MCQ Practice Unit 1

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Objective Questions

1) Which of the following set is a group under indicated binary operation,

(a) $(\mathbb{N}, +)$ (b) $(\mathbb{R}, \cdot)$

(c) $(\mathbb{R}^*, +)$ ✓(d) $(\mathbb{R}, +)$

(a) $(\mathbb{N},+)$ no identity, no inverse

(b) $(\mathbb{R},\cdot)$ having all but no inverse of 0

(c) $(\mathbb{R}^*,+)$ no identity

(d) $(\mathbb{R},+)$ having all identity and inverse

2) Let G be a group and $a_1, a_2, a_3, a_4 \in G$ then inverse of $a_1 a_2^{-1} a_3^{-1} a_4$ is

(a) $a_1^{-1} a_2 a_3 a_4^{-1}$ (b) $a_4^{-1} a_2 a_3 a_1^{-1}$

✓(c) $a_4^{-1} a_3 a_2 a_1^{-1}$ (d) None of these

$$(a_1 a_2^{-1} a_3^{-1} a_4)^{-1} = a_4^{-1} (a_3^{-1})^{-1} (a_2^{-1})^{-1} a_1^{-1}$$

$$[(a \cdot b)^{-1} = b^{-1} \cdot a^{-1}] \quad [(a^{-1})^{-1} = a]$$

$$(a_1 a_2^{-1} a_3^{-1} a_4)^{-1} = a_4^{-1} a_3 a_2 a_1^{-1}$$

3) Let G be a group and $a, b, c \in G$ then the solution of the equations $axb^{-1} = c$ and $a^{-1}y^{-1}b^{-1} = c$ are

(a) $x = a^{-1}c^{-1}b, y = a^{-1}c^{-1}b^{-1}$ ✓(b) $x = a^{-1}cb, y = b^{-1}c^{-1}a^{-1}$

(c) $x = a^{-1}c^{-1}b, y = b^{-1}c^{-1}a^{-1}$ (d) $x = a^{-1}c^{-1}b, y = b^{-1}c^{-1}a^{-1}$

$$axb^{-1} = c \Rightarrow a^{-1}(axb^{-1})b = a^{-1}cb \Rightarrow (a^{-1}a)x(b^{-1}b) = a^{-1}cb$$

$$\Rightarrow exe = a^{-1}cb \Rightarrow x = a^{-1}cb$$

$$a(a^{-1}y^{-1}b^{-1})b = acb \Rightarrow (aa^{-1})y^{-1}(b^{-1}b) = acb$$

$$\Rightarrow ey^{-1}e = acb \Rightarrow y = (acb)^{-1}$$

$$= b^{-1}c^{-1}a^{-1}$$

4) Let O denotes the set of odd integers. Then

(a) O forms group under multiplication (b) O forms group under addition

✓(c) O dosen't forms group under addition (d) None of these

(a) O = odd integer w.r.t $\times$ no inverse

(b) odd + odd = even $\notin O$ $\Rightarrow$ no closer proper

5) $(\mathbb{N}, +)$ is not a group as

(a) Associative law does not hold in $(\mathbb{N}, +)$

(b) Cancellation law does not hold in $(\mathbb{N}, +)$

✓ ~~(c)~~ $\mathbb{N}$ has no identity element with respect to $+$

(d) $\mathbb{N}$ has elements which have more than one inverse with respect to $+$

(a) $(N, +)$ have no identity

(b) $(N, +)$ have no inverse

6) Consider the set $G = \{\bar{5}, \overline{15}, \overline{25}, \overline{35}\}$ under multiplication of residue classes modulo 40. Then

(a) G is not a group as $\bar{1} \notin G$

✓ ~~(b)~~ G is a group with $\overline{25}$ as identity

(c) G is a group with $\bar{5}$ as identity

(d) None of these

$\overline{25}$ is identity $(G, \times_{40})$ is a group

7) Consider the group $(\mathbb{Z}, *)$, where $\mathbb{Z}$ is set of integers and $a * b = a + b - 6$

(a) -6 is identity of $(\mathbb{Z}, *)$ and inverse of $a \in \mathbb{Z}$ is $6 - a$.

✓ ~~(b)~~ 6 is identity of $(\mathbb{Z}, *)$ and inverse of $a \in \mathbb{Z}$ is $12 - a$.

(c) 6 is identity of $(\mathbb{Z}, *)$ and inverse of $a \in \mathbb{Z}$ is $6 - a$.

(d) None of these

$a * b = a + b - 6$

$a * e = a + e - 6 = a$

$\Rightarrow e - 6 = 0$

$\Rightarrow e = 6$

$a * a^{-1} = e = 6$

$\Rightarrow a + a^{-1} - 6 = 6$

$\Rightarrow a^{-1} = 6 + 6 - a$

$= 12 - a$

8) Consider the sets (i) $(\mathbb{Z}, \cdot)$ (ii) $(\mathbb{N}, +)$ (iii) $(\mathbb{R}, \cdot)$ (iv) $(G, \cdot)$ where $G = \{2^m 3^n : m, n \in \mathbb{Z}\}$

(a) (i) and (iv) are groups.

✓ ~~(b)~~ Only (iv) is group.

(c) (ii) and (iii) are groups.

(d) None of these

(i) $(\mathbb{Z}, \cdot) \Rightarrow$ No inverse

(ii) $(N, +) \Rightarrow$ No inverse, no identity

(iii) $(\mathbb{R}, \cdot) \Rightarrow$ zero does not have inverse

(iv) $G = \{2^m 3^n : m, n \in \mathbb{Z}\}$

(a) $m = 0, n = 0$, $2^0 . 3^0 = 1 \in G$ identity

(b) $(2^m . 3^n)(2^a . 3^b) = 2^{(m+a)} . 3^{(n+b)} \in G$, $(m+a), (n+b) \in \mathbb{Z}$

(c) $(2^m . 3^n) \times (2^{-m} . 3^{-n})$

$= 2^{m-m} . 3^{n-n}$

$= 2^0 . 3^0 = 1 \Rightarrow (2^{-m} . 3^{-n})$ is the inverse of $2^m . 3^n$

9) Which of the following group is non–Abelian

(a) $(\mathbb{Q}, +)$ ✓(b) S_3

(c) μ_4 , the fourth root of unity (d) None of these

S_3 does not satisfy commutative

$\Rightarrow$ (b)

10) The identity of $G = \left\{\begin{pmatrix} a & a \\ a & a \end{pmatrix} : a \in \mathbb{Q}, a \neq 0\right\}$ under multiplication of 2×2 matrices is

(a) $\begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ ✓(b) $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

(a) $\begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}\begin{pmatrix} a & a \\ a & a \end{pmatrix} = \begin{pmatrix} \frac{a}{4}+\frac{a}{4} & \frac{a}{4}+\frac{a}{4} \\ \frac{a}{4}+\frac{a}{4} & \frac{a}{4}+\frac{a}{4} \end{pmatrix} = \begin{pmatrix} \frac{a}{2} & \frac{a}{2} \\ \frac{a}{2} & \frac{a}{2} \end{pmatrix}$

(b) $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} a & a \\ a & a \end{pmatrix} = \begin{pmatrix} \frac{a}{2}+\frac{a}{2} & \frac{a}{2}+\frac{a}{2} \\ \frac{a}{2}+\frac{a}{2} & \frac{a}{2}+\frac{a}{2} \end{pmatrix} = \begin{pmatrix} a & a \\ a & a \end{pmatrix}$

11) Let $G_1 = \{\bar{1}, \bar{2}, \bar{3}\}$ under multiplication of residue classes modulo 4 and $G_2 = \{\bar{1}, \bar{2}\}$ under multiplication of residue classes modulo 3. Then

(a) G_1 and G_2 are groups (b) G_1 is a group but G_2 is not a group

✓(c) G_1 is not a group but G_2 is a groups (d) None of these

$\times_4$	1	2	3
1	1	2	3
2	2	0	2
3	3	2	1

✗ No Group

$\times_3$	1	2
1	1	2
2	2	1

12) Let $G = \{\bar{1}, \bar{3}, \bar{9}, \overline{11}, \bar{a}, \overline{19}, \overline{27}, \overline{33}\}$ under multiplication of residue classes modulo 40 where $1 \leq a \leq 40$. Then

(a) $a = 13$ ✓(b) $a = 17$ (c) $a = 23$ (d) $a = 31$

	$\bar{1}$	$\bar{3}$	$\bar{9}$	$\bar{1}$	$\bar{a}$	$\overline{19}$	$\overline{27}$	$\overline{33}$
$\bar{1}$	$\bar{1}$	$\bar{3}$	$\bar{9}$	$\overline{11}$	$\bar{a}$	$\overline{19}$	$\overline{27}$	$\overline{33}$
$\bar{3}$	$\bar{3}$	$\bar{9}$	$\overline{27}$	$\overline{33}$	$\overline{11}$	$\overline{17}$	$\bar{1}$	$\overline{19}$
$\bar{9}$								
$\overline{11}$								
$\bar{a}$								
$\overline{19}$								
$\overline{27}$								
$\overline{33}$								

$3 \times 27 = 81 \equiv 1 \pmod{40}$

$99 \equiv 19$ (mod

$3 \times 19 = 57 \equiv 17 \pmod{40}$

$\overline{27}$

$\overline{33}$

14) Let $G = GL_n(\mathbb{R})$. Then

(a) G is an infinite Abelian group for $n \geq 2$
(b) G is a finite Abelian group for $n = 2$
(c) G is an infinite non-Abelian group for $n \geq 2$ ✓
(d) None of these

As $(GL_n(\mathbb{R}), \cdot)$ does not satisfy commutative prop.

15) Let G be a group $a, b \in G$ such that $ab = ba, o(a) = m, o(b) = n$. Then

(a) $o(ab) = mn$
(b) $o(ab) = l.c.m[m, n]$ ✓
(c) $o(ab)$ divides l.c.m$[m, n]$ but may not be equal to $l.c.m[m, n]$
(d) None of these

$o(ab) = mn$ only $(m, n) = 1$

16) Let G be a group, $a \in G$ a such that $o(a) = n$. If $a^r = a^s$ then

(a) $r = s$
(b) $r + s = n$
(c) $r \equiv s \bmod n$ ✓
(d) None of these

$o(a) = n \Rightarrow a^n = e$

$a^r = a^s \Rightarrow a^{r-s} = e$

$n \mid (r-s)$

$\Rightarrow r \equiv s \pmod n$

17) The number of elements in $U(n)$, the group of prime residue class modulo n is

(a) n
(b) $n - 1$
(c) $\phi(n)$
(d) None of these

$\phi(n) =$ Euler's ϕ function

$U(n) = \phi(n)$

$\phi(p) = p-1$, $|U(p)| = p-1$

18) The set $\mathbb{Z}^*_n$ under multiplication of residue class modulo n is a group if and only if

(a) n is odd
(b) n is prime ✓
(c) n is even
(d) None of these

Z_n^* is a group only when n is prime

19) In the group $(\mathbb{Z}_{42}, +)$, order of $\overline{18}$ is

(a) 7 ✓
(b) 18
(c) 6
(d) None of these

$(42, 18) = 6$, $o(Z_{42}) = 42$

$42 = 3 \times 2 \times 7$

$18 + 18 + 18 = 54 \not\equiv 0 \pmod{42}$

$18 \times 6 = 108 \not\equiv 0 \pmod{42}$

$42 = 3 \times 2 \times 7$

$18 + 18 = 36 \not\equiv 0 \pmod{42}$

$18 \times 6 = 108 \not\equiv 0 \pmod{42}$

$18 \times 7 = 126 \equiv 0 \pmod{42}$

$o(18) = 7$

$$o(a^k) = \frac{n}{(n,k)} = \frac{42}{6} = 7$$

20) The inverse of $\bar{3}$ in the group $(\mathbb{Z}^*_7, \cdot)$ is

(a) $\bar{3}$ ✓(b) $\bar{5}$ (c) $\bar{2}$ (d) None of these

$o(\mathbb{Z}^*_7) = 6$, $\bar{3} \times 5 = 15 \equiv 1 \pmod{7}$

21) Which of the following is a Klein-4 group.

(a) $U(10)$ (b) μ_4, fourth root of unity ✓(c) $U(8)$ (d) None of these

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22) Let G be a group and $a, b, \in G$. Let e be the identity element of G. If $a^4 = e, ab = ba^2$ then

✓(a) $a = e$ (b) $a = b$ (c) $a = b^2$ (d) $b = a^2$

$a^4 = e$, $ab = ba^2$

(a) $a = e$, $e^4 = e$, $e.b = b.e^2 \Rightarrow b = b$ true

(b) $a = b$, $b^4 = e$, $b.b = b.b^2 \Rightarrow b^2 = b^3$ which may not true

(c) $a = b^2$, $b^8 = e$, $b^2.b = b.(b^2)^2 \Rightarrow b^3 = b^5$ which may not be true

23) Let G be a finite group. Then the number of solutions of the equation $x^3 = e$

(a) is one (b) is a multiple of 3

✓(c) is always odd number (d) is always even number

$x^3 = e$

Take G = 3rd root of unity

$x^3 = 1$ ——— (1)

Solution of (1) $= 1, \omega, \omega^2 \Rightarrow$ odd solution.

25) Let G be a group and $a \in G$. If $o(a) = 20$, then $o(a^4)$ is

(a) 15 ✓(b) 5 (c) 12 (d) 20

$o(a) = 20$

(a) 15 (b) 5 (c) 12 (d) 20

$o(a) = 20$

$\Rightarrow a^{20} = e$

$\Rightarrow (a^4)^5 = e$

$\Rightarrow o(a^4) = 5$

26) Consider the group (Q^+, o) where Q^+ is set of positive rational numbers and $a \; o \; b = \frac{ab}{3}$ for $a, b \in Q^+$. Then

(a) $\frac{1}{3}$ is the identity element of the group and for $a \in Q^+, a^{-1} = \frac{1}{3a}$

(b) 1 is the identity element of the group and for $a \in Q^+, a^{-1} = \frac{3}{a}$

(c) 3 is identity element of the group and $a^{-1} = \frac{3}{a}$ for $a \in Q^+$

(d) None of the above

$a \circ b = \frac{ab}{3}$, $a, b \in Q^+$

$a \circ e = a$

$\frac{ae}{3} = a$

$\frac{e}{3} = 1$

$e = 3$

$a \circ a^{-1} = e$

$\frac{aa^{-1}}{3} = 3$

$\Rightarrow a^{-1} = \frac{9}{a}$

27) Let G be a group and $a \in G$. If $o(a) = 17$, then $o(a^8)$ is

(a) 17 (b) 16 (c) 8 (d) 5

$o(a) = 17 \Rightarrow a^{17} = e$

$o(a^8) = \frac{17}{(8, 17)}$

$= \frac{17}{1}$

$= 17$

28) Suppose a group G contains elements a and b such that $o(a) = 4, o(b) = 2, a^3 b = ba$. Then $o(ab)$ is

(a) 2 (b) 5 (c) Infinite (d) 6

$o(a) = 4 \Rightarrow a^4 = e$

$o(b) = 2 \Rightarrow b^2 = e$

$o(ab) = 2$

$o(b) = 2 \Rightarrow b^2 = e$

$a^3 b = b a$

$a(a^3 b) b = a(ba) b$

$\Rightarrow (a a^3)(bb) = (ab) \cdot (ab)$

$\Rightarrow a^4 . b^2 = (ab)^2$

$\Rightarrow e = (ab)^2$

$O(ab) = 2$

29) In a group G, the number of elements $a \in G$ such that $a^2 = a$ is

(a) 0 ✓(b) 1 (c) 2 (d) None of these

$e^2 = e$

Practical - 02 Objectives

Objective Questions

(1) The group of symmetries of a rectangle which is not a square is

(a) $(\mathbb{Z}_4, +)$ ✓(b) V_4 –Klein's Four group (c) D_4 (d) None of these

Group of symmetries of rectangle $= V_4$

(2) The group of symmetries of a square has order

(a) 4 (b) 24 ✓(c) 8 (d) None of these

(3) The group of symmetries of

(a) a square is Abelian (b) an equilateral triangle is Abelian

✓(c) a rectangle is Abelian (d) None of these

(4) The group of symmetries of a regular n –gon $(n > 3)$ has

(a) n elements of order 2 and $n - 1$ elements of order n.

(b) $n$elements of order 2 if n is odd.

✓(c) exactly 2 elements of order n.

(d) None of the above.

$O(R_1) = 4$, $O(R_2) = 2$, $O(R_3) = 4$, $O(R_4) = O(R_5)$
$= O(R_6)$
$= O(R_7) = 2$

So the set G = { R_0 , R_1 , R_2 , R_3 , R_4 , R_5, R_6, R_7 }

	R_0	R_1	R_2	R_3	R_4	R_5	R_6	R_7
R_0	R_0	R_1	R_2	R_3	R_4	R_5	R_6	R_7
R_1	R_1	R_2	R_3	R_0	R_6	R_7	R_5	R_4

$R_1 \circ R_1 = R_2$

$(R_1 \circ R_1) \circ R_1 = R_2 \circ R_1$
$= R_3$

R_1	R_1	R_2	R_3	R_0	R_6	R_7	R_5	R_4
R_2	R_2	R_3	R_0	R_1	R_5	R_4	R_7	R_6
R_3	R_3	R_0	R_1	R_2	R_7	R_6	R_4	R_5
R_4	R_4	R_7	R_5	R_6	R_0	R_2	R_3	R_1
R_5	R_5	R_6	R_4	R_7	R_2	R_0	R_1	R_3
R_6	R_6	R_4	R_7	R_5	R_1	R_3	R_0	R_2
R_7	R_7	R_5	R_6	R_4	R_3	R_1	R_2	R_0

$(R_1 \circ R_1) \circ R_1 = R_2 \circ R_1$
$= R_3$
$(R_1 \circ R_1) \circ R_1 \circ R_1 = R_3 \circ R_1$
$= R_0$
$(R_1)^4 = R_0$

$R_2 \circ R_2 = R_0$
$(R_2)^2 = 2$

(5) Let G be the group of symmetries of a square. Then center of G has

(a) only one element. (b) four elements.

✓(c) exactly two elements. (d) None of these

$Z(G) = \{x \in G \mid xg = gx, \forall g \in G\}$

$R_2, R_0 \in Z(G)$

(6) Let G be the group of symmetries of a regular pentagon. Then G has

✓(a) 5 reflections and 5 rotations. (b) no reflections and 10 rotations.

(c) 10 reflections and 10 rotations. (d) None of these

(7) Let $\sigma \in S_n$ be a cycle of length r. Then σ^k is a cycle of length r if

(a) k is odd ✓(b) $(k, r) = 1$ (c) k is even (d) None of these

$\sigma \in S_n$ be a cycle of len $= r$
$\text{len}(\sigma^k) = r$ only $(k, r) = 1$

(8) The number of elements of order 2 in S_4 is

(a) 8 (b) 6 ✓(c) 9 (d) None of these

(9) Let $H = \{\alpha \in S_5 : \alpha(1) = 1\}$. Then

(a) H has only identity element. (b) H has only even permutations.

✓(c) $o(H) = 24$ (d) $o(H) = 12$

$o(S_5) = 5! = 120$

$o(S_5) = 5! = 120$

$\sigma(1) = 1$, $4! = 24$ | $3!$

$\Rightarrow o(H) = 24$

(10) The orders of elements in the group A_4 are

✓(a) 1 , 2 and 3. (b) 1 , 2 and 4. (c) 1 , 3 and 4. (d) 1 , 2.

$o(A_4) = \frac{4!}{2} = \frac{24}{2} = 12$, $1, 2, 3, 4, 6, 12$

(11) The maximum order of an element in S_{10} is

(a) 10 (b) 24 (c) 25 ✓(d) 30

$o(S_{10}) = 10!$

$(1\,2\,3)(4\,5\,6\,7\,8)(9\,8) \in S_{10}$

$o(123) = 3$

$o(98) = 2$

$o(45678) = 5$ ✓

$\therefore o((123)(45678)(98)) = \text{lcm}(3, 2, 5)$

$= 30 \quad (3 \times 2 \times 5)$

(12) The normalizer $N((123))$ of the element (123) in S_3 is

(a) $\{I, (123)\}$ ✓(b) $\{I, (123), (132)\}$

(c) $\{I, (132)\}$ (d) None of these

$(123)\sigma = \sigma(123)$, $\sigma \in S_3$

$(123) I = I (123) \Rightarrow I \in N(123)$

$(123)(123) = (132) = (123)(123) \Rightarrow (123) \in N(123)$

$(123)(132) = I = (132)(123) \Rightarrow (132) \in N(123)$

(13) For what value of n , $D_n = S_n$

✓(a) $n = 3$ (b) $n = 4$ (c) $n = 5$ (d) there is no such n exists

(14) $o(\alpha \circ \beta)$ is where $o(\alpha) = m$, $o(\beta) = k$

(14) $O(\alpha \circ \beta)$ is where $O(\alpha) = m$, $O(\beta) = K$

(a) always less than n. (b) mk (c) $l.c.m[m, k]$ (d) None of these

(15) If $\sigma \in S_n$ has odd order then

(a) σ is odd permutation. (b) σ is even permutation.

(c) σ may be even or odd permutation. (d) None of these

$\text{sig}(1\,2\,3) = (-1)^{3-1} = (-1)^2 = +1$ even

(16) Every permutation in A_n can be written as product of

(a) even number of transpositions (b) p transpositions, where p is an odd prime

(c) odd number of transpositions (d) None of the above

A_3, $(1\,2\,3) \in A_3$

$(1\,2\,3) = (1\,2)(2\,3)$

(17) Let $\beta \in S_7$ such that $\beta^4 = (2143567)$ then β is equal to

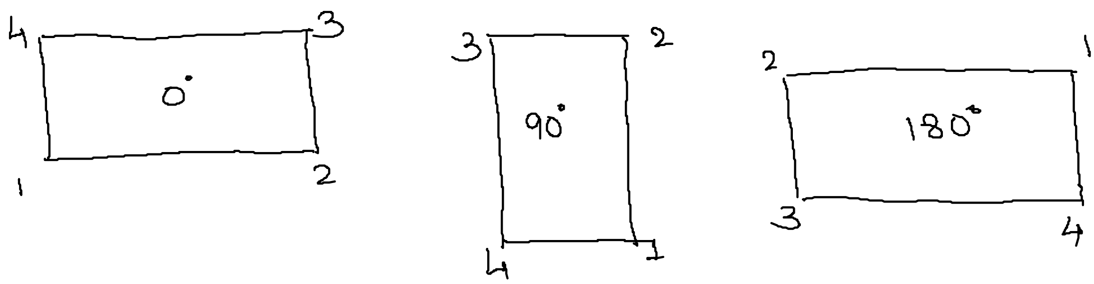
(a) (2516473) (b) (2457136) (c) (2631754) (d) None of these

$(2457\,136)(2457\,136)(2457\,136)(2457\,136)$

$= (21\,4\,3\,5\,67)$

(18) In the group of symmetry of a rectangle (which is not a square) , the element obtain in rotation of rectangle by angle $180°$ is

(a) $(12)(34)$ (b) $(13)(24)$ (c) $(14)(23)$ (d) None of these



$(1\,3)(2\,4)$

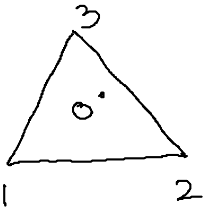
(19) The group of symmetric of an equilateral triangle is

(a) S_3 (b) A_3 (c) A_4 (d) S_4

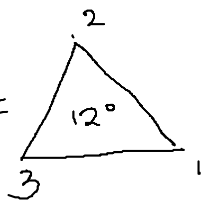
(20) In the group of symmetries of an equilateral triangle with vertices. 1, 2,3, let σ denote clockwise rotation through 120° about center and τ denote reflection about the line joining vertex 1 and mid point of opposite side then

(a) σ does not commute with τ. (b) σ commutes with every element of the group

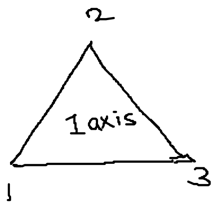
(c) σ, τ have order 2 (d) σ, τ have order 3.



$\sigma =$



$(1\,3\,2)$



$\tau = (1)\,(23)$

$\sigma\tau = (132)(23) = (13)\,(2)$

$\tau\sigma = (23)\,(132) = (12)\,(3)$

$\sigma\tau \neq \tau\sigma$

(21) Let G_1, G_2, G_3 denote the group of symmetries of a equilateral rectangle, rectangle and square respectively. Then,

(a) G_1, G_2, G_3 are all abelian (b) G_1, G_2, G_3 are all non-abelian

(c) G_1 and G_3 are non-abelian and G_2 is abelian (d) G_1, G_2 are abelian and G_3 is non-abelian

(22) Let G denote the group of symmetries of a square then the number of elements in the centre of G is

(a) 1 (b) 2 (c) 3 (d) 4

(23) The group of symmetries of a regular $n - gon$ is

(a) S_n (b) S_{2n} (c) $\mathbb{Z}_n$ (d) D_n

(24) Consider the groups A_3, A_4, S_3, S_4. Among them

(a) A_3 is abelian, S_3, S_4 are non-abelian (b) A_3, A_4 are abelian, S_3, S_4 are non-abelian

(c) All the groups are non-abelian (d) A_3, S_3 are abelian, A_4, S_4 are non-abelian

$A_3 = \{ I, (123), (132) \}$

$I\,(123) = (123)\,I$, $(123)\,(132) = 1 = (132)(123)$

$(132)\,(I) = I\,(132)$

(25) S_3 has no element of order

(a) 1 (b) 2 ✓(c) 4 (d) 3

1 , 2 , 3

Practical - 03 Objectives

1. Let H be a proper subgroup of the group $(\mathbb{Z},+)$ such that $18, 30, 40$ belong to H then
(a) $H = 10\mathbb{Z}$ ✓(b) $H = 2\mathbb{Z}$ (c) $H = \mathbb{Z}$ (d) $H = 360\mathbb{Z}$

$$\begin{array}{r|l} 2 & 18, 30, 40 \\ \hline 3 & 9, 15, 20 \\ \hline 5 & 3, 5, 20 \\ \hline & 3, 1, 4 \end{array}$$

lcm = 360

gcd = 2

2. Let G be a group having elements of order $1, 2, 3, 4, 5, 6$. The minimal possible order of G is
(a) 100 (b) 30 ✓(c) 60 (d) 1

lcm $= \{1, 2, 3, 4, 5, 6\} = 2 \times 3 \times 2 \times 5 = 60$

$$\begin{array}{r|l} 2 & 1, 2, 3, 4, 5, 6 \\ \hline 3 & 1, 1, 3, 2, 5, 3 \\ \hline & 1, 1, 1, 2, 5, 1 \end{array}$$

$o(a) \mid o(G)$

3. Let G be a group of odd order and e be an identity element of G. The equation $x^2 = e$ has
✓(a) a unique solution in G (b) no solution in G (c) two solutions in G (d) Cannot say

$x^2 = e \Rightarrow x = e$, $e^2 = e$

$\therefore$ $x = e$ is a soln

4. Let G be a non-Abelian group $Z(G) = \{x \in G \mid ax = xa \ \forall a \in G\}$ then
✓(a) $Z(G) = \{e\}$ (b) $Z(G) \neq G$ and $Z(G)$ is Abelian (c) $Z(G) = G$ (d) $Z(G)$ is non-Abelian

5. A group G has subgroups of order 45 and 75. If $0(G) < 400$ then
(a) $0(G) = 150$ ✓(b) $0(G) = 225$ (c) $0(G) = 375$ (d) $0(G) = 150$ or 225

$45 \nmid 150$, $45 \mid 225$ ✓ , $75 \mid 225$ ✓

$o(H) \mid o(G)$

6. A_4 has no subgroup of order

(a) 2 (b) 3 (c) 4 (d) 6

7. Let $\alpha = (13)(24) \in S_4$. Let $N(\alpha) = \{\sigma \in S_4 \mid \sigma\alpha = \alpha\sigma\}$. Then order of subgroup $N(\alpha)$is

(a) 4 (b) 6 (c) 12 (d) 24

$\alpha = (13)(24) \in S_4$ $\quad I \in N(\alpha)$

$(13)(24)(13)(24) = (13)(24)(13)(24)$, $(13)(24) \in N(\alpha)$

$(13)(24)(12)(34) = (14)(23)$

$(12)(34)(13)(24) = (14)(23)$, $(12)(34) \in N(\alpha)$

$(14)(23) \in N(\alpha)$

8. Let $\alpha = (1\ 2\ 3) \in S_3$ .Let $N(\alpha) = \{\sigma \in S_3 \mid \sigma\alpha = \alpha\sigma\}$. Then order of subgroup $N(\alpha)$ is

(a) 3 (b) 2 (c) 1 (d) 6

10. $\mathbb{C}^*$ under multiplication has

(a) No non-trivial finite cyclic subgroup

(b) Only one non-trivial finite cyclic subgroup

(c) Infinitely many non-trivial finite cyclic subgroups

(d) None of these.

11. Let p be prime. If a group G has more than $p-1$ elements of order p, then

(a) G is cyclic

(b) G is not cyclic

(c) G has a unique subgroup of order p.

(d) None of these.

(e)

In that subgroup of **order p**, there are ϕ(**p**)=**p−1 elements of order p**. Hence, there are only **p−1 elements of order p** in **G**. **If** there are **more than p−1 elements of order p**, **then**

there must be one **more** subgroups of **order p**,

12. $U(12)$, the multiplicative group of units in $\mathbb{Z}_{12}$, has

(a) Only one proper subgroup
(b) Two proper subgroup. ✓
(c) Three proper subgroups
(d) None of these.

$U(12) = \{1, 5, 7, 11\}$ $\quad O(U(12)) = 4$

$H_1 = \{1\}$, $H_2 = \{1, 5\}$

13. $\{\bar{1}, \bar{4}, \bar{7}\}$ in $U(9)$ is

(a) Not a subgroup of $U(9)$
(b) Is a subgroup of $U(9)$
(c) Is a cyclic subgroup of $U(9)$ ✓
(d) None of these.

$\times_9$	1	4	7
1	1	4	7
4	4	7	1
7	7	1	4

$\{1, 4, 7\}$ is a subgroup $U(9)$

$4^3 = 64 \equiv 1$

14. $\{\sigma \in S_6 : \sigma(6) = 1\}$ in S_6 is

(a) Not a subgroup of S_6 ✓
(b) Is a subgroup of S_6
(c) Is a cyclic subgroup of S_6
(d) None of these.

let us take any two elements of the set
$(1\,2\,3\,4\,5\,6)$, $(1\,6)(2\,3\,4\,5) \in$ set
Now let us see by subgroup test

$(1\,6)(2\,3\,4\,5)(1\,2\,3\,4\,5\,6)^{-1} = (1\,6)(2\,3\,4\,5)(6\,5\,4\,3\,2\,1)$
$= (1)(2\,6)(3)(4)(5) = (2\,6) \notin$ set

15. If a group G has only one element a of order n, then which of the following is true;
(i) $a \in Z(G)$ (ii) $n = 2$ (iii) a is inverse of itself

(a) Only (i)
(b) Only (ii) and (iii)
(c) All the three ✓
(d) None of these.

If a group has only one element a of order n, then a belongs to $Z(G)$ and $n = 2$.

and $a^2 = e \Rightarrow a$ is inverse of itself