 100^{20}$

$= 40\%$

3) If No. of Male workers are 60 then find the no of female worker.

$2x = 60$

$x = 20$

$\therefore$ Female worker $= 2x$

$= 2 \times 20$

$= 40$

Weighted mean.

W	N	WN
5	3	15
8	5	40
11	7	77
14	5	70
	N=20	$\Sigma WN = 202$

Weighted mean $= \frac{\Sigma WN}{N} = \frac{202}{20} = 10.1$

Median (Partitioned Avg)

Raw data < can be even / can be odd.

Step 1:- Arrange the data in ascending or descending order.

Step 2:- If no. of observations are

1) odd - median $= \left(\frac{N+1}{2}\right)^{th}$ observation.

2) Even =

$$\text{median} = \frac{\left(\frac{N}{2}\right)^{th}\text{obv} + \left(\frac{N}{2}+1\right)^{th}\text{obv}}{2}$$

Find median of

1) 12, 10, 18, 5, 9, 16, 14, 18, 12

5, 9, 10, 12, 12, 14, 16, 18, 18

(odd) median $= \left(\frac{9+1}{2}\right)^{th}$

$= 5^{th}$ obv

$= 12$

2) 24, 36, 18, 35, 12, 9, 18, 36, 32, 45

9, 12, 18, 18, 24, 32, 35, 36, 36, 45

even median $= \frac{\left(\frac{10}{2}\right)^{th} + (6)^{th}\text{obv}}{2}$ avg median $= \frac{\left(\frac{n}{2}\right)^{th}\text{obv} + \left(\frac{N}{2}+1\right)^{th}}{2}$

$= \frac{5^{th} + 6^{th}\text{obv}}{2}$

$= \frac{24+32}{2}$

$= \frac{56}{2}$

$= 28$

If the median of the given data arranged in ascending order is 31 then find the value of x. Hence find mean also.

10, 18, 23, 25, 4x+5, 4x+9, 35, 38, 47, 60

$\text{median} = \frac{4x+5+4x+9}{2}$

$31 = \frac{4x+5+4x+9}{2}$

$62 = 8x + 14$

$8x = 62 - 14$

$8x = 48$

$x = 6$

$\therefore 5^{th} = 4x6+5$ $\quad 6^{th} = 4x+9$

$= 24+5$ $\quad = 24+9$

$5^{th} = 29$ $\quad 6^{th} = 33$

mean.

$= \frac{320}{10}$

mean $= 32$

Tabulated data

marks x	No of students f	cumulative freq
4	5	5
7	4	9
8	9	18
9	7	25
12	9	34
16	7	41
18	8	49
20	6	55
	55	

27.5

55

2

= 25, 34

Find median of

x	f	Cumulative Freq
4	9	9
7	5	14
8	7	21
10	8	29
11	4	33
16	8	41
18	4	45

N=45

Median (odd) = $\left(\frac{n+1}{2}\right)^{th}$

$= \left(\frac{46}{2}\right)^{th}$

$= 23^{th}$

$= 10$

short cut

- Find "$\frac{N}{2}$"
- locate 2 Adj. values in C.F column in such a way that one should be $< \frac{n}{2}$ other should be $> \frac{n}{2}$
- The value in the x column corresp. to the ">" sign is the value of median.

M	F	C.F
15	17	17
26	23	40
27	18	58
32	15	73
38	29	102
71	40	142
40	33	175

median = $\frac{N}{2}$

$= \frac{172}{2} = 86$ / $\frac{175}{2} = 87.5$

median = 38

Grouped data

CI	F	CF
0-4	9	9
4-8	8	17
8-12	6	23
12-16	12	35 <
16-20	18	53 >
20-24	7	60
24-28	15	75
	N=75	

$\frac{N}{2} = \frac{37.5}{2}$

median = $16 + \frac{37.5 - 35}{18} \times 4$

$= 16 + \frac{2.5}{18} \times 4$

$= 16 + \frac{1}{2}$

$= \frac{33}{2}$

$= 16.56$

median CI is the CI correspond to "> $\frac{N}{2}$"

Formula:-

median = $L_1 + \frac{\frac{N}{2} - CF}{F} \times (L_2 - L_1)$

Where L_1 = L.L of median class

L_2 = U.L of median class

F = Freq of median CI

CF = C.F of pre median CI

Grouped data

CI	F	CF
5-10	18	18
10-15	17	35 < $\frac{N}{2}$
15-20	12	47 > $\frac{N}{2}$
20-25	13	60
25-30	16	76
30-35	14	90

N = 90

$\frac{N}{2} = \frac{90}{2} = 45$

L_1 L_2

15-20

F = 12

CF = 35

median = $L_1 + \frac{\frac{N}{2} - CF}{F} \times (L_2 - L_1)$

$= 15 + \frac{45 - 35}{12} \times 5$

$= 15 + \frac{50}{12}$

$= 15 + \frac{25}{6}$

$= 19.16$

CI	F	CF
3.5 - 7.5	20	20
7.5 - 11.5	10	30 $< \frac{N}{2}$
11.5 - 15.5	18	48 $> \frac{N}{2}$
15.5 - 19.5	15	63
~~19.5~~ 17.5 - 23.5	14	77
23.5 - 27.5	16	93

$N = 93$

$\frac{N}{2} = \frac{46.5}{93}$

l_1 l_2

11.5 - 15.5

$F = 18$

$CF = 30$

$$\text{Median} = l_1 + \frac{\frac{N}{2} - CF}{F} \times (l_2 - l_1)$$

$$= 11.5 + \frac{46.5 - 30}{18} \times (15.5 - 11.5)$$

$$= 11.5 + \frac{16.5}{4.5} \times 4$$

$$= 11.5 + 3.6$$

$$= 15.17$$

Quartiles :- dividing data in 4 equal parts.

0 — 25% of N — 50% of N — 75% of N — 100

$\frac{25}{100}$ of N, $\frac{50}{100}$ of N, $\frac{75}{100}$ of N

$\frac{1}{4}(N)$ → lower quartile Q_1

$\frac{N}{2}$ → central / median Q_2

$\frac{3N}{4}$ → Upper quartile Q_3

formula :-

$$Q_1 = l_1 + \frac{\frac{N}{4} - CF}{F} \times (l_2 - l_1)$$

$$Q_3 = l_1 + \frac{\frac{3N}{4} - CF}{F} \times (l_2 - l_1) = 7.5 + \frac{69.75 - 20}{10} \times 4 = 17.5$$

Ex :- $Q_1 = \frac{N}{4} = \frac{93}{4} = 23.25$

$$Q_1 = 7.5 + \frac{23.25 - 20}{10} \times (11.5 - 7.5)$$

$$= 7.5 + \frac{3.25 \times 4}{10}$$

$$= 7.5 + \frac{13}{10}$$

$$= 7.5 + 1.3$$

$$= 8.8$$

CI	F	CF	(U.L, CF) X, Y
18-22	8	8	22, 8
22-26	28	36 $\leq (\frac{N}{4})$	26, 36
26-30	16	52 $> (\frac{N}{4})$	30, 52
30-34	19	71 $< (\frac{N}{2})$	34, 71
34-38	36	107 $> (\frac{3N}{4})$	38, 107
38-42	45	152 $\leq (\frac{3N}{4})$	42, 152
42-46	38	190 $\leq (N)$	46, 190

$N = 190$ $\quad \frac{N}{4} = 47.5$, $\quad \frac{N}{2} = 95$, $\quad \frac{3N}{4} = 142.5$

K^{th} quartile $= l_1 + \frac{\frac{KN}{4} - CF}{F} \times (l_2 - l_1)$

$Q_1 = 26 + \frac{47.5 - 36}{16} \times 4$

$= 26 + \frac{11.5}{16} \times 4$

$= 26 + 2.875$

$= 28.875$

$Q_2 = 34 + \frac{95 - 71}{36} \times 4$

$= 34 + \frac{24}{36} \times 4$

$= 34 + 2.67$

$= 36.67$

~~$Q_3 = 142.5 - 1$~~

$Q_3 = 38 + \frac{142.5 - 107}{45} \times 4$

$= 38 + \frac{35.5}{45} \times 4$

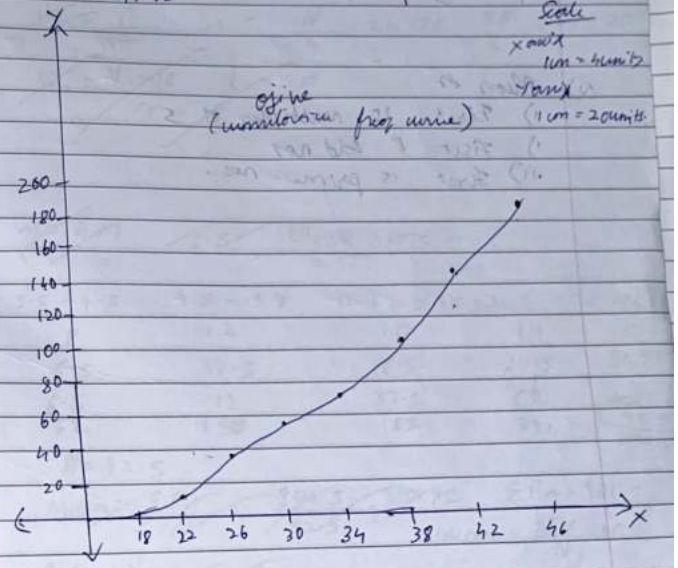
$= 38 + 3.16$

~~$= 38 + 35.5$~~

~~$= 73.5$~~

$= 41.16$

Median can be found graphically by the ojive or cumulative freq curve.
(All "iles)



Find mean & median of

1)

X	6	7	10	12	15	18
f	18	12	16	14	25	14
fx	108	84	160	168	375	252
CF	18	30	46	60	85	99

$\Sigma fx = 1147$

2)

CI	4-9	10-15	16-21	22-27	28-33
f	8	12	10	16	5

3)

CI	0-8	8-16	16-24	24-32	32-40
f	5	4	3	8	5

Q) Mean of
i) First 10 multiples of 5
ii) First 8 odd nos
iii) First 15 prime nos.

1)

$\Sigma fx = 1147$

$\Sigma f = 99$

$\bar{x} = \frac{1147}{99} = 11.58$

median $= \left(\frac{n+1}{2}\right)^{th}$ obs

$= \frac{99+1}{2} = \frac{100}{2} = 50$

median $= 12$

$\frac{n}{2} = 49.5 \quad \frac{N}{4} = 24.75, \quad \frac{3N}{4} = 74.25$

2)

CI	3.5-9.5	9.5-15.5	15.5-21.5	21.5-27.5	27.5-33.5
F	8	12	10	16	5
X	6.5	12.5	18.5	24.5	30.5
CF	6.5	19	37.5	62	92.5
FX	52	150	185	392	152.5

$N = 92.5$

$\Sigma FX = 931.5$

$\text{mean} = \frac{\Sigma FX}{N} = 10$

$\text{median} = 21 + \frac{46.25 - 37.5}{16.5} \times (27.5 - 21.5) = 24.78$

Q1 →

	0-8	8-16	16-24	24-32	32-40
F	5	4	3	8	5
x	4	12	20	28	36
Fx	20	48	60	224	180 = 532
CF	5	9	12	20	25
				mid	N

$$\text{Mean} = \frac{\Sigma Fx}{N} = \frac{532}{25} = 21.28$$

$$\frac{N}{2} = 12.5, \quad \frac{N}{4} = 6.25 \quad \frac{3N}{4} = 18.75$$

$$\text{Median} = 24 + \frac{12.5 - 12}{8} \times 8$$

$$= 24 + 0.5$$

$$= 24.5$$

$$Q_1 = 8 + \frac{6.25 - 5}{4} \times 8$$

$$= 8 + 1.25 \times 2$$

$$= 8 + 3$$

$$= 11$$

$$Q_3 = 24 + \frac{18.75 - 12}{8} \times 8$$

$$= 24 + 6.75$$

$$= 30.75$$

K^{th} Quartile : $l_1 + \frac{\frac{KN}{4} - CF}{F} \times (l_2 - l_1)$

K^{th} decile (1 to 9) : $l_1 + \frac{\frac{KN}{10} - CF}{F} \times (l_2 - l_1)$

K^{th} percentile (1 to 99) : $l_1 + \frac{\frac{KN}{100} - CF}{F} \times (l_2 - l_1)$

CE	F	CF
4-8	16	16
8-12	26	42
12-16	38	80 <
16-20	30	110 < 130
20-24	42	152 >
24-28	24	176 <
28-32	34	210 >
32-36	50	260

$N = 260$

$\frac{75N}{100} = 195$

$\frac{50N}{100} = 130$

$\frac{4N}{10} = 104$ $\frac{7N}{10} = 182$

$$D_4 = 16 + \frac{104 - 80}{30} \times (20 - 16)$$

$$= 16 + 3.2$$

$$= 19.2$$

$$D_7 = 28 + \frac{182 - 176}{34} \times 4$$

$D_7 = 28.7$

$$P_{50} = 20 + \frac{130 - 110}{13} \times 4$$

$$= 20 + \frac{20}{13} = 20 + 1.53$$

$$= 21.53$$

$$P_{75} = 28 + \frac{195 - 176}{34} \times 4$$

$$= 28 + \frac{19}{34} \times 4$$

$$= 28 + 2.24$$

$$= 30.24$$

Mode

The obv. with the highest freq. is called Mode. When all the obs have equal modal freq no mode no.

Uni-Modal - single modal.
bi. Modal - 2 modal
Multi-Modal - 3 or > modal.

x	10	12	16	18	24
f	18	27	20	26	17

Mode = 12
(B'cos freq is max)

The value of x corresp to the highest freq is called as Mode.

Grouped data:-

CI	F
0-5	10
5-10	18
10-15	12 → F_0
15-20	20 → F_1 (highest freq)
20-25	13 → F_2
25-30	18

$$\text{Mode} = l_1 + \frac{F_1 - F_0}{(F_1 - F_0) + (F_1 - F_2)} \times l_2 - l_1$$

$$= 15 + \frac{20 - 12}{(20-12)+(20-13)} \times (20 - 15)$$

$$= 15 + \frac{8}{8+7} \times 5$$

$$= 15 + \frac{8}{15} \times 5$$

$$= 15 + 2.67$$

$$= 17.67$$

CI	F
0-5	10
5-10	18 → F_0
10-15	22 → F_1
15-20	16 → F_2
20-25	13
25-30	18

$$\text{Mode} = 10 + \frac{22 - 18}{(22-18)+(22-16)} \times 5$$

$$= 10 + \frac{4}{4+6} \times 5$$

$$= 10 + \frac{4^2 \times 5^1}{10_2}$$

$$= 12$$

If the mode of the given data is 12

$Z = 12$

since the value of mode = 12 lies in the range of interval 10 - 15, hence it is the modal CI.

CI	F
0 - 5	16
5 - 10	$18 - F_0$
10 - 15	$a - F_1$
15 - 20	$16 - F_2$
20 - 25	19
25 - 30	21

$$Z = l_1 + \frac{(F_1 - F_0)}{(F_1 - F_0) + (F_1 - F_2)} \times l_2 - l_1$$

$$12 = 10 + \frac{(a - 18)}{(a-18) + (a-16)} \times 5$$

$$2 = \frac{(a-18)}{(a-18)+(a-16)} \times 5$$

$$\frac{a-18}{(a-18)+(a-16)} = \frac{2}{5}$$

$$\frac{a-18}{a-18+a-16} = \frac{2}{5}$$

$$\frac{a-18}{2a-34} = \frac{2}{5}$$

$$5(a-18) = 2(2a-34)$$

$$5a - 90 = 4a - 68$$

$$5a - 4a = 90 - 68$$

$$a = 22$$

If mode = 22, find a.

CI	F
10-15	21
15-20	$a - F_0$
20-25	$22 \rightarrow F_1$
25-30	$16 - F_2$
30-35	20
35-40	19

$$Z = l_1 + \frac{(F_1 - F_0)}{(F_1 - F_0) + (F_1 - F_2)} \times l_2 - l_1$$

$$22 = 20 + \frac{(22-a)}{(22-a)+(22-16)} \times 5$$

$$2 = \frac{(22-a)}{(22-a)+6} \times 5$$

$$\frac{2}{5} = \frac{22-a}{22-a+6}$$

$$\frac{2}{5} = \frac{22-a}{28-a}$$

$$2(28-a) = 5(22-a)$$

$$56 - 2a = 110 - 5a$$

$$5a - 2a = 110 - 56$$

$$3a = 54$$

$$a = 18$$

CI	F
0-10	$f_0 \rightarrow F_0$
10-20	$28 \rightarrow F_1$
20-30	$18 \rightarrow F_2$
30-40	26
40-50	27
50-60	25

$$L_1 + \frac{28-0}{(28-0)+(28+18)} \times 10$$

$$10 + \frac{28}{28+46} \times 10$$

$$10 + \frac{280}{74}$$

$$10 + \frac{28}{28+(28-18)} \times 10$$

$$10 + \frac{28}{28+10} \times 10$$

$$10 + \frac{280}{38}$$

$$10 + 7.36$$

$$17.36$$

Measures of dispersions:

1) Range & coeff.
2) Quartile deviations & coeff.
3) Standard deviation & variance.
4) Combined std. deviation.
5) App. based sums.
6) Mean deviation about
   i) Mean
   ii) Median
   iii) Mode.
   & its deviation.

1) Range = H - L or max val - Min. val.

coeff = $\left(\frac{H-L}{H+L}\right) \times 100$ if < 1 = not x ed by 100.

Range = 37

coeff $\frac{40-3}{43}$ $\frac{37}{43} = 0.86$

2) $Q.d = \frac{Q_3 - Q_1}{2}$

coeff of $Q.d = \frac{Q_3 - Q_1}{Q_3 + Q_1}$

Find Q.d & its Coeff.

CI	F	CF
0-5	10	10
5-10	8	18 <
10-15	18	36 >
15-20	32	68 <
20-25	20	88 <
25-30	26	114

$N = 114$ $\quad \frac{3N}{4} = 85.5$ $\quad \frac{N}{4} = 28.5$

$Q_3 = 20 + \frac{85.5 - 68}{20} \times 5$

$= 20 + \frac{17.5 \times 5}{20}$

$= 20 + \frac{87.5}{20}$

$= 20 + 4.38$

$= 24.38$

$Q_1 = 10 + \frac{28.5 - 18}{18} \times 5$

$= 10 + \frac{10.5}{18} \times 5$

$= 10 + 2.91$

$= 12.91$

$Q.d = \frac{Q_3 - Q_1}{2} = \frac{24.38 - 12.91}{2} = 5.74$

Coeff:- $\frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{11.47}{37.29} = 0.3$

Find Quartile deviation & its coeff.

18, 12, 16, 26, 20, 30, 28, 32, 25

12, 16, 18, 20, 25, 26, 28, 30, 32

$Q_1 = \frac{(N+1)^{th}}{4}$

$= \frac{9+1}{4}$

$= \frac{10}{4}$

$= 2.5^{th}$

2^{nd} term + 0.5 (3^{rd} term - 2^{nd} term)

12 + 0.5 (16 - 12)

12 + 0.5 (4)

16 + 0.5 (18 - 16)

16 + 0.5 (2)

16 + 1

$Q_1 = 17$

$Q_3 = 3\left(\frac{N+1}{4}\right)^{th}$

$= 3\left(\frac{9+1}{4}\right)^{th}$

$= 3\left(\frac{10}{4}\right)^{th}$

$= 3(2.5)$

$= 7.5$

$Q_3 = 28 + 0.5(30 - 28)$

$= 28 + 0.5(2)$

$= 28 + 1$

$= 29$

Q1) 1) 18, 26, 20, 24, 15, 26, 38, 35, 15, 24, 32

3) 45, 30, 36, 48, 28, 31, 37, 18, 27, 36

1) 15, 17, 18, 20, 24, 24, 26, 26, 32, 35, 38.

$N = 11$

$Q_1 = \left(\frac{N+1}{4}\right)^{th}$ obv

$= \left(\frac{11+1}{4}\right)^{th}$

$= 3^{rd}$ obv

$Q_1 = 18$

$Q_3 = 9^{th}$ obv

$Q_3 = 32$

$Q.D = \frac{Q_3 - Q_1}{2} = \frac{32-18}{2} = \frac{14}{2} = 7$

coeff: $\frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{32-18}{32+18} = \frac{7}{50} = 0.14$

$= \frac{14}{50} = 0.28.$

2) 18, 27, 28, 30, 36, 36, 37, 39, 45, 48.

$N = 10$ (even)

$Q_1 = \left(\frac{N}{4}\right)^{th}$ obv

$= \frac{10}{4} = 2.5^{th}$ obv

$\therefore Q_1 = 2^{nd}$ obv $+ 0.5(3^{rd}$ obv $- 2^{nd}$ obv$)$

$= 27 + 0.5(28-27)$

$= 27 + 0.5$

$Q_1 = 27.5$

$Q_3 = 3\left(\frac{N}{4}\right)^{th}$ obv

$= 3 \times 10.5$

$= 7.5$ obv

$= 38.$

$Q.D = \frac{38 - 27.5}{2} \left(\frac{Q_3 - Q_1}{2}\right)$

$= \frac{10.5}{2}$

$= 5.25$

coeff $= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{38 - 27.5}{38 + 27.5}$

$= \frac{10.5}{65.5} = 0.16$

$Q_d = 7$

$Q_1 = 18$

$Q_3 = x$

$Q_d = \frac{Q_3 - Q_1}{2}$

$7 = \frac{x - 18}{2}$

$14 = x - 18$

$x = 18 + 14$

$x = 32$

---

coeff $= 0.28$

$Q_3 = 32$

$Q_1 = x$

$\text{coeff} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$

$0.28 = \frac{32 - x}{32 + x}$

$0.28(32 + x) = 32 - x$

$8.96 + 0.28x = 32 - x$

$0.28x + x = 32 - 8.96$

$1.28x = 23.04$

$x = 18$

Standard deviation

Raw data $= \sqrt{\frac{\Sigma x^2}{N} - (\bar{x})^2}$ $\quad \bar{x}$ = mean

Find SD of

5, 8, 7, 12, 16, 14, 18, 20 $\quad N = 8$

x^2 = 25, 64, 49, 144, 256, 196, 324, 400

$\Sigma x = 100 \quad \Sigma x^2 = 1458$

$\bar{x} = \frac{\Sigma x}{N}$

$= \frac{100}{8}$

$\bar{x} = 12.5$

$SD = \sqrt{\frac{1458}{8} - (12.5)^2}$

$= \sqrt{182.25 - 156.25}$

$= \sqrt{26}$

$= 5.09$

Mean deviation from mean.

Raw

$M.D = \frac{\Sigma |x - \bar{x}|}{N}$ where $\bar{x}$ = mean.

Coeff on mean = $\frac{M.D}{mean}$

Tabulated Data

(3) Mean (2) deviation about mean / (1) median

x	f	fx	$\|x - \bar{x}\|$	$f\|x - \bar{x}\|$	CF	
5	6	30	5.24	31.44	6	
8	4	32	2.24	8.96	10	≤
10	5	50	0.24	1.2	15	>
13	3	39	2.76	8.28	18	
15	7	105	4.76	33.32	25	
S	$\Sigma F = 25$	$\Sigma Fx = 256$	16.2	$\Sigma f(x - \bar{x}) = 83.2$		

$\bar{x} = \frac{\Sigma Fx}{\Sigma F} = \frac{256}{25} = 10.24$

$M.D = \frac{\Sigma |x - \bar{x}|}{N} = \frac{16.2}{5} = 3.24$

$M.D_{mean} = \frac{\Sigma f(x - \bar{x})}{\Sigma F}$

$= \frac{83.2}{25}$

$= 3.328$

$Coeff = \frac{M.D}{\bar{x}} = \frac{3.24}{10.24} = 0.316$

$Coeff = \frac{M.D_{mean}}{mean}$

$= \frac{3.328}{} = 0.32$

$\Sigma 204$ 101 105

median = 10

$M.D_{mean} = \Sigma F($

x	F	CF	$\|x - \text{Median}\|$	$f\|x - med\|$
12	6	6	11	66
18	18	24	5	90
20	12	36 <	3	36
23	9	45 >	0	0
25	7	52	2	14
28	12	64	5	60
34	15	79	11	165
	N = 79			$\Sigma F\|x - med\| = 431$

$\frac{N}{2} = \frac{79}{2} = 39.5$

$M.D_{median} = \frac{\Sigma F(x - med)}{\Sigma F}$

$= \frac{431}{79}$

$= 5.45$

$Coeff = \frac{M.D_{med}}{Med.}$

$= \frac{5.45}{431}$

$= 0.012$

Mean deviation about Median.

CI	f	CF	x	$\lvert x-med \rvert$	$F\lvert x-med \rvert$
0-4	8	8	2	11	88
4-8	18	26	6	7	126
8-12	14	40 <	10	3	42
12-16	20	60 >	14	1	20
16-20	18	78	18	5	90
20-24	12	90	22	9	108
					$\Sigma F\lvert x-med \rvert$ = 474

$\frac{N}{2} = \frac{90}{2} = 45$

$Median = l_1 + \left(\frac{\frac{N}{2} - CF}{F}\right) \times l_2 - l_1$

$= 12 + \left(\frac{45-40}{20}\right) \times 4$

$= 12 + \left(\frac{5}{20}\right) \times 4$

$= 12 + 1$

$= 13$

$M.D_{median} = \frac{\Sigma F\lvert x-med \rvert}{\Sigma F} = \frac{474}{90} = 5.27$

$Coeff\ of\ M.D_{med} = \frac{M.D_{med}}{Median}$

$= \frac{5.27}{13}$

$= 0.405$

CI	f	CF	x	$\lvert x-med \rvert$	$f.\lvert x-med \rvert$
10-15	12	12	12.5	13.93	167.16
15-20	18	30	17.5	8.93	160.74
20-25	16	46 <	22.5	3.93	62.88
25-30	14	60 >	27.5	1.07	14.98
30-35	10	70	32.5	6.07	60.7
35-40	30	100	37.5	11.07	332.1
	N=100				$\Sigma F.\lvert x-med \rvert = 798.56$

$\frac{N}{2} = \frac{100}{2} = 50$

$Med = l_1 + \left(\frac{\frac{N}{2} - CF}{F}\right) \times l_2 - l_1$

$= 25 + \left(\frac{50-46}{14}\right) \times 5$

$= 25 + \left(\frac{4 \times 5}{14}\right)$

$= 25 + \frac{20}{14}$

$= \frac{350+20}{14}$

$= \frac{370}{14}$

$= 26.43$

$M.D_{med} = \frac{\Sigma F\lvert x-med \rvert}{\Sigma F}$

$= \frac{798.56}{100}$

$= 7.9856$

$Coeff = \frac{M.D_{med}}{Med}$

$= \frac{7.9856}{26.43}$

$= 0.302$

Mean dev About Mode

12, 18, 16, 12, 19, 12, 10, 17, 12, 18

Mode = 12

X	|X - mode|
12	0
18	6
16	4
12	0
19	7
12	
10	
17	
12	
18	

$\Sigma |x - mode| = 30$

$N = 10$

$M.D_{mode} = \frac{\Sigma |x - mode|}{N}$

$= \frac{30}{10}$

$= 3$

$Coeff = \frac{MD_{mode}}{Mode} = \frac{3}{12} = 0.25$

Find

Mean, Median, mode, S.D, Q_1, Q_3, D_7, P_{60}, P_{40}.
MD mean, MD med, M D mode

C.I	F
0-4	4
4-8	6
8-12	5
12-16	3
16-20	7
20-24	9
24-28	8

CI	F
2-6	5
6-10	8
10-14	7
14-18	9
18-22	11
22-26	12
26-30	18

Com. obs:- $\sqrt{\frac{N_1(\sigma_1^2 + d_1^2) + N_2(\sigma_2^2 + d_2^2)}{N_1 + N_2}}$

The mean & S.D of 100 obs. was 40 & 5.1 respectively by a student who took by mistake 50 instead of 40 by for 1 obs. find corrected mean & S.D.

$\bar{X} = 40$

S.D = 5.1

$\bar{X} = \frac{\Sigma x}{N}$

Wrong

$\Sigma x = N \cdot \bar{X}$

$= 40 \cdot 100$

$= 4000$

Correct Σx = Incorrect Σx − wrong obs + correct obs

$= 4000 - 50 + 40$

$= 3990$

correct mean = $\frac{\text{Correct } \Sigma x}{N}$

$= \frac{3990}{100}$

$= 39.9$

short cut

correct − wrong

$-50 + 40 = -0.1$

− correct + incorrect = the diff

N = 50

$\bar{X} = 60$

wrong obs = 55

correct obs = 75

$SD = \sqrt{\frac{\Sigma x^2}{N} - (\bar{x})^2}$

$(SD)^2 = \frac{\Sigma x^2}{N} - (\bar{x})^2$

$(5.1)^2 = \frac{\Sigma x^2}{100} - (40)^2$

$26.01 + 1600 = \frac{\Sigma x^2}{100}$

$1626.01 \times 100 = \Sigma x^2$

Incorrect $\Sigma x^2 = 162601$

correct Σx^2 = Incorrect $\Sigma x^2 - (\text{wrong obs})^2 + (\text{correct obs})^2$

$= 162601 - (50)^2 + (40)^2$

$= 161701$

correct SD = $\sqrt{\frac{161701}{100} - (39.90)^2}$

$= \sqrt{1617.01 - 1592.01}$

$= \sqrt{25}$

$= 5$

Avg marks of 50 students in the class is 64 while their S.D is 6.6. If student having marks 65 left the class then find the change in S.D.

$\bar{X} = 64$

$S.D = 6.6$

Wrong $\bar{X} = 64$

$$\bar{X} = \frac{\Sigma X}{N}$$

$$\Sigma X = \bar{X} \cdot N$$

$$\Sigma X = 64 \times 50$$

$$\Sigma X = 3200$$

Correct ΣX = Incorrect ΣX

New $\Sigma X = 3200 - 65$

$= 3135$

New no of students $= 50 - 1$

$= 49$

New $\bar{X} = \frac{3135}{49}$

$= 63.98$

$$S.D = \sqrt{\frac{\Sigma x^2}{N} - (\bar{x})^2}$$

$$(S.D)^2 = \frac{\Sigma x^2}{N} - (\bar{x})^2$$

$$(6.6)^2 = \frac{\Sigma x^2}{50} - (64)^2$$

$$(6.6)^2 = \frac{\Sigma x^2}{50} - 4096$$

$$21.16 + 4096 = \frac{\Sigma x^2}{50}$$

$$4117.16 \times 50 = \Sigma x^2$$

$\Sigma x = 453.7$

Incorrect $\Sigma x^2 = 205858$

Correct Σx^2 = incorrect Σx^2 − (original D)^2

$= 205858 - (65)^2$

$= 205858 - 4225$

$= 201633$

Correct $\Sigma x = 449.03$

Correct $SD = \frac{\Sigma x^2}{49} - (63.98)^2$

$= \frac{201633}{49} - 4093.4$

$= 4114.95 - 4093.4$

$= 21.55$

$1023x + 105y = 521$ — (i)

$1029x + 1034y = 519$ — (ii)

$-1105x + 1034y = 2$

$-2x + 2y = 1$ — (iv)

$10609 + 10815 = 53603$

add (i) & (iii)

$2088x + 2084 = 1040$

÷ by 208

$\therefore x + y = 5$ — (iii)

$\therefore x + y = 5$

$-x + y = 1$

$\therefore 2y = 6$

$y = 3$

$\therefore x + 3 = 5$

$\therefore x = 2$

$\frac{10}{x+y} - \frac{1}{x-y} = 3$

$\frac{25}{x+y} + \frac{10}{x-y} = -5$

let $(x+y) = a$ $(x-y) = b$

$\therefore \frac{10}{a} - \frac{1}{b} = 3$

$\therefore \frac{25}{a} + \frac{10}{b} = -5$

$\therefore \frac{10b - a}{ab} = 3$

$\therefore 10b - a = 3ab$

$\therefore -a - 3ab + 10b = 0$

$\therefore \frac{25b + 10a}{ab} = -5$

$\therefore 25b + 10a = -5ab$

$\therefore 10a + 5ab + 25b = 0$

÷ by 5

$2a + 1ab + 5b = 0$

$\therefore -a + 3ab + 10b = 0$

$2a + ab + 5b = 0$

$-2a - 6ab + 20b = 0$

$-5ab + 25b = 0$

$-ab + 5b = 0$

$\therefore b = \frac{ab}{5}$

let $\frac{1}{x+y} = a$, $\frac{1}{x-y} = b$

$\therefore 10a - b = 3$

$25a + 10b = -5$

$\therefore 100a - 10b = 30$

$\therefore -5a + 10b = -5$

$125a = 25$

$a = \frac{25}{125} = \frac{1}{5}$ $\boxed{a = \frac{1}{5}}$

$\therefore \frac{10}{5} - b = 3$

$2 - b = 3$ $\therefore -b = 1$

$\boxed{b = -1}$

$\therefore \frac{1}{x+y} = \frac{1}{5}$ $\therefore x + y = 5$

$\frac{1}{x-y} = -1$ $\therefore -x + y = 1$

$2y = 6$

$\boxed{y = 3}$

$x + 3 = 5$

$\boxed{x = 2}$

sub in (iv)

$-a - 3a\left(\frac{ab}{5}\right) + 10b = 0$

$-a - \frac{3a^2b}{5} + 10b = 0$

The score of 2 batsmen A & B in 10 innings of a match during a certain tournament are as under.

	1	2	3	4	5	6	7	8	9	10
A	32	28	47	63	71	39	10	60	96	14
B	19	31	48	53	67	90	10	62	40	80

Identify which batsman is more consistent & reliable.

A

x	x^2
32	1024
28	784
47	2209
63	3969
71	5041
39	1521
10	100
60	3600
96	9216
14	196
$\Sigma x = 460$	$\Sigma x^2 = 27660$

B

x	x^2
19	361
31	961
48	2304
53	2809
67	4489
90	8100
10	100
62	3844
40	1600
80	6400
$\Sigma x = 500$	30968

$$\bar{x} = \frac{\Sigma x}{N}$$

$$= \frac{460}{10}$$

$$\bar{x} = 46$$

$$S.D = \sqrt{\frac{\Sigma x^2}{N} - (\bar{x})^2}$$

$$= \sqrt{\frac{27660}{10} - 2116}$$

$$= \sqrt{2766 - 2116}$$

$$= \sqrt{650}$$

$$S.D = 25.495$$

$$C.V = \frac{S.D}{\text{mean}}$$

$$= \frac{25.495}{46}$$

$$C.V = 0.554$$

A

$$\bar{x}_b = \frac{500}{10}$$

$$\bar{x}_b = 50$$

$$S.D_b = \sqrt{\frac{\Sigma x^2}{N} - (\bar{x})^2}$$

$$= \sqrt{\frac{30968}{10} - 2500}$$

$$= \sqrt{3096.8 - 2500}$$

$$= \sqrt{596.8}$$

$$= 24.429$$

$$\therefore CV_b = \frac{S.D_b}{\bar{x}_b}$$

$$= \frac{24.429}{50}$$

$$CV_b = 0.488$$

$\therefore CV_A > CV_b$

$\therefore C.V_A$ is the more reliable player.

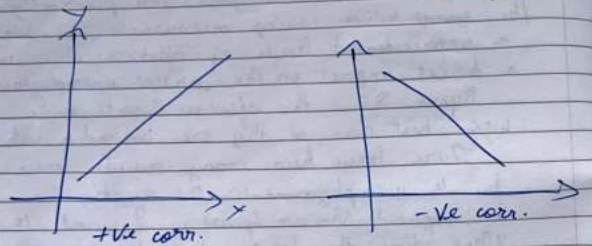
Bivariate linear correlation Analysis

Scatter diagram Method

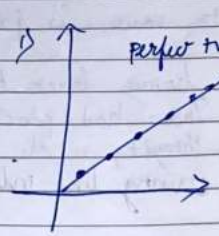
-> Qualitative Analysis

-> Quantitative Analysis.

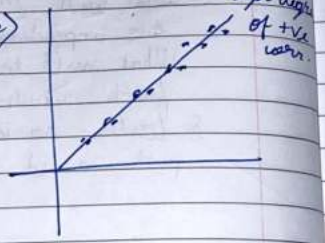
- Karl-pearson's coeff of correlation.
- Spearman's Rank correlation
  - (i) repeated ranks
  - (ii) Non repeated ranks.



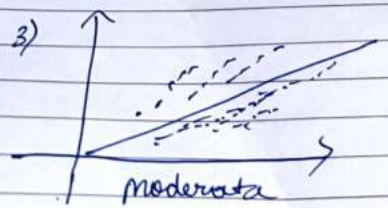
1) perfect +ve correlation.

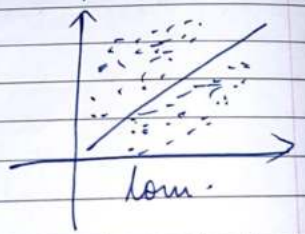


2) high degree of +ve corr.

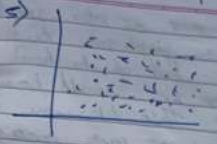


3)





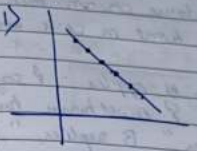
5)



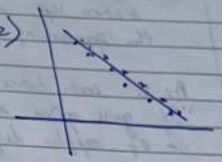
No correlation.

-Ve correlation

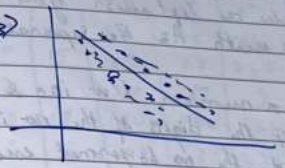
1)



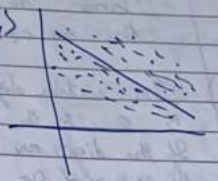
2)



3)



4)



... numerator becomes ... Then determine the prob.

Karl - Pearson's coeff. of correlation.

$$R = \frac{N\Sigma xy - \Sigma x \cdot \Sigma y}{\sqrt{N\Sigma x^2 - (\Sigma x)^2}\sqrt{N\Sigma y^2 - (\Sigma y)^2}}$$

$N = 8$

x	y	xy	x^2	y^2
5	8	40	25	64
7	15	105	49	225
9	3	27	81	9
10	18	180	100	324
6	15	90	36	225
8	5	40	64	25
7	7	49	49	49
14	16	224	196	256
66	87	755	600	1177

$$R = \frac{N\Sigma(xy) - \Sigma x \cdot \Sigma y}{\sqrt{N\Sigma x^2 - (\Sigma x)^2}\sqrt{N\Sigma y^2 - (\Sigma y)^2}}$$

$$= \frac{8 \times 755 - (66 \times 87)}{\sqrt{8 \times 600 - (66)^2}\sqrt{8(1177) - (87)^2}}$$

$$= \frac{6040 - 5742}{\sqrt{4800 - 4356}\sqrt{9416 - 7569}}$$

$$= \frac{298}{\sqrt{444}\sqrt{1847}}$$

$$= \frac{298}{21.07 \times 42.98}$$

$$= \frac{298}{905.50}$$

$$= 0.329$$

Spearman's Rank Correlation

Type I :- Ranks are not repeated i.e in a column x or y all the values are distinct or unique.

Eg :- Assign Rank 1 to the smallest or the largest value for both the col^n individually. If we are assigning Rank 1 to the smallest value for Variable x then rank 1 has to be assigned to the smallest value for variable y as well. Similar case with highest.

A/c (x) Sco (y)

Alk Sco

x	y	R_x	R_y	$D = R_x - R_y$	D^2
15	20	5	1	4	16
18	10	2	8	-6	36
16	17	4	3	1	1
20	15	1	4	-3	9
14	11	6	7	-1	1
13	14	7	2	5	25
10	8	10	9	1	1
17	14	3	5	-2	4
12	13	8	6	2	4
11	6	9	10	-1	1
					$\Sigma D^2 = 98$

$$R_s = 1 - \frac{6 \Sigma D^2}{N(N^2-1)}$$

$$R_s = 1 - \frac{6 \times 98}{10(10^2-1)} = 1 - \frac{588}{10 \times 99}$$

$$= 1 - \frac{588}{990}$$

$$= 1 - 0.5939$$

$$= 0.406$$

x	y	R_x	R_y	$D = R_x - R_y$	D^2
75	38	4	8	-4	16
38	78	9	2	7	49
95	40	1	7	-6	36
64	96	5	1	4	16
76	37	3	9	-6	36
55	74	7	3	4	16
90	69	2	5	-3	9
37	70	10	4	6	36
48	35	8	10	-2	4
56	65	6	6	0	0
					$\Sigma D^2 = 218$

$$R_s = 1 - \frac{6 \Sigma D^2}{N(N^2-1)}$$

$$= 1 - \frac{6 \times 218}{10(100-1)}$$

$$= 1 - \frac{1308}{990}$$

$$= 1 - 1.3212$$

$$R_s = -0.3212$$

Range

$$-1 \leq R \leq +1$$

X	Y	R_x	R_y	$D = R_x - R_y$	D^2
40	15	3	6	-3	9
38	10	4	8	-4	16
30	18	8	5	3	9
32	24	7	3	4	16
45	19	2	4	-2	4
36	30	5	1	4	16
47	27	1	2	-1	1
34	13	6	7	-1	1
					$\Sigma D^2 = 72$

$$R_s = 1 - \frac{6\Sigma D^2}{N(N^2-1)}$$

$$= 1 - \frac{6 \times 72}{8(64-1)}$$

$$= 1 - \frac{6 \times 72}{8(63)}$$

$$= 1 - \frac{432}{504}$$

$$= 1 - 0.8571$$

$$R_s = 0.1428$$

Type 2

Repeated Rank

x	y	R_x	R_y	D	D^2
(13)	28	7.5	2	5.5	30.25
10	20	10	8	2	4
16	(27)	6	3.5	2.5	6.25
18	26	4	5	-1	1
(15)	24	7.5	6	1.5	2.25
20	(27)	2	3.5	-1.5	2.25
17	30	5	1	4	16
19	15	3	10	-7	49
28	19	1	9	-8	64
14	21	9	7	2	4
					$\Sigma D^2 = 179$

Correction factor $= \frac{M_1(M_1^2-1) + M_2(M_2^2-1)}{12}$

$M_1 = 2$ (repeated twice)

$M_2 = 2$ (repeated twice)

$$= \frac{2(2^2-1) + 2(2^2-1)}{12}$$

$$= \frac{2(3) + 2(3)}{12}$$

$$= \frac{6+6}{12} = \frac{12}{12}$$

$$= 1$$

$$R_s = \frac{1 - 6\{\Sigma D^2 + CF\}}{N(N^2-1)} = \frac{1 - 6\{179 + 1\}}{10(10^2-1)}$$

$$= \frac{1 - 6(180)}{990}$$

$\frac{6+7+2}{5}$

$= 1 - \frac{1080}{990}$

$= 1 - 1.0909$

$= -0.09$

X	Y	R_x	R_y	D	D^2
36	29	1	9	-8	64
24	32	5	6.5	-1.5	2.25
(18)	46	7	2	5	25
27	39	4	3	1	1
(18)	37	7	5	2	4
35	(33)	2	6.5	-4.5	20.25
29	49	3	1	2	4
10	31	~~10~~	~~8~~	2	4
(18)	25	7	10	-3	9
14	38	9	4	5	25

$\Sigma D^2 = 158.5$

~~$R_s = 1 - R$~~

Correction factor $= \frac{m_1(m_1^2-1) + m_2(m_2^2-1)}{12}$

$= \frac{3(3^2-1) + 2(2^2-1)}{12}$

$= \frac{3(9-1) + 2(4-1)}{12}$

$= \frac{3(8) + 2(3)}{12}$

$= \frac{24+6}{12}$

$= \frac{30}{12}$

$= 2.5$

$R_s = 1 - \frac{6\{\Sigma D^2 + 2.5\}}{N(N^2-1)}$

$= 1 - \frac{6\{158.5 + 2.5\}}{10(100-1)}$

$= 1 - \frac{6\{161.7\}}{990}$

$= 1 - \frac{966}{990}$

$= 1 - 0.9757$

$R_s = 0.024$

X	Y	R_x	R_y	D	D^2
36	29	1	9	-8	64
24	32	6	7	-1	1
(14)	46	8.5	2	6.5	42.25
(27)	(39)	4.5	3.5	1	1
18	37	7	6	1	1
35	(39)	2	3.5	-1.5	2.25
29	49	3	1	2	4
10	31	10	8	2	4
(27)	25	4.5	10	-5.5	30.25
(14)	38	8.5	5	3.5	12.25

$\Sigma D^2 = 162$

Correction factor $= \frac{m_1(m_1^2-1) + m_2(m_2^2-1) + m_3(m_3^2-1)}{12}$

$= \frac{2(2^2-1) + 2(2^2-1) + 2(2^2-1)}{12}$

$= \frac{2(3) + 2(3) + 2(3)}{12}$

$= \frac{18}{12} = 1.5$

$R_s = 1 - \frac{6\{\Sigma D^2 + 1 + 5^2\}}{n(n^2-1)}$

$= 1 - \frac{6\{162 + 1 + 5^2\}}{10(10^2-1)}$

$= 1 - \frac{6\{163 - 5^2\}}{990}$

$= 1 - \frac{981}{990}$

$= 1 - 0.990$

$R_s = 0.009$

The coeff of ~~no~~ rank correlation of marks obtained by 10 students in Arithmetic & economics were found to be 0.5. It was later discovered those the differences in ranks in 2 subs obtained by 1 student was wrongly taken as 3 instead of 7. Then find the correct coeff of Rank corr.

$R_s = 1 - \frac{6\Sigma D^2}{10(10^2-1)}$

$= 1 - \frac{6\Sigma D^2}{990}$

$0.5 - 1 = -\frac{6\Sigma D^2}{990}$

$-0.5 \times 990 = -6\Sigma D^2$

$\frac{-495}{-6} = \Sigma D^2$

$82.5 = \Sigma D^2$

Correct $= 82.5 - (3)^2 + (7)^2$

$= 82.5 - 9 + 49$

$= 73.5 + 49$

Correct $\Sigma D^2 = 122.5$

$\therefore$ Correct $R_s = 1 - \frac{6\Sigma D^2}{n(n^2-1)}$

$= 1 - \frac{6(122.5)}{990}$

$= 1 - \frac{735}{990}$

$= 1 - 0.742$

$= 0.258$

The coeff of rank corr. b/w marks in 2 subs obtained by a group of students is 0.8. If the sum of square of the diff in ranks is 33 then find the no of students in the gp.

$R_s = 1 - \frac{6\Sigma D^2}{n(n^2-1)}$

$0.8 = 1 - \frac{6\Sigma D^2}{n(n^2-1)}$ $\quad 1 - \frac{6(33)}{n(n^2-1)}$

$0.8 = 1 - \frac{198}{n(n^2-1)}$

$-0.2 = -\frac{198}{n(n^2-1)}$

$n(n^2-1) = \frac{198}{0.2}$

$n(n^2-1) = 990$

$n(n^2-1^2) = 990$

$\therefore n(n-1)(n+1) = 990$

$[a^2 - b^2 = (a+b)(a-b)]$

$\therefore (n-1)(n)(n+1) = 990$

$\therefore$ taking $n = 10$

$(10-1)(10)(10+1) = 990$

$9 \times 10 \times 11$

$\therefore n = 10$

8 9 10 = 720

9 10 11 = 990

10 11 12 = 1320

In a Music competition, there were 10 singers who were being judged by 2 judges A & B. The Ranks assigned to these singers are as given

Participant	1	2	3	4	5	6	7	8	9	10
Judge A	18	16	19	15	17	14	15	12	10	20
Judge B	10	8	18	12	14	10	7	18	20	5

Determine if the 2 judges were consistent

Participant	Judge A	Judge B	RA	RB	D	D^2
1	18	(10)	3	6.5	-3.5	12
2	16	8	5	8	-3	
3	19	(18)	2	2.5	-0.5	0
4	(15)	12	6.5	5	1.5	2
5	17	14	4	4	0	
6	14	(10)	8	6.5	1.5	2
7	(15)	7	6.5	9	-2.5	6
8	12	(18)	9	2.5	6.5	42
9	10	20	10	1	9	
10	20	5	1	10	-9	

$\Sigma D^2 =$

Correction fact: $m_1 = 2$

$m_2 = 2$

$m_3 = 2$

$$= \frac{2(2^2-1) + 2(2^2-1) + 2(2^2-1)}{12}$$

$$= \frac{2(3) + 2(3) + 2(3)}{12} = \frac{18}{12} =$$

$$R.S = \frac{1 - 6(\Sigma D^2 + 1)}{n^2(n^2-1)}$$

$$= \frac{1 - 6(236.5 + 1)}{10(10^2-1)}$$

$$= \frac{1 - 6(238)}{990}$$

$$= \frac{1 - 1428}{999}$$

$$= 1 - 1.442$$

$$R.S = -0.442$$

No	Judge A	Judge B	RA	RB	D	D^2
1	10	8	1	3	-2	4
2	5	10	6	1	5	25
3	9	7	2	4	-2	4
4	8	6	3	5	-2	4
5	(6)	(5)	4.5	6.5	-2	4
6	4	1	7	6.5	0.5	0.25
7	(6)	3	4.5	10	-5.5	30.25
8	3	2	8	8	0	0
9	1	9	10	9	1	1
10	2		9	2	7	49
						$\Sigma D^2 = 121$

Correction $F = m_1 = 2$
$m_2 = 2$

$$CF = \frac{m_1(m_1^2-1) + m_2(m_2^2-1)}{12}$$

$$= \frac{2(2^2-1) + 2(2^2-1)}{12}$$

$$= \frac{6+6}{12}$$

$$CF = 1$$

$$RS = \frac{1 - 6(\Sigma D^2 + \frac{1}{12})}{990}$$

$$= 1 - \frac{6(121.5 + \frac{1}{12})}{990}$$

$$= 1 - \frac{6(122.5)}{990}$$

$$= 1 - \frac{735}{990}$$

$$= 1 - 0.742$$

$$R.S = 0.257$$

HCF × LCM = 162 × x

$\frac{1782 \times 18}{162} = x$; $x = 198$

1) HCF of 2 nos. is 18 & the LCM is 1782. If 1 no is 162 then find the other no.

2) Father's age is 6 years more than 3 times the age of his son. Then express the age of the son.

3) Comment on the nature of the roots of given eq.
$x^2 + 6x - 1 = 0$
$x^2 + 9x + 9 = 0$
$x^2 + 5x + 6 = 0$

4) Solve for x -
$$\frac{2x}{3} - \frac{3x}{5} = 8$$

5) $$\frac{x-2}{x-3} = \frac{x-1}{x+1}$$

6) Sum of the digits of a 2 digit no is 7. Then write an exp to write the no.

7) Length of the rectangle is 4x the width. If the perimeter is 80 m then find the length & width.

8) Find a no which is 5 less than its square by 30

9) The product of digits of a 2 digit no is 12. If 36 is added to the no the number is obtained which is same as the no obtained by reversing the digits of the original no. Then find the original no.

Ans 7) Let the width of □ be x

$\therefore$ Length $= 4x$

given, perimeter $= 80$ m

$\therefore$ perimeter $= 2(l+b)$

$\therefore 80 = 2(4x + x)$

$\frac{80}{2} = 5x$

$40 = 5x$

$\therefore x = 8$

$\therefore$ Length $= 4x$

$= 4 \times 8$

$= 32$

$\therefore$ Length is 32, width is 8.

Ans 1) Let the no be x

given, HCF $= 18$

LCM $= 1782$

$\therefore 162 \times x = 18 \times 1782$

$x = \frac{18 \times 1782}{162}$

$x = 198$

Ans 2) Let the son's age be y & father's age be x.

$\therefore$ the father's age $= 6+$

given

$\therefore x = 6 + 3y$

$\therefore x - 6 = 3y$

$\therefore y = \frac{x-6}{3}$

$\therefore$ Son's age $= \frac{x-6}{3}$

Ans 3) $b^2 - 4ac < 0$ Roots are imaginary

$= 0$ Root " real & equal

> 0 " " " & unequal.

$x^2 + 4x + 4 = 0$

$a = 1, b = 4, c = 4$

$b^2 - 4ac$

$4^2 - 4(1 \times 4)$

$16 - 16 = 0$

$\rightarrow$ real & equal.

$x^2 - 5x + 9 = 0$

$a = 1, b = -5, c = 9$

$(-5)^2 - 4(1 \times 9)$

$25 - 36$

$= -11$

$\rightarrow$ roots are imaginary

Ans 4 $\frac{2x}{3} - \frac{3x}{5} = 8$

$\frac{10x - 9x}{15} = 8$

$x = 8 \times 15$

$x = 120$

Ans 5 $\frac{x-2}{x-3} = \frac{x-1}{x+1}$

$(x+1)(x-2) = (x-3)(x-1)$

$x(x-2) + 1(x-2) = x(x-1) - 3(x-1)$

$x^2 - 2x + x - 2 = x^2 - x - 3x + 3$

$-x - 2 = -4x + 3$

$4x - x = 3 + 2$

$3x = 5$

$x = \frac{5}{3}$

Ans 6 Let the digit in tens place be x & units place be y.

∴ given:

∴ ~~$10x + y = 7$~~ ∴ $x + y = 7$

Let the no be x

∴ given

number = $x^2 - 30$

∴ $x = x^2 - 30$

∴ $x^2 - x - 30 = 0$

∴ $x^2 - 6x + 5x - 30 = 0$

$x(x-6) + 5(x-6) = 0$

∴ $(x-6)(x+5) = 0$

∴ $x = 6$ or $x = -5$

↑ the no is 6

Let the digit in tens place be x, units place be y

when no is $10x + y$

here $x \times y = 12$

$x = 12y$ $x = \frac{12}{y}$

$10x + y + 36 = 10y + x$

∴ ~~10×10~~

∴ $10(12y) + y + 36 = 10y + 12y$

$120y + y + 36 = 22y$

$121y + 22y = -36$

$99y = -36$

$10x + y + 36 = 10y + x$

$\frac{10 \times 12}{y} + \frac{y}{1} + 36 = 10y + \frac{12}{y}$

$\frac{120}{y} + \frac{y^2}{y} + \frac{36y}{y}$

$\frac{120 + y^2 + 36y}{y} = \frac{10y^2 + 12}{y}$

$y^2 - 10y^2 + 36y + 120 - 12 = 0$

$-9y^2 + 36y + 108 = 0$

$\div$ by 9

∴ $-y^2 + 4y + 12 = 0$

$-(-y^2 + 4y + 12) = 0$

$y^2 - 4y - 12 = 0$

$y^2 - 6y + 2y - 12 = 0$

$y(y-6) + 2(y-6) = 0$

$(y-6)(y+2) = 0$

$y = 6$ or $y = -2$

∴ $x = \frac{12}{6}$

$x = 2$

∴ no is 26

For the given data find mean, median, lower & upper quartile, coeff of QD, 7th dec. 40th percentile MD about Mean, SD, Mode.

CI	F	X	FX	CF	$\lvert X-\bar{X}\rvert$
0-10	8	5	40	8	31.75
10-20	4	15	60	12	21.75
20-30	3	25	75	15	11.75
30-40	$6-F_0$	35	210	21	1.75
40-50	$7-F_1$	45	315	28	8.25
50-60	$4-F_2$	55	220	32	18.25
60-70	5	65	325	37	28.25
70-80	3	75	225	40	38.25
	$N=40$		$\Sigma Fx=1470$		159.75

If mean is 36.75 then find missing freq. A

$34-A$

$\text{Mean}(\bar{x}) = \frac{1470}{40}$

$= 36.75$

$\frac{7N}{10} = 28$

$\frac{40+A}{100} = \frac{46\times 40}{100}$

$40+A = 46\times 4$

$A = 184-40$

$A = 144$

$\frac{N}{2} = 20$

$\text{median} = L_1 + \left(\frac{\frac{N}{2}-CF}{F}\right)\times L_2 - L_1$

$= 30 + \left(\frac{20-15}{6}\right)\times 10$

$= 30 + \frac{50}{6}$

$= 38.34$

$\frac{N}{4} = \frac{40}{4} = 10$

$\frac{3N}{4} = \frac{3\times 40}{4} = 30$

$Q_1 = L_1 + \left(\frac{\frac{N}{4}-CF}{F}\right)\times L_2 - L_1$

$= 10 + \left(\frac{10-8}{4}\right)\times 10$

$= 10 + \frac{20}{4}$

$= 10 + 5$

$Q_1 = 15$

$Q_3 = L_1 + \left(\frac{\frac{3N}{4}-CF}{F}\right)\times L_2 - L_1$

$= 50 + \left(\frac{30-28}{4}\right)\times(10)$

$= 50 + \left(\frac{2}{4}\right)\times 10$

$= 50 + 5$

$= 55$

$\text{coeff} :- \frac{Q_3-Q_1}{Q_3+Q_1}$

$= \frac{55-15}{55+15}$

$= \frac{40}{70}$

$= 0.57$

$\text{Mode} = L_1 + \frac{(F_1-F_0)}{(F_1-F_0)+(F_1-F_2)}$

$= 40 + \frac{(7-6)}{(7-6)+(7-4)}$

$= 40 + \frac{10}{1+3}$

$= 40 + \frac{10}{3}$

$= 40 + 3.34$

$= 43.34$

Regression Analysis

$y = mx + c$

$(y - y_1) = m(x - x_1)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

(y dependent on value of x)

$y = \frac{2}{5}x + 5$ → Regression eq of y on x

$x = 5y + 15$ → Regression eq of x on y
(x dependent on value of y)

Regression coefficients eq

for y on x

$(y - \bar{y}) = b_{yx}(x - \bar{x})$

for x on y

$(x - \bar{x}) = b_{xy}(y - \bar{y})$

where b_{yx} & b_{xy} are called Regression coeff of y on x & x on y resp.

$$b_{yx} = \frac{N\Sigma xy - \Sigma x \cdot \Sigma y}{N\Sigma x^2 - (\Sigma x)^2}$$

$$b_{xy} = \frac{N\Sigma xy - \Sigma x \cdot \Sigma y}{N\Sigma y^2 - (\Sigma y)^2}$$

$$\bar{x} = \frac{\Sigma x}{N}$$

$$\bar{y} = \frac{\Sigma y}{N}$$

For the given data find ① mean values of x & y
② Regression coeff.
③ Regression eqs.
④ Hence, estimate the value of y when $x = 15$ & estimate the value of x when $y = -2$

x	y	xy	x^2	y^2
1	10	10	1	100
3	8	24	9	64
5	6	30	25	36
7	4	28	49	16
9	2	18	81	4
$\Sigma x = 25$	$\Sigma y = 30$	$\Sigma = 110$	$\Sigma = 165$	$\Sigma y^2 = 220$

$$\bar{x} = \frac{\Sigma x}{N} = \frac{25}{5} = 5$$

$$\bar{y} = \frac{\Sigma y}{N} = \frac{30}{5} = 6$$

$$b_{yx} = \frac{N\Sigma xy - \Sigma x \cdot \Sigma y}{N\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{5 \times 110 - (25 \times 30)}{5 \times 165 - 625}$$

$$= \frac{550 - 750}{825 - 625}$$

$$= \frac{-200}{200} = -1$$

$$b_{xy} = \frac{N\Sigma xy - \Sigma x \cdot \Sigma y}{N\Sigma y^2 - (\Sigma y)^2}$$

$$= \frac{5 \times 110 - 25 \times 30}{5(220) - (30)^2}$$

$$= \frac{550 - 750}{1100 - 900}$$

$$= \frac{-100}{100} = -1$$

eq of y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - 6 = -1(x - 5)$$

$$y = 6 - x + 5$$

$$\boxed{y = 11 - x}$$

when $x = 15$

$$y = 11 - 15$$

$$y = -4$$

eq of x on y

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x - 5 = -1(y - 6)$$

$$x - 5 = -y + 6$$

$$x = -y + 5 + 6$$

$$\boxed{x = -y + 11}$$

when $y = -2$

$$x = -(-2) + 11$$

$$x = 2 + 11$$

$$\boxed{x = 13}$$

x	y	xy	x^2	y^2
4	12	48	16	144
5	14	70	25	196
7	10	70	49	100
9	18	162	81	324
15	12	180	225	144
$\Sigma x = 40$	$\Sigma y = 66$	$\Sigma xy = 530$	$\Sigma x^2 = 396$	$\Sigma y^2 = 908$

$$\bar{x} = \frac{\Sigma x}{N} = \frac{40}{5} = 8$$

$$\bar{y} = \frac{\Sigma y}{N} = \frac{66}{5} = 13.2$$

$$b_{yx} = \frac{N\Sigma xy - \Sigma x \cdot \Sigma y}{N\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{5(530) - (40) \cdot (66)}{5(396) - (40)^2}$$

$$= \frac{2650 - 2640}{1980 - 1600}$$

$$= \frac{10}{380}$$

$$= \frac{1}{38} = 0.026$$

$$b_{xy} = \frac{N\Sigma xy - \Sigma x \cdot \Sigma y}{N\Sigma y^2 - (\Sigma y)^2}$$

$$= \frac{10}{5(908) - (66)^2}$$

$$= \frac{10}{4540 - 4356} = \frac{10}{184} = 0.054$$

finding regression equation by method of least square

i) Regression eqn of y on x

$y = a + bx$ / $x = a + by$

where.

$\Sigma y = aN + b\Sigma x$

$\Sigma xy = a\Sigma x + b\Sigma x^2$

Substitute the values of known terms from the table and solve the eq for a & b.

subs. values of a & b in given eq.

x	y	xy	x^2	y^2
2	4	8	4	16
3	5	15	9	25
5	7	35	25	21
6	4	24	36	16
8	6	48	64	36
Σx = 24	Σy = 26	130	138	142

$26 = 5a + 24b \times 24$

$130 = 24a + 138b \times 5$

$624 = 120a + 576b$

$650 = 120a + 690b$

$-26 = -144b$

$b = \frac{26}{144} = 0.228 = 0.23$

Subst $b = 0.23$ in eq^n (i)

$26 = 5a + 24 \times 0.23$

$26 = 5a + 5.52$

$26 - 5.52 = 5a$

$5a = 20.48$

$a = \frac{20.48}{5}$

$a = 4.096$

req eq of y on x

$y = 4.096 + 0.23x$.

ii) $24 = 5a + 26b \times 26$

$130 = 26a + 142b \times 5$

$624 = 130a + 676b$

$650 = 130a + 710b$

$-26 = -34b$

$b = \frac{26}{34}$

$b = 0.76$.

$x = 0.848 + 0.76y$.

$R = \sqrt{byx \times bxy}$

$= \sqrt{0.76 \times 0.23}$

$= 0.41$

pearson coeff

$r = \frac{N\Sigma xy - \Sigma x \cdot \Sigma y}{\sqrt{N\Sigma x^2 - (\Sigma x)^2}\sqrt{N\Sigma y^2 - (\Sigma y)^2}}$

$= \frac{5(130) - 24 \times 26}{\sqrt{5(138) - (24)^2}\sqrt{5(142) - (26)^2}}$

$= \frac{650 - 624}{\sqrt{690 - 576}\sqrt{710 - 676}}$

$= \frac{26}{\sqrt{114} \cdot \sqrt{34}}$

$= \frac{26}{\sqrt{114 \times 34}}$

$= \frac{26}{\sqrt{3876}}$

$= \frac{26}{62.25} = 0.417$

$b_{xy} = 0.76$, $b_{yx} = 0.33$

For the given data, find or estimate the value of y when $x = 18$ & the value of x when $y = 20$

	x	y
No of obs	10	12
Sum of the	40	60
S.D	2	3

Corr. coeff $R = 0.36$

Reg. coeff. of y on x

$$b_{yx} = R \cdot \frac{\sigma y}{\sigma x} = \frac{0.36 \times 3}{2} = 0.54$$

x on y

$$b_{xy} = R \times \frac{\sigma x}{\sigma y} = 0.36 \times \frac{2}{3} = 0.24$$

$$\bar{x} = \frac{\Sigma x}{N} \qquad \bar{y} = \frac{\Sigma y}{N}$$

$$\bar{x} = 4 \qquad \bar{y} = 5$$

Reg eq of y on x

$$(y - \bar{y}) = b_{yx}(x - \bar{x})$$

$$y - 5 = 0.54(x - 4)$$

$$y - 5 = 0.54x - 2.16$$

$$y = 0.54x + 2.84$$

When $x = 18$

$$y = 0.54(18) + 2.84$$

$$= 9.72 + 2.84$$

$$y = 12.56.$$

Reg eq of x on y

$$(x - \bar{x}) = b_{xy}(y - \bar{y})$$

$$(x - 4) = 0.24(y - 5)$$

$$x - 4 = 0.24y - 1.20$$

$$x = 0.24y - 1.2 + 4$$

$$x = 0.24y + 2.8$$

When $y = 20$

$$x = 0.24(20) + 2.8$$

$$= 4.8 + 2.8$$

$$x = 7.6$$

Q) From the fall reg. eq^{ns} find mean values of x & y, σx, Reg. coeffs, corr. coeff. Also estimate the value of y when $x = 6$ & value of x when $y = 10$.

$$3x - 2y - 10 = 0 \quad \times 8$$

$$24x - 25y + 145 = 0$$

$$\therefore 24x - 16y - 80 = 0$$

$$\Rightarrow 24x - 25y + 145 = 0$$

$$9y - 225 = 0$$

$$9y = 225$$

$$y = 25$$

Sub $y = 25$

$$3x - 50 - 10 = 0$$

$$3x - 60 = 0$$

$$x = 20$$

x on y

$$(y - \bar{y}) = b_{yx}(x - \bar{x})$$