MOTION IN ONE DIMENSION & NEWTON'S LAWS OF MOTION

(i) **Displacement**: | displacement | ≤ distance covered

(ii) **Average speed**: $\bar{v} = \frac{s_1 + s_2}{t_1 + t_2} = \frac{s_1 + s_2}{\frac{s_1}{v_1} + \frac{s_2}{v_2}}$

(a) If $s_1 = s_2 = d$, then $\bar{v} = \frac{2 v_1 v_2}{v_1 + v_2}$ = Harmonic mean

(b) If $t_1 = t_2$, then $\bar{v} = \frac{v_1 + v_2}{2}$ =arithmetic mean

(iii) **Average velocity**: (a) $\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$; (b) $|\vec{v}_{av}| \le \bar{v}$

(iv) **Instantaneous velocity**: $\vec{v} = \frac{d\vec{r}}{dt}$ and $|\vec{v}|$ = v = instantaneous speed

(v) **Average acceleration**: $\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$

(vi) **Instantaneous acceleration**: $\vec{a} = d\vec{v}/dt$

In one – dimension, a = (dv/dt) = $\left(\frac{dv}{dx}\right)v$

(vii) **Equations of motion in one dimension**:

(a) $v = u + at$;

(b) $x = ut + \frac{1}{2}at^2$;

(c) $v_2\ u_2 + 2ax$;

(d) $x = vt - \frac{1}{2}at^2$;

(e) $x = \left(\frac{v+u}{2}\right)t$;

(f) $s = x - x_0 = ut + \frac{1}{2}at^2$;

(g) $v^2 = u^2 + 2a(x - x_0)$

(viii) **Distance travelled in nth second**: $d_n = u + \frac{a}{2}(2n-1)$

(ix) **Motion of a ball**: (a) when thrown up: $h = (u^2/2g)$ and $t = (u/g)$
(b) when dropped: $v = \sqrt{(2gh)}$ and $t = \sqrt{(2h/g)}$

(x) **Resultant force**: $F = \sqrt{(F_1^2 + F_2^2 + 2F_1F_2 \cos\theta)}$

(xi) **Condition for equilibrium**: (a) $\vec{F}_3 = -(\vec{F}_1 + \vec{F}_2)$; (b) $F_1 + F_2 \ge F_3 \ge |F_1 - F_2|$

(xii) **Lami's Theorem**: $\frac{P}{\sin(\pi - \alpha)} = \frac{Q}{\sin(\pi - \beta)} = \frac{R}{\sin(\pi - \gamma)}$

(xiii) **Newton's second law**: $\vec{F} = m\vec{a}$; $\vec{F} = \left(d\vec{p}/dt\right)$

(xiv) **Impulse**: $\Delta\vec{p} = \vec{F}\Delta t$ and $p_2 - p_1 \int_1^2$ Fdt

(xv) **Newton's third law**:

Spectrum

(a) $\vec{F}_{12} = -\vec{F}_{12}$

(b) Contact force: $F_{12} = \frac{m}{M+m} F = F_{21}$

(c) Acceleration: a = $\frac{F}{M+m}$

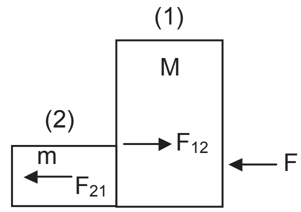

Fig. 1

(xvi) **Inertial mass**: m_I = F/a

(xvii) **Gravitational mass**: m_G = $\frac{F}{g} = \frac{FR^2}{GM}; m_I = m_G$

(xviii) **Non inertial frame**: If $\vec{a}_0$ be the acceleration of frame, then pseudo force $\vec{F} = -m\vec{a}_0$

Example: Centrifugal force = $\frac{mv^2}{r} = m\omega^2 r$

(xix) **Lift problems**: Apparent weight = M(g ± a_0)
(+ sign is used when lift is moving up while – sign when lift is moving down)

(xx) **Pulley Problems**:

(a) For figure (2):

Tension in the string, T = $\frac{m_1 m_2}{m_1 + m_2} g$

Acceleration of the system, a = $\frac{m_2}{m_1 + m_2} g$

The force on the pulley, F = $\frac{\sqrt{2} m_1 m_2}{m_1 + m_2} g$

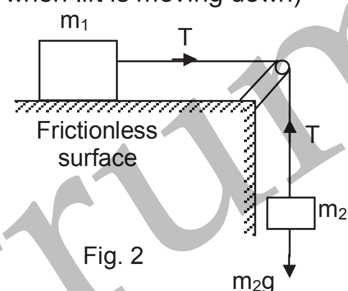

Fig. 2

(b) For figure (3):

Tension in the string, $T = \frac{2m_1 m_2}{m_1 + m_2} g$

Acceleration of the system, $a = \frac{m_2 - m_1}{m_2 + m_1} g$

The force on the pulley, $F = \frac{4m_1 m_2}{m_1 + m_2} g$

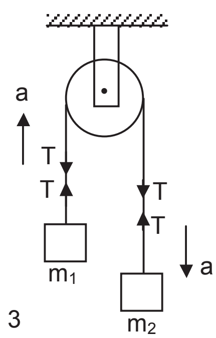

Fig. 3

Spectrum

VECTORS

(i) **Vector addition**: $\vec{R} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$ and $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

(ii) **Unit vector**: $\hat{A} = (\vec{A}/A)$

(iii) **Magnitude**: $A = \sqrt{(A_x^2 + A_y^2 + A_z^2)}$

(iv) **Direction cosines**: cos α = (A_x/A), cos β = (A_y/A), cos γ = (A_z/A)

(v) **Projection**:

(a) Component of $\vec{A}$ along $\vec{B}$ = $\vec{A}.\hat{B}$

(b) Component of $\vec{B}$ along $\vec{A}$ = $\hat{A}.\vec{B}$

(c) If $\vec{A} = A_x\hat{i} + A_y\hat{j}$, then its angle with the x–axis is $\theta = \tan^{-1}(A_y/A_x)$

(vi) **Dot product**:

(a) $\vec{A}.\vec{B}$ = AB cos θ, (b) $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$

(vii) **Cross product**:

(a) $\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n}$;

(b) $\vec{A} \times \vec{A} = 0$;

(c) $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

(viii) **Examples**:

(a) $W = \vec{F}.\vec{r}$; (b) $P = \vec{F}.\vec{v}$; (c) $\phi_E = \vec{E}.\vec{A}$; (d) $\phi_B = \vec{B}.\vec{A}$;

(e) $\vec{v} = \vec{w} \times \vec{r}$; (f) $\vec{\tau} = \vec{\tau} \times \vec{F}$; (g) $\vec{F}_m = q\left(\vec{v} \times \vec{B}\right)$

(ix) **Area of a parallelogram**: Area = $|\vec{A} \times \vec{B}|$

(x) **Area of a triangle**: Area = $\frac{1}{2}|\vec{A} \times \vec{B}|$

(xi) **Gradient operator**: $\overline{V} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

(xii) **Volume of a parallelopiped**: $V = \vec{A}.\left(\vec{B} \times \vec{C}\right)$

Spectrum

CIRCULAR MOTION, RELATIVE MOTION & PROJECTILE MOTION

(i) **Uniform Circular Motion**:
- (a) $v = \omega r$;
- (b) $a = (v^2/r) = \omega^2 r$;
- (c) $F = (mv^2/r)$;
- (d) $\vec{r} \cdot \vec{v} = 0$;
- (e) $\vec{v} \cdot \vec{a} = 0$

(ii) **Cyclist taking a turn**: $\tan\theta = (v^2/rg)$

(iii) **Car taking a turn on level road**: $v = \sqrt{(\mu_s rg)}$

(iv) **Banking of Roads**: $\tan\theta = v^2/rg$

(v) **Air plane taking a turn**: $\tan\theta = v^2/r\,g$

(vi) **Overloaded truck**:
- (a) $R_{inner\ wheel} < R_{outer\ wheel}$
- (b) maximum safe velocity on turn, $v = \sqrt{(gdr/2h)}$

(vii) **Non–uniform Circular Motion**:
- (a) Centripetal acceleration $a_r = (v^2/r)$;
- (b) Tangential acceleration $a_t = (dv/dt)$;
- (c) Resultant acceleration $a = \sqrt{(a_r^2 + a_t^2)}$

(viii) **Motion in a vertical Circle**:
- (a) For lowest point A and highest point B, $T_A - T_B = 6\,mg$; $v_A^2 = v_B^2 + 4g\ell$; $v_A \geq \sqrt{(5g\ell)}$; and $v_B \geq \sqrt{(g\ell)}$
- (b) Condition for Oscillation: $v_A < \sqrt{(2g\ell)}$
- (c) Condition for leaving Circular path: $\sqrt{(2g\ell)} < v_A < \sqrt{(5g\ell)}$

(ix) **Relative velocity**: $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

(x) **Condition for Collision of ships**: $(\vec{r}_A - \vec{v}_B) \times (\vec{v}_A - \vec{v}_B) = 0$

(xi) **Crossing a River**:
- (a) Beat Keeps its direction perpendicular to water current
- (1) $v_R = \sqrt{((v_w^2 + v_b^2)}$; (2) $\theta = \tan^{-1}(v_w/v_b)$;
- (3) $t = (x/v_b)$ (it is minimum) (4) Drift on opposite bank = $(v_w/v_b)x$
- (b) Boat to reach directly opposite to starting point:

  (1) $\sin\theta = (v_w/v_b)$; (2) $v_{resultant} = v_b \cos\theta$; (3) $t = \frac{x}{v_b \cos\theta}$

(xii) **Projectile thrown from the ground**:
- (a) equation of trajectory: $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$
- (b) time of flight: $T = \frac{2u\sin\theta}{g}$
- (c) Horizontal range, $R = (u^2 \sin 2\theta/g)$
- (d) Maximum height attained, $H = (u^2 \sin^2\theta/2g)$

Spectrum

(e) Range is maximum when $\theta = 45^0$
(f) Ranges are same for projection angles θ and $(90^0 - \theta)$
(g) Velocity at the top most point is = $u \cos \theta$
(h) $\tan \theta = gT^2/2R$
(i) $(H/T^2) = (g/8)$

(xiii) **Projectile thrown from a height h in horizontal direction**:
(a) $T = \sqrt{(2h/g)}$;
(b) $R = v\sqrt{(2h/g)}$;
(c) $y = h - (gx^2/2u^2)$

(d) Magnitude of velocity at the ground = $\sqrt{(u^2 + 2gh)}$
(e) Angle at which projectiles strikes the ground, $\theta = \tan^{-1} \sqrt{\frac{2gh}{u}}$

(xiv) **Projectile on an inclined plane**:
(a) Time of flight, $T = \frac{2u \sin(\theta - \theta_0)}{g \cos \theta_0}$

(b) Horizontal range, $R = \frac{2u^2 \sin(\theta - \theta_0) \cos \theta}{g \cos^2 \theta_0}$

Spectrum

FRICTION

(i) **Force of friction**:
- (a) $f_s \le \mu_s N$ (self adjusting); $(f_s)_{max} = \mu_s N$
- (b) $\mu_k = \mu_k N$ (μ_k = coefficient of kinetic friction)
- (c) $\mu_k < \mu_s$

(ii) **Acceleration on a horizontal plane**: $a = (F - \mu_k N)/M$

(iii) **Acceleration of a body sliding on an inclined plane**: $a = g \sin\theta\,(1 - \mu_k \cot t^2)$

(iv) **Force required to balance an object against wall**: $F = (Mg/\mu_s)$

(v) **Angle of friction**: $\tan\theta = \mu_s$ (μ_s = coefficient of static friction)

DYNAMICS OF RIGID BODIES

(i) **Average angular velocity**: $\overline{\omega} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$

(ii) **Instantaneous angular velocity**: $\omega = (d\theta/dt)$

(iii) **Relation between v, ω and r** : $v = \omega r$; In vector form $\vec{v} = \vec{\omega} \text{ x } \vec{r}$; In general form, $v = \omega r \sin\theta$

(iv) **Average angular acceleration**: $\overline{\alpha} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$

(v) **Instantaneous angular acceleration**: $\alpha = (d\omega/dt) = (d^2\theta/dt^2)$

(vi) **Relation between linear and angular acceleration**:
- (a) $a_T = \alpha r$ and $a_R = (v^2/r) = \omega^2 R$
- (b) Resultant acceleration, $a = \sqrt{(a_T^2 + a_R^2)}$
- (c) In vector form,

$$\vec{a} = \vec{a}_T + \vec{a}_R, \text{ where } \vec{a}_T = \vec{\alpha} \text{ x } \vec{r} \text{ and } \vec{a}_R = \vec{\omega} \text{ x } \vec{u} = \vec{\omega} \text{ x } \left(\vec{\omega} \text{ x } \vec{r}\right)$$

(vii) **Equations for rotational motion**:
- (a) $\omega = \omega_0 + \alpha t$;
- (b) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$;
- (c) $\omega^2 - \omega_0^2 = 2\alpha\theta$

(viii) **Centre of mass**: For two particle system:
- (a) $x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$;
- (b) $v_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$
- (c) $a_{CM} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}$

Also $v_{CM} = \frac{dx_{CM}}{dt}$ and $a_{CM} = \frac{dv_{CM}}{dt} = \frac{d^2 x_{CM}}{dt^2}$

(ix) **Centre of mass**: For many particle system:

Spectrum

(a) $X_{CM} = \frac{\Sigma m_i x_i}{M}$;

(b) $\vec{r}_{CM} = \frac{\Sigma m_i \vec{r}_i}{M}$;

(c) $\vec{v}_{CM} = \frac{d\vec{r}_{CM}}{dt}$;

(d) $\vec{a}_{CM} = \frac{d\vec{v}_{CM}}{dt}$;

(e) $\vec{P}_{CM} = M\vec{v}_{CM} = \Sigma m_i \vec{v}_i$;

(f) $\vec{F}_{ext} = M\vec{a}_{CM} = \Sigma m_i \vec{a}_i = \Sigma \vec{F}_i$. If $\vec{F}_{ext} = 0, \vec{a}_{CM} = 0, \vec{V}_{CM} = \text{constant}$;

(g) Also, moment of masses about CM is zero, i.e., $\Sigma m_i \vec{r}_i = 0$ or $m_1 r_1 = m_2 r_2$

(x) **Moment of Inertia**: (a) $I = \Sigma m_i r_i^2$
(b) $I = \mu r^2$, where $\mu = m_1 m_2/(m_1 + m_2)$

(xi) **Radius of gyration**: (a) $K = \sqrt{(I/M)}$; (b) $K = \sqrt{[(r_1^2 + r_2^2 + \dots + r_n^2)/n]}$ = root mean square distance.

(xii) **Kinetic energy of rotation**: $K = \frac{1}{2} I\omega^2$ or $I = (2K/\omega^2)$

(xiii) **Angular momentum**: (a) $\vec{L} = \vec{r} \times \vec{p}$; (b) $L = rp\sin\theta$; (c) mvd

(xiv) **Torque**: (a) $\vec{\tau} = \vec{r} \times \vec{F}$; (b) $\tau = rF\sin\theta$

(xv) **Relation between τ and L**: $\vec{\tau} = \left(d\vec{L}/dt\right)$;

(xvi) **Relation between L and I**: (a) $L = I\omega$; (b) $K = \frac{1}{2} I\omega^2 = L^2/2I$

(xvii) **Relation between τ and α**:
(a) $\tau = I\alpha$,
(b) If $\tau = 0$, then $(dL/dt)=0$ or L=constant or, $I\omega$=constant i.e., $I_1\omega_1 = I_2\omega_2$
(Laws of conservation of angular momentum)

(xviii) **Angular impulse**: $\Delta\vec{L} = \vec{\tau}\Delta t$

(xix) **Rotational work done**: $W = \int \tau\, d\theta = \tau_{av}\theta$

(xx) **Rotational Power**: $P = \vec{\tau}.\vec{\omega}$

(xxi) **(a) Perpendicular axes theorem**: $I_z = I_x + I_y$
(b) Parallel axes theorem: $I = I_c + Md^2$

(xxii) Moment of Inertia of some objects

(a) Ring: $I = MR^2$ (axis); $I = \frac{1}{2}MR^2$ (Diameter);
$I = 2\,MR^2$ (tangential to rim, perpendicular to plane);
$I = (3/2)\,MR^2$ (tangential to rim and parallel to diameter)

Spectrum

(b) Disc: $I = \frac{1}{2} MR^2$ (axis); $I = \frac{1}{4} MR^2$ (diameter)

(c) Cylinder: $I = \frac{1}{2} MR^2$ (axis)

(d) Thin rod: $I = (ML^2/12)$ (about centre); $I = (ML^2/3)$ (about one end)

(e) Hollow sphere : $I_{dia} = (2/3)\ MR^2$; $I_{tangential} = (5/3)\ MR^2$

(f) Solid sphere: $I_{dia} = (2/5)\ MR^2$; $I_{tangential} = (7/5)\ MR^2$

(g) Rectangular: $I_C = \frac{M(\ell^2 + b^2)}{12}$ (centre)

(h) Cube: $I = (1/6)\ Ma^2$

(i) Annular disc: $I = (1/2)\ M\ (R_1^2 + R_2^2)$

(j) Right circular cone: $I = (3/10)\ MR^2$

(k) Triangular lamina: $I = (1/6)\ Mh^2$ (about base axis)

(l) Elliptical lamina: $I = (1/4)\ Ma^2$ (about minor axis) and $I = (1/4)\ Mb^2$ (about major axis)

(xxiii) **Rolling without slipping on a horizontal surface**:

$$K = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}MV^2\left[1 + \frac{K^2}{R^2}\right] \qquad (\because\ V = R\omega \text{ and } I = MK^2)$$

For inclined plane

(a) Velocity at the bottom, $v = \sqrt{2gh \Big/ \left(1 + \frac{K^2}{R^2}\right)}$

(b) Acceleration, $a = g \sin\theta \Big/ \left(1 + \frac{K^2}{r^2}\right)$

(c) Time taken to reach the bottom, $t = \sqrt{2s\left(1 + \frac{K^2}{R^2}\right) \Big/ g\sin\theta}$

(xxiv) **Simple pendulum**: = $T = 2\pi\sqrt{(L/g)}$

(xxv) **Compound Pendulum**: $T = 2\pi\sqrt{(I/Mg\ \ell)}$, where $\ell = M\ (K^2 + \ell^2)$
Minimum time period, $T_0 = 2\pi\sqrt{(2K/g)}$

(xxvi) **Time period for disc**: $T = 2\pi\sqrt{(3R/2g)}$
Minimum time period for disc, $T = 2\pi\sqrt{(1.414R/g)}$

(xxvii) **Time period for a rod of length L pivoted at one end**: $T = 2\pi\sqrt{(2L/3g}$

Spectrum

CONSERVATION LAWS AND COLLISIONS

(i) **Work done**: (a) $W = \vec{F}.\vec{d}$; (b) $W = Fd\cos\theta$; (c) $W = \int_{x_1}^{x_2}$

(ii) **Conservation forces**: (a) $\int_a^b \vec{F}.d\vec{r}$ (Path 1) $=$ (b) $\int_a^b \vec{F}.d\vec{r}$ (Path 2); (c) $\oint_{\text{closed path}} \vec{F}.d\vec{r} = 0$

For conservative forces, one must have: $\vec{\nabla} \times \vec{F} = 0$

(iii) **Potential energy**: (a) $\vec{\nabla}U = -W$; (b) $F = -(dU/dX)$; (c) $\vec{F} = -\vec{\nabla}U$

(iv) **Gravitational potential energy**: (a) U = mgh ; (b) $U = -\dfrac{GMm}{(R+h)}$

(v) **Spring potential energy**: (a) $U = \frac{1}{2}Kx^2$; (b) $\Delta U = \frac{1}{2}K\left(x_2^2 - x_1^2\right)$

(vi) **Kinetic energy**: (a) $\Delta K = W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$; (b) $K = \frac{1}{2}mv^2$

(vii) **Total mechanical energy**: = E = K + U

(viii) **Conservation of energy**: $\Delta K = -\Delta U$ or, $K_f + U_f = K_i + U_i$

In an isolated system, E_{total} = constant

(ix) **Power**: (a) P = (dw/dt) ; (b) P = (dw/dt) ; (c) $P = \vec{F}.\vec{v}$

(x) **Tractive force**: F = (P/v)

(xi) **Equilibrium Conditions**:
- (a) For equilibrium, (dU/dx) = 0
- (b) For stable equilibrium: U(x) = minimum, (dU/dx) = 0 and (d^2U/dx^2) is positive
- (c) For unstable equilibrium: U(x) = maximum, (dU/dx) = 0 and (d^2U/dx^2) is negative
- (d) For neutral equilibrium: U(x) = constant, (dU/dx) = 0 and (d^2U/dx^2) is zero

(xii) **Velocity of a particle in terms of U(x)**: $v = \pm\sqrt{\dfrac{2}{m}[E - U(x)]}$

(xiii) **Momentum**:

(a) $\vec{p} = m\vec{v}$; (b) $\vec{F} = \left(d\vec{p}/dt\right)$,

(b) Conservation of momentum: If $\vec{F}_{net} = 0$, then $\vec{p}_f = \vec{p}_i$,

(c) Recoil speed of gun, $v_G = \dfrac{m_B}{m_G} \times v_B$

(xiv) **Impulse**: $\Delta\vec{p} = \vec{F}_{av}\,\Delta t$

(xv) **Collision in one dimension**:
- (a) Momentum conservation : $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
- (b) For elastic collision, e = 1 = coefficient of restitution
- (c) Energy conservation: $\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$
- (d) Velocities of 1st and 2nd body after collision are:

Spectrum

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_2 + m_1}\right)u_2; \quad v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_2 + m_1}\right)u_2$$

(e) If $m_1 = m_2 = m$, then $v_1 = u_2$ and $v_2 = u_1$

(f) Coefficient of restitution, $e = (v_2 – v_1/u_1 = u_2)$

(g) e = 1 for perfectly elastic collision and e=0 for perfectly inelastic collision. For inelastic collision $0 < e < 1$

(xvi) Inelastic collision of a ball dropped from height h_0

(a) Height attained after nth impact, $h_n = e^{2n}h_0$

(b) Total distance traveled when the ball finally comes to rest, $s = h_0 (1+e^2)/(1–e^2)$

(c) Total time taken, $t = \sqrt{\frac{2h_0}{g}}\left(\frac{1+e}{1-e}\right)$

(xvii) **Loss of KE in elastic collision**: For the first incident particle

$$\frac{K_f}{K_i} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 \text{ and } \frac{\Delta K_{lost}}{K_i} = \frac{4m_1m_2}{(m_1 + m_2)^2}; \text{ If } m_1 = m_2, \frac{\Delta K_{lost}}{K_i} = 100\%$$

(xviii) **Loss of KE in inelastic collision**: $\Delta K_{lost} = K_i – K_f = \frac{1}{2}\frac{m_1m_2}{m_1 + m_2}(u_1 – u_2)^2 (1–e^2)$

Velocity after inelastic collision (with target at rest)

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 \quad \text{and} \quad v_2 = \frac{m_1(1+e)}{m_1 + m_2}u_1$$

(xix) **Oblique Collision (target at rest)**:

$m_1u_1 = m_1v_1 \cos\theta_1 + m_2v_2 \cos\theta_2$ and $m_1v_1 \sin\theta_1 = m_2v_2 \sin\theta_2$

Solving, we get: $m_1u_1^2 = m_1v_1^2 + m_2v_2^2$

(xx) **Rocket equation**: (a) $M\frac{dV}{dt} = -v_{el}\frac{dM}{dt}$

(b) $V = – v_{rel} \log_e\left(\frac{M_0 - m_b}{M_0}\right)$ [M_0 = original mass of rocket plus fuel and m_b = mass of fuel burnt]

(c) If we write $M = M_0 – m_b$ = mass of the rocket and full at any time, than velocity of rocks at that time is:
$V = v_{rel} \log_e (M_0/M)$

(xxi) **Conservation of angular momentum**:

(a) If $\tau_{ext} = 0$, then $L_f = L_i$

(b) For planets, $\frac{v_{max}}{v_{min}} = \frac{r_{max}}{r_{min}}$

(c) Spinning skater, $I_1\omega_1 = I_2W_2$ or $\omega_f = \omega_i\left(\frac{I_i}{I_f}\right)$

Spectrum

SIMPLE HARMONIC MOTION AND LISSAJOUS FIGURES

(i) Simple Harmonic Motion:

(a) $F = -Kx$;

(b) $a = -\frac{K}{m}x$ or $a = -\omega^2 x$, where $\omega = \sqrt{(K/m)}$;

(c) $F_{max} = \pm KA$ and $a_{max} = \pm\omega^2 A$

(ii) **Equation of motion**: $\frac{d^2x}{dt^2} + \omega^2 x = 0$

(iii) **Displacement**: $x = A \sin(\omega t + \phi)$

(a) If $\phi = 0$, $x = A \sin \omega t$;

(b) If $\phi = \pi/2$, $x = A \cos \omega t$

(c) If $x = C \sin \omega t + D \cos \omega t$, then $x = A \sin(\omega t + \phi)$ with $A = \sqrt{(C^2+D^2)}$ and $\phi = \tan^{-1}(D/C)$

(iv) **Velocity**:

(a) $v = A\,\omega \cos(\omega + \phi)$;

(b) If $\phi=0$, $v = A\,\omega \cos \omega t$;

(c) $v_{max} = \pm\omega A$

(d) $v = \pm\,\omega\sqrt{(A^2 - x^2)}$;

(e) $\frac{x^2}{A^2} + \frac{v^2}{\omega^2 A^2} = 1$

(v) **Acceleration**:

(a) $a = -\omega^2 x = -\omega^2 A \sin(\omega t+\phi)$;

(b) If $\phi=0$, $a = -\omega^2 A \sin \omega t$

(c) $|a_{max}| = \omega^2 A$;

(d) $F_{max} = \pm m\,\omega^2 A$

(vi) **Frequency and Time period**:

(a) $\omega = \sqrt{(K/m)}$;

(b) $f = \frac{1}{2\pi}\sqrt{(K/m)}$;

(c) $T = 2\pi\sqrt{\frac{m}{K}}$

(vii) **Energy in SHM: Potential Energy**:

(a) $U = \frac{1}{2} Kx^2$;

(b) $F = -\frac{dU}{dx}$;

(c) $U_{max} = \frac{1}{2} m\omega^2 A^2$;

(d) $U = \frac{1}{2} m\omega^2 A^2 \sin^2 \omega t$

(viii) **Energy in SHM: Kinetic energy**:

(a) $K = \frac{1}{2} mv^2$;

(b) $K = \frac{1}{2} m\omega^2 (A^2 - x^2)$;

(c) $K = \frac{1}{2} m\omega^2 A^2 \cos^2 \omega t$;

(d) $K_{max} = \frac{1}{2} m\omega^2 A^2$

Spectrum

(ix) **Total energy**:
(a) $E = K + U$ = conserved;
(b) $E = (1/2)\, m\omega^2 A^2$;
(c) $E = K_{max} = U_{max}$

(x) **Average PE and KE**:
(a) $< U > = (1/4)\, m\omega^2 A^2$;
(b) $< K > = (1/4)\, m\omega^2 A^2$;
(c) $(E/2) = < U > = < K >$

(xi) Some relations:
(a) $\omega = \sqrt{\dfrac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}$; (b) $T = 2\pi\sqrt{\dfrac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$; (c) $A = \sqrt{\dfrac{(v_1 x_2)^2 - (v_2 x_1)^2}{v_1^2 - v_2^2}}$

(xii) Spring– mass system:
(a) $mg = Kx_0$;
(b) $T = 2\pi\sqrt{\dfrac{m}{K}} = 2\pi\sqrt{\dfrac{x_0}{g}}$

(xiii) **Massive spring**: $T = 2\pi\sqrt{\dfrac{m + (m_s/3)}{K}}$

(xiv) **Cutting a spring**:
(a) $K' = nK$;
(b) $T' = T_0/\sqrt{(n)}$;
(c) $f' = \sqrt{(n)}\, f_0$
(d) If spring is cut into two pieces of lengths ℓ_1 and ℓ_2 such that $\ell_1 = n\ell_2$, then $K_1 = \left(\dfrac{n+1}{n}\right)K$, $K_2 = (n+1)\,K$ and $K_1\ell_1 = K_2\ell_2$

(xv) **Springs in parallel**:
(a) $K = K_1 + K_2$;
(b) $T = 2\pi\sqrt{[m/(K_1 + K_2)]}$
(c) If $T_1 = 2\pi\sqrt{(m/K_1)}$ and $T_2 = 2\pi\sqrt{(m/K_2)}$, then for the parallel combination:

$$\frac{1}{T^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2} \quad \text{or} \quad T = \frac{T_1 T_2}{\sqrt{T_1^2 + T_2^2}} \quad \text{and} \quad \omega^2 = \omega_1^2 + \omega_2^2$$

(xvi) **Springs in series**:
(a) $K_1x_1 = K_2x_2 = Kx = F$ applied
(b) $\dfrac{1}{K} = \dfrac{1}{K_1} + \dfrac{1}{K_2}$ or $K = \dfrac{K_1K_2}{K_1 + K_2}$
(c) $\dfrac{1}{\omega^2} = \dfrac{1}{\omega_1^2} + \dfrac{1}{\omega_2^2}$ or $T^2 - T_1^2 + T_2^2$
(d) $T = 2\pi\sqrt{\dfrac{m(K_1 + K_2)}{K_1K_2}}$ or $f = \dfrac{1}{2\pi}\sqrt{\dfrac{K_1K_2}{m(K_1 + K_2)}}$

(xvii) **Torsional pendulum**:
(a) $I\alpha = \tau - C\theta$ or $\dfrac{d^2\theta}{dt^2} + \dfrac{C}{I}\theta = 0$;
(b) $\theta = \theta_0 \sin(\omega t + \phi)$;
(c) $\omega = \sqrt{(C/I)}$;

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(d) $f = \frac{1}{2\pi}\sqrt{\frac{C}{I}}$;

(e) $T = 2\pi\sqrt{(I/C)}$, where $C = \pi\eta r^4/2\ell$

(xviii) **Simple pendulum**:

(a) $I\alpha = \tau = -mg\ell \sin\theta$ or $\frac{d^2\theta}{dt^2} + \left(\frac{g}{\ell}\right)\sin\theta = 0$ or $\frac{d^2\theta}{dt^2} + \frac{g}{\ell}\theta = 0$;

(b) $\omega = \sqrt{(g/\ell)}$;

(c) $f = \frac{1}{2\pi}\sqrt{(g/\ell)}$;

(d) $T = 2\pi\sqrt{(\ell/g)}$

(xix) **Second pendulum**:

(a) T = 2 sec ;

(b) ℓ = 99.3 cm

(xx) **Infinite length pendulum**:

(a) $T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{\ell} + \frac{1}{R_e}\right)}}$;

(b) $T=2\pi\sqrt{\frac{R_e}{g}}$ (when $\ell \to \infty$)

(xxi) **Anharmonic pendulum**: $T \cong T_0\left(1 + \frac{\theta_0^2}{16}\right) \cong T_0\left(1 + \frac{A^2}{16\ell^2}\right)$

(xxii) **Tension in string of a simple pendulum**: $T = (3\ mg\cos\theta - 2\ mg\cos\theta_0)$

(xxiii) **Conical Pendulum**:

(a) $v = \sqrt{(gR\tan\theta)}$;

(b) $T = 2\pi\sqrt{(L\cos\theta/g)}$

(xxiv) **Compound pendulum**: $T = 2\pi\sqrt{\frac{(\ell + K^2/\ell)}{2}}$

(a) For a bar: $T = 2\pi\sqrt{(2L/3g)}$;

(b) For a disc : $T = 2\pi\sqrt{(3R/2g)}$

(xxv) **Floating cylinder**:

(a) $K = A\rho g$;

(b) $T = 2\pi\sqrt{(m/A\rho g)} = 2\pi\sqrt{(Ld/\rho g)}$

(xxvi) **Liquid in U–tube**:

(a) $K = 2A\rho g$ and $m = AL\rho$;

(b) $T = 2\pi\sqrt{(L/2g)} = 2\pi\sqrt{(h/g)}$

(xxvii) **Ball in bowl**: $T = 2\pi\sqrt{[(R - r)/g]}$

(xxviii) **Piston in a gas cylinder**:

(a) $K = \frac{A^2E}{V}$;

(b) $T = 2\pi\sqrt{\frac{mV}{A^2E}}$;

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(c) $T = 2\pi\sqrt{\frac{V_m}{A^2P}}$ (E–P for Isothermal process);

(d) $T = 2\pi\sqrt{\frac{V_m}{A^2\gamma P}}$ (E = γ P for adiabatic process)

(xxix) **Elastic wire**:

(a) $K = \frac{AY}{\ell}$;

(b) $T = 2\pi\sqrt{\frac{\ell m}{AY}}$

(xxx) **Tunnel across earth**: $T = 2\pi\sqrt{(R_e/g)}$

(xxxi) **Magnetic dipole in magnetic field**: $T = 2\pi\sqrt{(I/MB)}$

(xxxii) **Electrical LC circuit**: $T = 2\pi\sqrt{LC}$ or $f = \frac{1}{2\pi\sqrt{LC}}$

(xxxiii) **Lissajous figures** –

Case (a): $\omega_1 = \omega_2 = \omega$ or $\omega_1 : \omega_2 = 1 : 1$

General equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\phi = \sin^2\phi$

For $\phi = 0$: y = (b/a) x ; straight line with positive slope

For $\phi = \pi/4$: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{\sqrt{2}xy}{ab} = \frac{1}{2}$; oblique ellipse

For $\phi = \pi/2$: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; symmetrical ellipse

For $\phi = \pi$: y = –(b/a) x ; straight line with negative slope.

Case (b): For $\omega_1 : \omega_2$ = 2:1 with x = a sin ($2\omega t + \phi$) and y = b sin ωt

For $\phi = 0, \pi$: Figure of eight

For $\phi = \frac{\pi}{4}, \frac{3\pi}{4}$: Double parabola

For $\phi = \frac{\pi}{2}, \frac{3\pi}{2}$: Single parabola

Spectrum

GRAVITATION

(i) **Newton's law of gravitation**:

(a) $F = G\, m_1 m_2/r^2$; (b) $a = 6.67 \times 10^{-11}$ K.m^2/(kg)^2 ; (c) $\frac{dF}{F} = -\frac{2\,dr}{r}$

(ii) **Acceleration due to gravity** (a) $g = GM/R^2$; (b) Weight $W = mg$

(iii) **Variation of g**:

(a) due to shape ; $g_{equator} < g_{pole}$

(b) due to rotation of earth: (i) $g_{pole} = GM/R^2$ (No effect)

(ii) $g_{equator} = \frac{GM}{R^2} - \omega^2 R$

(iii) $g_{equator} < g_{pole}$

(iv) $\omega^2 R = 0.034$ m/s^2

(v) If $\omega \cong 17\ \omega_0$ or $T = (T_0/17) = (24/17)h = 1.4$ h, then object would float on equator

(c) At a height h above earth's surface $g' = g\left(1 - \frac{2h}{g}\right)$, if $h << R$

(d) At a depth of below earth's surface: $g' = g\left(1 - \frac{d}{R}\right)$

(iv) **Acceleration on moon**: $g_m = \frac{GM_m}{R_m^2} \cong \frac{1}{6} g_{earth}$

(v) **Gravitational field**: (a) $\vec{g} = -\frac{GM}{r^2}\hat{r}$ (outside); (b) $\vec{g} = -\frac{GM}{R^3} r\hat{r}$ (inside)

(vi) **Gravitational potential energy of mass m**:

(a) At a distance r : $U(r) = -GMm/r$

(b) At the surface of the earth: $U_0 = -GMm/R$

(c) At any height h above earth's surface: $U - U_0 = mgh$ (for $h << R$)

or $U = mgh$ (if origin of potential energy is shifted to the surface of earth)

(vii) **Potential energy and gravitational force**: $F = -(dU/dR)$

(viii) **Gravitational potential**: $V(r) = -GM/r$

(ix) **Gravitational potential energy of system of masses**:

(a) Two particles: $U = -Gm_1m_2/r$

(b) Three particles: $U = -\frac{Gm_1m_2}{r_{12}} - \frac{Gm_1m_3}{r_{13}} - \frac{Gm_2m_3}{r_{23}}$

(x) **Escape velocity**:

(a) $v_e = \sqrt{\frac{2GM}{R}}$ or $v_e = \sqrt{(2gR)} = \sqrt{(gD)}$

(b) $v_e = R\sqrt{\frac{8\pi G\rho}{3}}$

(xi) **Maximum height attained by a projectile**:

$h = \frac{R}{(v_e/v)^2 - 1}$ or $v = v_e\sqrt{\frac{h}{R+h}} \cong v_e\sqrt{\frac{h}{R}}$ (if $h << R$)

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Spectrum

(xii) **Orbital velocity of satellite**:

$(a)\, v_0 = \sqrt{\frac{GM}{r}}$; $(b)\; v_0 = v_e\sqrt{\frac{R}{2(R+h)}}$; (c) $v_0 \cong v_e/\sqrt{2}$ (if h<<R)

(xiii) **Time period of satellite**: (a) $T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$; (b) $T = 2\pi\sqrt{\frac{R}{g}}$ (if h << R)

(xiv) **Energy of satellite**: (a) Kinetic energy K = $\frac{1}{2}mv_0^2 = \frac{1}{2}\frac{GMm}{r}$

(b) Potential energy U = $-\frac{GMm}{r} = -2K$;

(c) Total energy E=K + U= $-\frac{1}{2}\frac{GMm}{r}$;

(d) E = U/2 = – K ; (e) BE = –E = $\frac{1}{2}\frac{GMm}{r}$

(xv) **Geosynchronous satellite**: (a) T = 24 hours ; (b) $T^2 = \frac{4\pi^2}{GM}(R+h)^3$;

(c) $h = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} - R$; (d) $h \cong 36{,}000$ km.

(xvi) **Kepler's law**:

(a) Law of orbits: Orbits are elliptical
(b) Law of areas: Equal area is swept in equal time
(c) Law of period: $T^2 \propto r^3$; $T^2 = (4\pi^2/GM)r^3$

SURFACE TENSION

(i) (a) $T = \frac{\text{Force}}{\text{Length}} = \frac{F}{\ell}$; (b) $T = \frac{\text{Surface energy}}{\text{Surface area}} = \frac{W}{A}$

(ii) **Combination of n drops into one big drop**: (a) $R = n^{1/3}r$

(b) $E_i = n(4\pi r^2 T)$, $E_f = 4\pi R^2 T$, $(E_f/E_i) = n^{-1/3}$, $\frac{\Delta E}{E_i} = \left(1 - \frac{1}{n^{1/3}}\right)$

(c) $\Delta E = 4\pi R^2 T\,(n^{1/3} - 1) = 4\pi R^3 T \left(\frac{1}{r} - \frac{1}{R}\right)$

(iii) **Increase in temperature**: $\Delta\theta = \frac{3T}{\rho s}\left(\frac{1}{r} - \frac{1}{R}\right)$ or $\frac{3T}{\rho s J}\left(\frac{1}{r} - \frac{1}{R}\right)$

(iv) **Shape of liquid surface**:

(a) Plane surface (as for water – silver) if $F_{adhesive} > \frac{F_{cohesive}}{\sqrt{2}}$

(b) Concave surface (as for water – glass) if $F_{adhesive} > \frac{F_{cohesive}}{\sqrt{2}}$

(c) Convex surface (as for mercury–glass) if $F_{adhesive} < \frac{F_{cohesive}}{\sqrt{2}}$

(v) **Angle of contact**:

(a) Acute: If $F_a > F_c/\sqrt{2}$;

(b) obtuse: if $F_a < F_c/\sqrt{2}$;

(c) $\theta_c = 90^0$: if $F_a = F_c\sqrt{}/2$

(d) $\cos\theta_c = \frac{T_{sa} - T_{s\ell}}{T_{\ell a}}$, (where T_{sa}, $T_{s\ell}$ and $T_{\ell a}$ represent solid-air, solid- liquid and liquid-air surface tensions respectively). Here θ_c is acute if $T_{s\ell} < T_{sa}$ while θ_c is obtuse if $T_{s\ell} > T_{sa}$

(vi) **Excess pressure**:

(a) General formula: $P_{excess} = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

(b) For a liquid drop: $P_{excess} = 2T/R$

(c) For an air bubble in liquid: $P_{excess} = 2T/R$

(d) For a soap bubble: $P_{excess} = 4T/R$

(e) Pressure inside an air bubble at a depth h in a liquid: $P_{in} = P_{atm} + hdg + (2T/R)$

(vii) **Forces between two plates with thin water film separating them**:

(a) $\Delta P = T\left(\frac{1}{r} - \frac{1}{R}\right)$;

(b) $F = AT\left(\frac{1}{r} - \frac{1}{R}\right)$;

(c) If separation between plates is d, then $\Delta P = 2T/d$ and $F = 2AT/d$

(viii) **Double bubble**: Radius of Curvature of common film $R_{common} = \frac{rR}{R - r}$

(ix) **Capillary rise**:

(a) $h = \frac{2T\cos\theta}{rdg}$;

(b) $h = \frac{2T}{rdg}$ (For water $\theta = 0^0$)

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(c) If weight of water in meniseus is taken into account then T = $\dfrac{rdg\left(h+\dfrac{r}{3}\right)}{2\cos\theta}$

(d) Capillary depression, $h - \dfrac{2T\cos(\pi-\theta)}{rdg}$

(x) **Combination of two soap bubbles**:

(a) If ΔV is the increase in volume and ΔS is the increase in surface area, then $3P_0\Delta V + 4T\Delta S = 0$ where P_0 is the atmospheric pressure

(b) If the bubbles combine in environment of zero outside pressure isothermally, then $\Delta S = 0$ or $R_3 = \sqrt{\left(R_1^2 + R_2^2\right)}$

ELASTICITY

(i) **Stress**: (a) Stress = [Deforming force/cross–sectional area];

(b) Tensile or longitudinal stress = $(F/\pi r^2)$;

(c) Tangential or shearing stress = (F/A);

(d) Hydrostatic stress = P

(ii) **Strain**: (a) Tensile or longitudinal strain = $(\Delta L/L)$;

(b) Shearing strain = ϕ;

(c) Volume strain = $(\Delta V/V)$

(iii) **Hook's law**:

(a) For stretching: Stress = Y x Strain or $Y = \dfrac{FL}{A(\Delta L)}$

(b) For shear: Stress = η x Strain or $\eta = F/A\phi$

(c) For volume elasticity: Stress = B x Strain or B = $-\dfrac{P}{(\Delta V/V)}$

(iv) **Compressibility**: K = (1/B)

(v) **Elongation of a wire due to its own weight**: $\Delta L = \dfrac{1}{2}\dfrac{MgL}{YA} = \dfrac{1}{2}\dfrac{L^2\rho g}{Y}$

(vi) **Bulk modulus of an idea gas**: $B_{isothermal} = P$ and $B_{adiabatic} = \gamma P$ (where $\gamma = C_p/C_v$)

(vii) Stress due to heating or cooling of a clamped rod
Thermal stress = $Y\alpha\,(\Delta t)$ and force = $YA\,\alpha\,(\Delta t)$

(viii) **Torsion of a cylinder**:

(a) $r\,\theta = \ell\phi$ (where θ = angle of twist and ϕ = angle of shear);

(b) restoring torque $\tau = c\theta$

(c) restoring Couple per unit twist, $c = \pi\eta r^4/2\ell$ (for solid cylinder)
and $C = \pi\eta\,(r_2^4 - r_1^4)/2\ell$ (for hollow cylinder)

(ix) **Work done in stretching**:

(a) W = $\dfrac{1}{2}$ x stress x strain x volume = $\dfrac{1}{2}Y$ (strain)^2 x volume = $\dfrac{1}{2}\dfrac{(\text{stress})^2}{Y}$ x volume

(b) Potential energy stored, U = W = $\dfrac{1}{2}$ x stress x strain x volume

(c) Potential energy stored per unit volume, u = $\dfrac{1}{2}$ x stress x strain

(x) **Loaded beam**:

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(a) depression, $\delta = \frac{W\ell^3}{4Ybd^3}(\text{rectangular})$

(b) Depression, $\delta = \frac{W\ell^3}{12Y\pi r^2}(\text{cylindrical})$

(xi) **Position's ratio**:

(a) Lateral strain = $-\frac{\Delta D}{D} = \frac{-\Delta r}{r}$

(b) Longitudinal strain = $(\Delta L/L)$

(c) Poisson's ratio $\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{-\Delta r/r}{\Delta L/L}$

(d) Theoretically, $-1 < \sigma < 0.5$ but experimentally $\sigma \cong 0.2 - 0.4$

(xii) **Relations between Y, η, B and σ**:

(a) $Y = 3B(1-2\sigma)$;

(b) $Y = 2\eta(1+\sigma)$;

(c) $\frac{1}{Y} = \frac{1}{9B} + \frac{1}{3\eta}$

(xiii) **Interatomic force constant**: $k = Yr_0$ (r_0 = equilibrium inter atomic separation)

KINETIC THEORY OF GASES

(i) **Boyle's law**: PV = constant or $P_1V_1 = P_2V_2$

(i) **Chare's law**: (V/T) = constant or $(V_1/T_1) = (V_2/T_2)$

(ii) **Pressure – temperature law**: $(P_1/T_1) = (P_2/T_2)$

(iii) **Avogadro's principle**: At constant temperature and pressure, Volume of gas,
$V \propto$ number of moles, μ
Where $\mu = N/Na$ [N = number of molecules in the sample
and N_A = Avogadro's number = 6.02×10^{23}/mole]

$= \frac{M_{sample}}{M}$ [M_{sample} = mass of gas sample and M = molecular weight]

(iv) **Kinetic Theory**:

(a) Momentum delivered to the wall perpendicular to the x–axis, $\Delta P = 2m\, v_x$

(b) Time taken between two successive collisions on the same wall by the same molecule: $\Delta t = (2L/v_x)$

(c) The frequency of collision: $\nu_{coll.} = (v_x/2L)$

(d) Total force exerted on the wall by collision of various molecules: $F = (MN/L) \langle v_x^2 \rangle$

(e) The pressure on the wall : $P = \frac{mN}{V} \langle v_x^2 \rangle = \frac{mN}{3V} \langle v^2 \rangle = \frac{1}{3}\frac{mN}{V} v_{rms}^2 = \frac{1}{3}\rho v_{rms}^2$

(v) **RMS speed**:

(a) $v_{rms} = \sqrt{(v_1^2 + v_2^2 + \ldots + v_N^2/N)}$;

(b) $v_{rms} = \sqrt{(3P/\rho)}$;

(c) $v_{rms} = \sqrt{(3KT/m)}$;

(d) $v_{rms} = \sqrt{(3RT/M)}$; (e) $\frac{(v_{rms})_1}{(v_{rms})_2} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{M_2}{M_1}}$

(vi) **Kinetic interpretation of temperature**:

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(a) $(1/2)\ Mv^2_{rms} = (3/2)\ RT$;
(b) $(1/2)\ mv^2_{rms} = (3/2)\ KT$
(c) Kinetic energy of one molecule = (3/2) KT ;
(d) kinetic energy of one mole of gas = (3/2) RT
(e) Kinetic energy of one gram of gas (3/2) (RT/M)

(ix) Maxwell molecular speed distribution:

(a) $n(v) = 4\pi N \left(\frac{m}{2\pi KT}\right)^{3/2} v^2 e^{-mv^2/2KT}$

(b) The average speed: $\bar{v} = \sqrt{\frac{8KT}{\pi m}} = \sqrt{\frac{8RT}{\pi M}} = 1.60\sqrt{\frac{RT}{M}}$

(c) The rms speed: $v_{rms} = \sqrt{\frac{3kT}{m}} = \frac{\sqrt{3RT}}{M} = 1.73\sqrt{\frac{RT}{M}}$

(d) The most probable speed: $v_p = \sqrt{\frac{2KT}{m}} = \sqrt{\frac{2RT}{M}} = 1.41\sqrt{\frac{RT}{M}}$

(e) Speed relations: (I) $v_p < \bar{v} < v_{rms}$
(II) $v_p : \bar{v} : v_{rms} = \sqrt{(2)} : \sqrt{(8/\pi)} : \sqrt{(3)} = 1.41 : 1.60 : 1.73$

(x) Internal energy:
(a) $E_{internal} = (3/2)RT$ (for one mole)
(b) $E_{internal} = (3/2\ \mu RT$ (for μ mole)
(c) Pressure exerted by a gas $P = \frac{2}{3}\frac{E}{V} = \frac{2}{3}\bar{E}$

(xi) Degrees of freedom:
(a) Ideal gas: 3 (all translational)
(b) Monoatomic gas : 3 (all translational)
(c) Diatomic gas: 5 (three translational plus two rotational)
(d) Polyatomic gas (linear molecule e.g. CO_2) : 7 (three translational plus two rotational plus two vibrational)
(e) Polyatomic gas (non–linear molecule, e.g., NH_3, H_2O etc): 6 (three translational plus three rotational)
(f) Internal energy of a gas: $E_{internal} = (f/2)\ \mu RT$. (where f = number of degrees of freedom)

(xii) Dalton's law: The pressure exerted by a mixture of perfect gases is the sum of the pressures exerted by the individual gases occupying the same volume alone i.e., $P = P_1 + P_2 + \ldots$.

(xiii) Van der Wall's gas equation:

(a) $\left(P + a\frac{\mu^2}{V^2}\right)(V - \mu b) = \mu RT$

(b) $\left(P + a\frac{\mu^2}{V_m^2}\right)(V_m - b) = RT$ (where $V_m = V/\mu$ = volume per mole);

(c) $b = 30\ cm^3/mole$

(d) Critical values: $P_c = \frac{a}{27b^2}, V_C = 3b, T_C = \frac{8a}{27Rb}$;

(e) $\frac{P_C V_C}{RT_C} = \frac{3}{8} = 0.375$

(xiv) Mean free path: $\lambda = \frac{1}{\sqrt{2\pi d^2 \rho n}}$,

Where ρ_n = (N/V) = number of gas molecules per unit volume and
d = diameter of molecules of the gas

Spectrum

FLUID MECHANICS

(i) The viscous force between two layers of area A having velocity gradient (dv/dx) is given by: $F = -\eta A\,(dv/dx)$, where η is called coefficient of viscosity

(i) In SI system, η is measured I Poiseiulle ($P\ell$) $1P\ell = 1Nsm^{-2}$ = 1 decapoise. In egs system, the unit of η is g/cm/sec and is called POISE

(ii) When a spherical body is allowed to fall through viscous medium, its velocity increases, till the sum of viscous drag and upthrust becomes equal to the weight of the body. After that the body moves with a constant velocity called terminal velocity.

(iii) According to STOKE's Law, the viscous drag on a spherical body moving in a fluid is given by: $F = 6\pi\eta r v$, where r is the radius and v is the velocity of the body.

(iv) The terminal velocity is given by: $v_T = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$

where ρ is the density of the material of the body and σ is the density of liquid

(v) Rate of flow of liquid through a capillary tube of radius r and length ℓ

$$V = \frac{\pi p r^4}{8\eta\ell} = \frac{p}{8\eta\ell/\pi r^4} = \frac{p}{R}$$

where p is the pressure difference between two ends of the capillary and R is the fluid resistance ($=8\,\eta\ell/\pi r^4$)

(vi) The matter which possess the property of flowing is called as FLUID (For example, gases and liquids)

(vii) Pressure exerted by a column of liquid of height h is : $P = h\rho g$ (ρ = density of the liquid)

(viii) Pressure at a point within the liquid, $P = P_0 + h\rho g$, where P_0 is atmospheric pressure and h is the depth of point w.r.t. free surface of liquid

(ix) Apparent weight of the body immersed in a liquid $Mg' = Mg - V\rho g$

(x) If W be the weight of a body and U be the upthrust force of the liquid on the body then

(a) the body sinks in the liquid of W > U

(b) the body floats just completely immersed if W = U

(c) the body floats with a part immersed in t

(xi) $\frac{\text{Volume of immersed part of a solid}}{\text{total volume of solid}} = \frac{\text{density of solid}}{\text{density of solid}}$

(xii) Equation of Continuity: $a_1v_1 = a_2v_2$

(xiii) Bernouilli's theorem: $(P/\rho) + gh + \frac{1}{2}v^2$ = constant

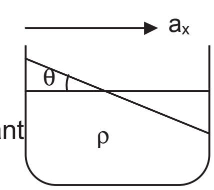


Fig. 4

(xiv) Accelerated fluid containers : $\tan\theta = \frac{a_x}{g}$

(xv) Volume of liquid flowing per second through a tube: $R = a_1v_1 = a_2v_2\sqrt{\frac{2gh}{(a_1^2 - a_2^2)}}$

(xvi) Velocity of efflux of liquid from a hole:

$v = \sqrt{(2gh)}$, where h is the depth of a hole from the free surface of liquid

Spectrum

HEAT AND THERMODYNAMICS

(i) $L_2 - L_1 = L_1\alpha(T_2 - T_1)$; $A_2 - A_1 = A_2\,\beta(T_2 - T_1)$; $V_2 - V_1 = V_1\gamma(T_2 - T_1)$
where, L_1, A_1, V_1 are the length, area and volume at temperature T_1; and L_2, A_2, V_2 are that at temperature T_2.α represents the coefficient of linear expansion, β the coefficient of superficial expansion and γ the coefficient of cubical expansion.

(ii) If d_t be the density at t^0C and d_0 be that at 0^0C, then: $d_t = d_0\,(1-\gamma\Delta T)$

(iii) $\alpha : \beta : \gamma = 1 : 2 : 3$

(iv) If γ_r, γ_a be the coefficients of real and apparent expansions of a liquid and γ_g be the coefficient of the cubical expansion for the containing vessel (say glass), then
$\gamma_r = \gamma_a + \gamma_g$

(v) The pressure of the gases varies with temperature as : $P_t = P_0\,(1+\gamma\Delta T)$, where $\gamma = (1/273)$ per ^{0}C

(vi) If temperature on Celsius scale is C, that on Fahrenheit scale is F, on Kelvin scale is K, and on Reaumer scale is R, then

(a) $\frac{C}{5} = \frac{F-32}{9} = \frac{K-273}{5} = \frac{R}{4}$ (b) $F = \frac{9}{5}C + 32$

(c) $C = \frac{5}{9}(F-32)$

(d) $K = C + 273$ (e) $K = \frac{5}{9}(F + 459.4)$

(vii) (a) Triple point of water = 273.16 K
(b) Absolute zero = 0 K = –273.15^{0}C
(c) For a gas thermometer, $T = (273.15)\,\frac{P}{P_{triple}}\,(\text{Kelvin})$
(d) For a resistance thermometer, $R_\theta = R_0\,[1+\alpha\theta]$

(viii) If mechanical work W produces the same temperature change as heat H, then we can write:
$W = JH$, where J is called mechanical equivalent of heat

(ix) The heat absorbed or given out by a body of mass m, when the temperature changes by ΔT is: $\Delta Q = mc\Delta T$, where c is a constant for a substance, called as SPECIFIC HEAT.

(x) HEAT CAPACITY of a body of mass m is defined as : $\Delta Q = mc$

(xi) WATER EQUIVALENT of a body is numerically equal to the product of its mass and specific heat i.e., $W = mc$

(xii) When the state of matter changes, the heat absorbed or evolved is given by: $Q = mL$, where L is called LATENT HEAT

(xiii) In case of gases, there are two types of specific heats i.e., c_p and c_v [c_p = specific heat at constant pressure and C_v = specific heat at constant volume]. Molar specific heats of a gas are: $C_p = Mc_p$ and $C_v = Mc_v$, where M = molecular weight of the gas.

(xiv) $C_p > C_v$ and according to Mayer's formula $C_p - C_v = R$

(xv) For all thermodynamic processes, equation of state for an ideal gas: $PV = \mu RT$

(a) For ISOBARIC process: P = Constant ; $\frac{V}{T}$ =Constant

(b) For ISOCHORIC (Isometric) process: V = Constant; $\frac{P}{T}$ =Constant

(c) For ISOTHERMAL process T = Constant ; PV= Constant

(d) For ADIABATIC process: PV^γ = Constant ; $TV^{\gamma-1}$=Constant
and $P^{(1-\gamma)}\,T^\gamma$ = Constant

(xvi) Slope on PV diagram

(a) For isobaric process: zero
(b) For isochoric process: infinite
(c) For isothermal process: slope = –(P/V)
(d) For adiabatic process: slope = –γ(P/V)
(e) Slope of adiabatic curve > slope of isothermal curve.

(xvii) Work done

(a) For isobaric process: $W = P(V_2 - V_1)$
(b) For isochoric process: W = 0
(c) For isothermal process: $W = \mu RT \log_e (V_2/V_1)$
$\mu RT \times 2.303 \times \log_{10}(V_2/V_1)$
$P_1V_1 \times 2.303 \times \log_{10}(V_2/V_1)$
$\mu RT \times 2.303 \times \log_{10}(P_1/P_2)$
(d) For adiabatic process: $W = \frac{\mu R(T_1 - T_2)}{(\gamma - 1)} = \frac{(P_1V_1 - P_2V_2)}{(\gamma - 1)}$
(e) In expansion from same initial state to same final volume

$$W_{adiabatic} < W_{isothermal} < W_{isobaric}$$

(f) In compression from same initial state to same final volume:

$$W_{adiabatic} < W_{isothermal} < W_{isobaric}$$

(xviii) **Heat added or removed**:

(a) For isobaric process: $Q = \mu C_p \Delta T$
(b) For isochoric process = $Q = \mu C_v \Delta T$
(c) For isothermal process = $Q = W = \mu Rt \log_e (V_2/V_1)$
(d) For adiabatic process: Q = 0

(xix) Change in internal energy

(a) For isobaric process = $\Delta U = \mu C_v \Delta T$
(b) For isochoric process = $\Delta U = \mu C_v \Delta T$
(c) For isothermal process = $\Delta U = 0$
(d) For adiabatic process: $\Delta U = -W = \frac{\mu R(T_2 - T_1)}{(\gamma - 1)}$

(xx) Elasticities of gases

(a) Isothermal bulk modulus = $B_I = P$
(b) Adiabatic bulk modulus $B_A = \gamma_P$

(xxi) For a CYCLIC process, work done ΔW = area enclosed in the cycle on PV diagram. Further, ΔU = 0 (as state of the system remains unchanged)
So, ΔQ = ΔW

(xxii) Internal energy and specific heats of an ideal gas (Monoatomic gas)

(a) $U = \frac{3}{2}RT$ (for one mole);

(b) $U = \frac{3}{2}\mu RT$ (for μ moles)

(c) $\Delta U = \frac{3}{2}\mu R\Delta T$ (for μ moles);

(d) $C_v = \frac{1}{\mu}\left(\frac{\Delta U}{\Delta T}\right) = \frac{3}{2}R$

Spectrum

(e) $C_p = Cv + R = \frac{3}{2}R + R = \frac{5}{2}R$

(f) $\gamma = \left(\frac{C_p}{C_v}\right) = \left(\frac{5}{2}R \Big/ \frac{3}{2}R\right) = \frac{5}{3} = 1.67$

(xxiii) Internal energy and specific heats of a diatomic gas

(a) $U = \frac{5}{2}\mu RT$ (for μ moles);

(b) $\Delta U = \frac{5}{2}\mu R\Delta T$ (for μ moles)

(c) $C_v = \frac{1}{\mu}\left(\frac{\Delta U}{\Delta T}\right) = \frac{5}{2}R;$

(d) $C_p = C_v + R = \frac{5}{2}R + R = \frac{7}{2}R$

(e) $\gamma = \left(\frac{C_p}{C_v}\right) = \left(\frac{7R}{2} \Big/ \frac{5R}{2}\right) = \frac{7}{5} = 1.4$

(xxiv) **Mixture of gases**: $\mu = \mu_1 + \mu_2$

$$M = \frac{\mu_1 M_1 + \mu_2 + M_2}{\mu_1 + \mu_2} = \frac{N_1 m_1 + N_2 m_2}{N_1 + N_2}$$

$$C_v = \frac{\mu_1 C_{v_1} + \mu_2 C_{v_2}}{\mu_1 + \mu_2} \quad \text{and} \quad C_p = \frac{\mu_1 C_{p_1} + \mu_2 C_{p_2}}{\mu_1 + \mu_2}$$

(xxv) First law of thermodynamics

(a) $\Delta Q = \Delta U + \Delta W$ or $\Delta U = \Delta Q - \Delta W$
(b) Both ΔQ, ΔW depends on path, but ΔU does not depend on the path
(c) For isothermal process: $\Delta Q = \Delta W = \mu RT \log | V_2/V_1|$, $\Delta U = 0$, T = Constant, PV = Constant and $C_{iso} = \pm\infty$
(d) For adiabatic process: $\Delta W = \frac{\mu R(T_2 - T_1)}{(1-\gamma)}$, $\Delta Q = 0$, $\Delta U = \mu C_v (T_2 - T_1)$, Q = 0,

PV^γ = constant, $C_{ad} = 0$ and $\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$

(where f is the degree of freedom)

(e) For isochoric process: $\Delta W = 0$, $\Delta Q = \Delta U = \mu C_v\Delta T$, V = constant, and $C_v = (R/\gamma-1)$
(f) For isobaric process: $\Delta Q = \mu C_p\Delta T$, $\Delta U = \mu C_v\Delta T$., $\Delta W = \mu R\Delta T$, P = constant and $C_p = (\gamma R/\gamma-1)$
(g) For cyclic process: $\Delta U = 0$, $\Delta Q = \Delta W$
(h) For free expansion: $\Delta U = 0$, $\Delta Q = 0$, $\Delta W = 0$
(i) For polytropic process: $\Delta W = [\mu R(T_2-T_1)/1-n]$, $\Delta Q = \mu C (T_2-T_1)$, PV^n = constant and

$$C = \frac{R}{\gamma - 1} + \frac{R}{1-n}$$

(xxvi) Second law of thermodynamics
(a) There are no perfect engines
(b) There are no perfect refrigerators

Spectrum

(c) Efficiency of carnot engine: $\eta = 1 - \frac{Q_2}{Q_1}, \frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

(d) Coefficient of performance of a refrigerator:

$$\beta = \frac{\text{Heat absorbed from cold reservoir}}{\text{Work done on refrigerator}} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

For a perfect refrigerator, $W = 0$ or $Q_1 = Q_2$ or $\beta = \infty$

(xxvii) **The amount of heat transmitted is given by**: $Q = -KA \frac{\Delta\theta}{\Delta x} t$, where K is coefficient of thermal conductivity, A is the area of cross section, $\Delta\theta$ is the difference in temperature, t is the time of heat flow and Δx is separation between two ends

(xxviii) **Thermal resistance of a conductor of length** $d = R_{Th} = \frac{d}{KA}$

(xxix) **Flow of heat through a composite conductor**:

(a) Temperature of interface, $\theta = \frac{(K_1\theta_1/d_1) + (K_2\theta_2/d_2)}{(K_1/d_1) + (K_2/d_2)}$

(b) Rate of flow of heat through the composite conductor: $H = \frac{Q}{t} = \frac{A(\theta_1 - \theta_2)}{(d_1/K_1) + (d_2/K_2)}$

(c) Thermal resistance of the composite conductor

$$R_{TH} = \frac{d_1}{K_1 A} + \frac{d_2}{K_2 A} = (R_{Th})_1 + (R_{Th})_2$$

(d) Equivalent thermal conductivity, $K = \frac{d_1 + d_2}{(d_1/K_1) + (d_2/K_2)}$

(xxx) (a) Radiation absorption coefficient: $a = Q_0/Q_0$

(b) Reflection coefficient: $r = Q_r/Q_0$

(c) Transmission coefficient: $t = Q_t/Q_0$

(d) Emissive power: e or $E = Q/A.t$ [t = time]

(e) Spectral emissive power: $e_\lambda = \frac{Q}{At(d\lambda)}$ and $e = e_\lambda e \int_0^\infty$

(f) Emissivity: $\varepsilon = e/E$; $0 \le \varepsilon \le 1$

(g) Absorptive power: $a = Q_a/Q_0$

(h) Kirchhoffs law: $(e_\lambda/a_\lambda)1 = (e_\lambda/a_\lambda)_B = \ldots\ldots\ldots = E_\lambda$

(i) Stefan's law: (a) $E = \sigma T^4$ (where $\sigma = 5.67 \times 10^{-8}\ Wm^{-2} K^{-4}$)

For a black body: $E = \sigma (T^4 - T_0^4)$

For a body: $e = \varepsilon\sigma (T^4 - T_0^4)$

(j) Rate of loss of heat: $-\frac{dQ}{dt} = \varepsilon A \sigma (\theta^4 - \theta_0^4)$

For spherical objects: $\frac{(dQ/dt)_1}{(dQ/dt)_2} = \frac{r_1^2}{r_2^2}$

(k) Rate of fall of temperature: $\frac{d\theta}{dt} = \frac{\varepsilon A \sigma}{ms}(\theta^4 - \theta_0^4) = \frac{\varepsilon A \sigma}{V\rho s}(\theta^4 - \theta_0^4)$

$\therefore \frac{(d\theta/dt)_1}{(d\theta/dt)_2} = \frac{A_1}{A_2} \times \frac{V_2}{V_1} = \frac{r_2}{r_1}$

Spectrum

(For spherical bodies)

(l) Newton's law of cooling: $\frac{d\theta}{dt} = -K(\theta - \theta_0)$ or $(\theta - \theta_0) \alpha e^{-KT}$

(m) Wein's displacement law: $\lambda_m T = b$ (where $b = 2.9 \times 10^{-3}$ m – K)

(n) Wein's radiation law: $E_\lambda d\lambda = \left(\frac{A}{\lambda^5}\right) f(\lambda T)\, d\lambda = \left(\frac{A}{\lambda^5}\right) e^{-a/\lambda T} d\lambda$

(o) Solar Constant: $S = \left(\frac{R_S}{R_{ES}}\right)^2 \sigma T^4$ or $T = \left(\frac{S}{\sigma}\right)^{1/4} \left(\frac{R_{ES}}{R_S}\right)^{1/2}$

Spectrum

WAVES

1. **Velocity**: $v = n\lambda$ and $n = (1/T)$

2. **Velocity of transverse waves in a string**: $v = \sqrt{\frac{T}{m}} = \sqrt{\frac{T}{\pi r^2 d}}$

3. **Velocity of longitudinal waves**:
   (a) In rods: $v = \sqrt{(Y/\rho)}$ (Y – Young's modulus, ρ = density)
   (b) In liquids: $v = \sqrt{(B/\rho)}$ (B = Bulk modulus)
   (c) In gases: $v = \sqrt{(\gamma P/\rho)}$ (Laplace formula)

4. **Effect of temperature**:
   (a) $v = v_0\sqrt{(T/273)}$ or $v = v_0 + 0.61t$
   (b) $(v_{sound}/v_{rms}) = \sqrt{(\gamma/3)}$

5. **Wave equation**: (a) $y = a \sin \frac{2\pi}{\lambda}(vt-x)$

   (b) $y = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$

   (c) $y = a \sin(\omega t - kx)$, where wave velocity $v = \frac{\omega}{k} = n\lambda$

6. **Particle velocity**: (a) $v_{particle} = (\partial y/\partial t)$
   (b) maximum particle velocity, $(v_{particle})max = \omega a$

7. **Strain in medium** (a) strain $= -(\partial y/\partial x) = ka \cos(\omega t - kx)$
   (b) Maximum strain $= (\partial y/dx)_{max} = ka$
   (c) $(v_{particle}/\text{strain}) = (\omega/k)$ = wave velocity
   i.e., $v_{particle}$ = wave velocity x strain in the medium

8. **Wave equation**: $\frac{\partial^2 y}{\partial t^2} = v^2\left(\frac{\partial^2 y}{\partial x^2}\right)$

9. **Intensity of sound waves**:
   (a) $I = (E/At)$
   (b) If ρ is the density of the medium; v the velocity of the wave; n the frequency and a the amplitude then $I = 2\pi^2 \rho v n^2 a^2$ i.e. $I \propto n^2a^2$
   (c) Intensity level is decibel: β 10 log (I/I_0). Where, I_0 =Threshold of hearing = 10^{-12} Watt/m^2

10. **Principle of superposition**: $y = y_1 + y_2$

11. **Resultant amplitude**: $a = \sqrt{(a_1^2 + a_2^2 + 2a_1a^2 \cos\phi)}$

12. **Resultant intensity**: $I = I_1 + I_2 + 2\sqrt{(I_1I_2 \cos\phi)}$
   (a) For constructive interference: $\phi = 2n\pi$, $a_{max} = a_1 + a_2$ and $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$
   (b) For destructive interference: $\phi = (2n-1)\pi$, $a_{min} = a_2 - a_2$ and $I_{min} = (\sqrt{I_1}=\sqrt{I_2})^2$

13. (a) Beat frequency $= n_1 - n_2$ and beat period $T = (T_1T_2/T_2-T_1)$
   (b) If there are N forks in successive order each giving x beat/sec with nearest neighbour, then
   $n_{last} = n_{first} + (N-1)x$

14. **Stationary waves**: The equation of stationary wave,
   (a) When the wave is reflected from a free boundary, is:

$$y = +\,2a \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi t}{T} = 2a \cos\, kx \sin \omega t$$

Spectrum

(b) When the wave is reflected from a rigid boundary, is:

$$Y + -2a \sin \frac{2\pi x}{\lambda} \cos \frac{2\pi t}{T} = -2a \sin kx \cos \omega t$$

15. **Vibrations of a stretched string**:

(a) For fundamental tone: $n_1 = \frac{1}{\lambda}\sqrt{\frac{T}{m}}$

(b) For p th harmonic : $n_p = \frac{p}{\lambda}\sqrt{\frac{T}{m}}$

(c) The ratio of successive harmonic frequencies: $n_1 : n_2 : n_3 : \ldots\ldots = 1 : 2 : 3 : \ldots\ldots$

(d) Sonometer: $n = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$ $(m = \pi r^2 d)$

(e) Melde's experiment: (i) Transverse mode: $n = \frac{p}{2\ell}\sqrt{\frac{T}{m}}$

(ii) Longitudinal mode: $n = \frac{2p}{2\ell}\sqrt{\frac{T}{m}}$

16. **Vibrations of closed organ pipe**

(a) For fundamental tone: $n_1 = \left(\frac{v}{4L}\right)$

(b) For first overtone (third harmonic): $n_2 = 3n_1$

(c) Only odd harmonics are found in the vibrations of a closed organ pipe and $n_1 : n_2 : n_3 : \ldots = 1 : 3 : 5 : \ldots\ldots$

17. **Vibrations of open organ pipe**:

(a) For fundamental tone: $n_1 = (v/2L)$

(b) For first overtone (second harmonic) : $n_2 = 2n_1$

(c) Both even and odd harmonics are found in the vibrations of an open organ pipe and $n_1 : n_2 : n_3 : \ldots\ldots = 1 : 2 : 3 : \ldots\ldots$

18. **End correction**: (a) Closed pipe : $L = L_{pipe} + 0.3d$

(b) Open pipe: $L = L_{pipe} + 0.6\,d$

where d = diameter = 2r

19. **Resonance column**: (a) $\ell_1 + e = \frac{\lambda}{4}$; (b) $\ell_2 + e = \frac{3\lambda}{4}$

(c) $e = \frac{\ell_2 - 3\ell_1}{2}$; (d) $n = \frac{v}{2(\ell_2 - \ell_1)}$ or $\lambda = 2(\ell_2 - \ell_1)$

20. **Kundt's tube**: $\frac{v_{air}}{v_{rod}} = \frac{\lambda_{air}}{\lambda_{rod}}$

21. **Longitudinal vibration of rods**

(a) Both ends open and clamped in middle:

(i) Fundamental frequency, $n_1 = (v/2\ell)$

(ii) Frequency of first overtone, $n_2 = 3n_1$

(iii) Ratio of frequencies, $n_1 : n_2 : n_3 : \ldots\ldots = 1\ 3 : 5 : \ldots\ldots$

(b) One end clamped

(i) Fundamental frequency, $n_1 = (v/4\ell)$

(ii) Frequency of first overtone, $n_2 = 3n_1$

Spectrum

(iii) Ratio of frequencies, $n_1 : n_2 : n_3 : ......= 1 : 3 : 5 : ....$

22. **Frequency of a turning fork**: $n \propto \frac{t}{\ell_2}\sqrt{\frac{E}{\rho}}$

Where t = thickness, ℓ = length of prong, E = Elastic constant and ρ = density

23. Doppler Effect for Sound

(a) Observer stationary and source moving:

(i) Source approaching: $n' = \frac{v}{v - v_s} \times n$ and $\lambda' = \frac{v - v_s}{v} \times \lambda$

(ii) Source receding: $n' = \frac{v}{v + v_s} \times n$ and $\lambda' = \frac{v + v_s}{v} \times \lambda$

(b) Source stationary and observer moving:

(i) Observer approaching the source: $n' = \frac{v + v_0}{v} \times n$ and $\lambda' = \lambda$

(ii) Observer receding away from source: $n' = \frac{v - v_0}{v} \times n$ and $\lambda' = \lambda$

(c) Source and observer both moving:

(i) S and O moving towards each other: $n' = \frac{v + v_0}{v - v_s} \times n$

(ii) S and O moving away from each other: $n' = \frac{v - v_0}{v + v_s} \times n$

(iii) S and O in same direction, S behind O : $n' = \frac{v - v_0}{v - v_s} \times n$

(iv) S and O in same direction, S ahead of O: $n' = \frac{v + v_0}{v + v_s} \times n$

(d) Effect of motion of medium: $n' = \frac{v \pm v_m \pm v_0}{v \pm v_m \pm v_s}$

(e) Change in frequency: (i) Moving source passes a stationary observer: $\Delta n = \frac{2vv_s}{v^2 - v_s^2} \times n$

For $v_s \ll v$, Δ, $= \frac{2v_s}{v} \times n$

(ii) Moving observer passes a stationary source: $\Delta n = \frac{2v_0}{v} \times n$

(f) Source moving towards or away from hill or wall

(i) Source moving towards wall

(a) Observer between source and wall

$n' = \frac{v}{v - v_s} \times n$ (for direct waves)

$n' = \frac{v}{v - v_s} \times n$ (for reflected waves)

(b) Source between observer and wall

$n' = \frac{v}{v+v_s} \times n$ (for direct waves)

$n' = \frac{v}{v-v_s} \times n$ (for reflected waves)

(ii) Source moving away from wall

(a) Observer between source and wall

$n' = \frac{v}{v+v_s} \times n$ (for direct waves)

$n' = \frac{v}{v+v_s} \times n$ (for reflected waves)

(b) Source between observer and wall

$n' = \frac{v}{v-v_s} \times n$ (for direct waves)

$n' = \frac{v}{v+v_s} \times n$ (for reflected waves)

(g) Moving Target:

(i) S and O stationary at the same place and target approaching with speed u

$n' = \left(\frac{v+u}{v-u}\right) \times n$ or $n' = \left(1+\frac{2u}{v}\right) \times n$ (for $u \ll v$)

(ii) S and O stationary at the same place and target receding with speed u

$n' = \left(\frac{v-u}{v+u}\right) \times n$ or $n' = \left(1-\frac{2u}{v}\right) \times n$ (for $u \ll v$)

(h) SONAR: $n' = \frac{v \pm v_{sub}}{v \pm v_{sub}} \times n \cong \left(1 \pm \frac{2\,v_{sub}}{v}\right) \times n$

(upper sign for approaching submarine while lower sign for receding submarine)

(i) Transverse Doppler effect: There is no transverse Doppler effect in sound. For velocity component $v_s \cos\theta$

$n' = \frac{v}{v \pm v_s \cos\theta} \times n$ (– sign for approaching and + sign for receding)

24. Doppler Effect for light

(a) Red shift (when light source is moving away):

$n' = \sqrt{\frac{1-v/c}{1+v/c}} \times n$ or $\lambda' = \sqrt{\frac{1+v/c}{1-v/c}} \times \lambda$

For $v \ll c$, $\Delta n = -\left(\frac{v}{c}\right) \times n$ or $\Delta\lambda' = \left(\frac{v}{c}\right) \times \lambda$

(b) Blue shift (when light source is approaching)

$n' = \sqrt{\frac{1+v/c}{1-v/c}} \times n$ or $\lambda' = \sqrt{\frac{1-v/c}{1+v/c}} \times \lambda$

For $v \ll c$, $\Delta n = \left(\frac{v}{c}\right) n$ or $\Delta\lambda' = -\left(\frac{v}{c}\right)\lambda$

Spectrum

(c) Doppler Broadening = $2\Delta\lambda = 2\left(\frac{v}{c}\right)\lambda$

(d) Transverse Doppler effect:

For light, n' = $\sqrt{1-\frac{v^2}{c^2}}\ x\ n = \left(1-\frac{1}{2}\frac{v^2}{c^2}\right) x\ n$ (for v << c)

(e) RADAR: $\Delta_n = \left(\frac{2v}{c}\right)n$

Spectrum

ELECTROSTATICS & CAPACITANCE

ELECTROSTATICS

1. Coulomb's Law

(a) $F_m = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{r^2}$, K = Dielectric constant or relative permittivity of the medium

(b) $F_m = \frac{F_0}{K}$ [F_0 – Force between point charges placed in vacuum]

(c) $[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$

(d) $\frac{F_e}{F_g} = 2.4 \times 10^{39}$ [For electron–proton pair)

$= 1.2 \times 10^{36}$ (For proton–proton pair)

2. Electric field

(a) $\vec{E} = \frac{\vec{F}}{q_0}$

(b) Electric field due to a point charge: (i) $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}.\hat{r}$ (if charge q is placed at the origin)

(ii) $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q(\vec{r}-\vec{r}_0)}{|\vec{r}-\vec{r}_0|^3}$ (if charge q is placed at some point having position vector $\vec{r}_0$)

(c) $[E] = [M^1L^1T^{-3}A^{-1}]$

3. Electric field on the axis of a uniformly charged ring

(a) $E_{axial} = \frac{1}{4\pi\varepsilon_0}\frac{qx}{(R^2+x^2)^{3/2}}$ (R = radius of the ring)

(b) $E_{centre} = 0$

4. Electric dipole

(a) dipole moment $p = q(2\ell)$ (where 2ℓ = length of the dipole)

(b) $E_{axial} = \frac{1}{4\pi\varepsilon_0}\frac{2pr}{[r^2-\ell^2]^2}$ (r = distance of axial point w.r.t. centre of dipole)

$\cong \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$ (if $r >> \ell$)

(c) $E_{equat.} = \frac{1}{4\pi\varepsilon_0}\frac{p}{[r^2+\ell^2]^{3/2}} \approx \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$ (if $r >> \ell$)

(d) $(E_{axial}/E_{equat.}) = 2/1$

(e) Dipole field at an arbitrary point (r, θ)

(i) $E_r = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}$; (ii) $E_\theta = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}$

(iii) $E = \sqrt{E_r^2 + E_\theta^2} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{1+3\cos^2\theta}$

(f) Dipole field component at (x, y, z) point

(i) $E_x = \frac{1}{4\pi\varepsilon_0}\frac{3xz\,p}{r^s}$; (ii) $E_y = \frac{1}{4\pi\varepsilon_0}\frac{3yz\,p}{r^5}$;

Spectrum

(iii) $E_z = \frac{1}{4\pi\varepsilon_0}\frac{p\left(3z^2 - r^2\right)}{r^5}$

(g) Torque on a dipole : (i) $\vec{\tau} = \vec{p} \times \vec{E}$; (ii) $\tau = pE \sin\theta$

(h) Potential energy of a dipole: (i) $U = -\vec{p}\,.\,\vec{E} = -pE\cos\theta$

(ii) Work done in rotating a dipole from angle θ_1 to angle θ_2

$W = U_2 - U_1 = pE(\cos\theta_1 - \cos\theta_2)$

5. Electric flux

(a) $d\phi = \vec{E}\,.\,\vec{dS}$

(b) $\phi \int \vec{E}\,.\,\vec{dS} = EA\cos\theta$ (If electric field is constant over the whole surface)

(c) Unit of ϕ = (Nm^2/Coulomb) = J.m/Couplomb

(d) $[\phi] = [M^1L^3T^{-3}A^{-1}]$

6. Gauss's Law: $\oint \vec{E}\,.\,\vec{dS} = \frac{q}{\varepsilon_0}$

7. Electric field due to various systems of charges

(a) Isolated Charge:

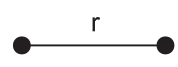


Fig. 1

$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

(b) Electric dipole:

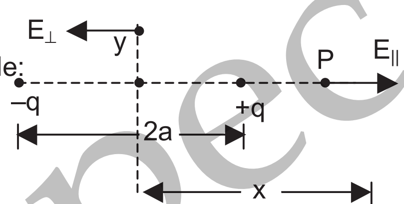


Fig. 2

(i) $\vec{E}_{||} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{x^3}$

(ii) $\vec{E}_{\perp} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{y^3}$

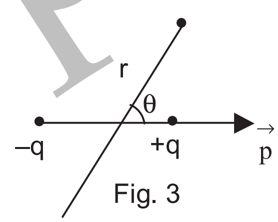


Fig. 3

(iii) $E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{1 + 3\cos^2\theta}$

(c) A ring of charge:

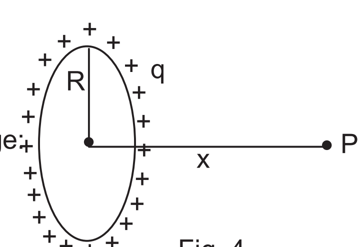


Fig. 4

$E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{\left(R^2 + x^2\right)^{3/2}}$

Spectrum

(d) A disc of charge:

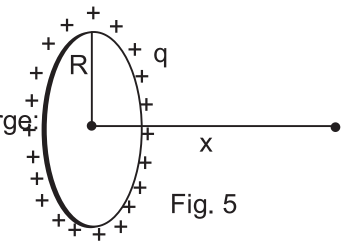


Fig. 5

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

(e) Infinite sheet of charge:

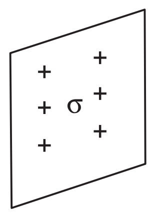


Fig. 6

$$E = \frac{\sigma}{2\varepsilon_0}$$

(f) Infinitely long line of charge:

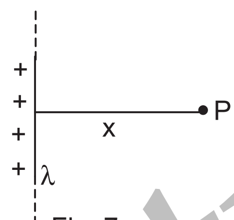


Fig. 7

$$E = \frac{\lambda}{2\pi\varepsilon_0 x}$$

(g) Finite line of charge:

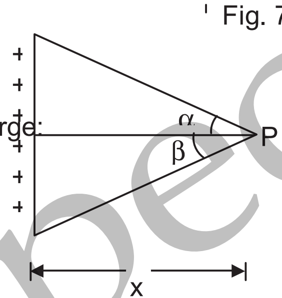


Fig. 8

$$E_{\perp} = \frac{\lambda}{4\pi\varepsilon_0 x}(\sin\alpha + \sin\beta)$$

$$E_{\parallel} = \frac{\lambda}{4\pi\varepsilon_0}(\cos\alpha + \cos\beta)$$

(h) Charged spherical shell:

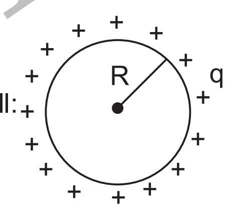


Fig. 9

(i) Inside: $0 \le r \le R$

$E = 0$

(ii) Outside: $r \ge R$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

(i) Solid sphere of charge:

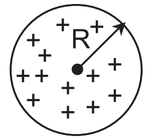


Fig. 10

(i) Inside: $0 \le r \le r$

$E = \rho r/3\varepsilon_0$

(ii) Outside : $r \ge R$

$$E = \frac{\rho R}{3\varepsilon_0}\left(\frac{R}{r}\right)^2$$

Spectrum

8. Force on a charged conductor: The force per unit area or electric pressure

$$P_{elec.} = \frac{dF}{dA} = \frac{\sigma^2}{2\varepsilon_0}$$

9. Charged soap bubble: (a) $P_{in} - P_{out} = \frac{4T}{r} - \frac{\sigma^2}{2\varepsilon_0}$

(b) If air pressure inside and outside are assumed equal then: $P_{in} = P_{out}$ and $\frac{4T}{r} = \frac{\sigma^2}{2\varepsilon_0}$

or $r = \frac{8\varepsilon_0 T}{\sigma^2}$ or $T = \frac{\sigma^2 r}{8\varepsilon_0}$ or $\sigma = \sqrt{(8\varepsilon_0 T/r)}$ or $Q = 8\pi r\sqrt{(2\varepsilon_0 rT)}$

or $r = [Q^2/128\pi^2\varepsilon_0 T]^{1/3}$

10. Electric potential:

(a) $V = (W/q)$

(b) Unit of V = Volt

(c) $[V] = [ML^2T^{-3}A^{-1}]$

(d) $\vec{E} = -\vec{\nabla} V$

(e) Potential due to a point charge, $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

(f) Potential due to a group of charges, $V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right]$

(g) Potential due to a dipole:

(i) Axial point, $V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$; (ii) equatorial point, V = 0;

(iii) $V(r, \theta) = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{\rho^2}$

(h) Potential due to a charged spherical shell

(i) outside: $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ (ii) surface: $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$;

(iii) inside : $V = V_{surface} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$

(i) Potential due to a charged spherical conductor is the same as that due to a charged spherical shell.

(j) Potential due to a uniformly charged nonconducting sphere

(i) outside: $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$; (ii) surface: $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$

(iii) inside: $V = \frac{1}{4\pi\varepsilon_0}\frac{q(3R^2 - r^2)}{2R^3}$; (iv) centre: $V = \frac{3}{2} \times \frac{1}{4\pi\varepsilon_0}\frac{q}{R} = 1.5\ V_{surface}$

(k) Common potential (Two spheres joined by thin wire)

(i) common potential $V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1 + Q_2}{r_1 + r_2}\right)$

(ii) $q_1 = \frac{r_1(Q_1 + Q_2)}{(r_1 + r_2)} = \frac{r_1 Q}{r_1 + r_2}$; $q_2 = \frac{r_2 Q}{r_1 + r_2}$

Spectrum

(iii) $\frac{q_1}{q_2} = \frac{r_1}{r_2}$ or $\frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1}$

11. Potential energy

(a) U = $\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = q_1 V_1$ (For a system of two charges)

(b) U= $\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}}$ (For a system of three charges)

(c) $U = -\vec{p}\,.\,\vec{E}$ (For an electric dipole)

12. If n drops coalesce to form one drop, then

(a) Q = nq; (b) R=$n^{1/3}$r ; (c) V = $n^{2/3}$ V_{small} ;
(d) $\sigma = n^{1/3}\,\sigma_{small}$ (e) E=$n^{1/3}$ E_{small}

13. Energy density of electrostatic field: $u = \frac{1}{2}\varepsilon_0 E^2$

CAPACITANCE

14. Capacitance:

(a) C = (q/V)
(b) Unit of C = farad (F)
(c) Dimensions of C = $[M^{-1}L^{-2}T^4A^2]$

15. Energy stored in a charged capacitor

(a) U=$\frac{1}{2}$ CV^2 ; (b) U=$\frac{1}{2}$QV; (c) $U = \frac{1}{2}\frac{Q^2}{C}$

16. Energy density: (a) u = $\frac{1}{2}\varepsilon_0 E^2$; (b) u = $\frac{1}{2}\frac{\sigma^2}{\varepsilon_0}$

17. Force of attraction between plates of a charged capacitor

(a) F=$\frac{1}{2}$ $\varepsilon_0 E^2 A$; (b) F = $\frac{\sigma^2 A}{2\varepsilon_0}$; (c) F=$\frac{Q^2}{2\varepsilon_0 A}$

18. Capacitance formulae

(a) Sphere: (i) $C_{air} = 4\pi\,\varepsilon_0 R$; (ii) $C_{med} = K\,(4\pi\,\varepsilon_0 R)$

(b) Spherical capacitor: (i) $C_{air} = \frac{4\pi\varepsilon_0 r_a r_b}{r_b - r_a}$; (ii) $C_{med} = \frac{4\pi\varepsilon_0 K\, r_a r_b}{(r_b - r_a)}$

(c) Parallel plate capacitor: (i) $C_{air} = \frac{\varepsilon_0 A}{d}$; (ii) $C_{med} = \frac{K\varepsilon_0 A}{d}$

(d) Cylindrical capacitor: (i) $C_{air} = \frac{2\pi\varepsilon_0 \ell}{\log_e (r_b / r_a)}$; (ii) $C_{med} = \frac{2\pi K\varepsilon_0 \ell}{\log_e (r_b / r_a)}$

(e) Two long parallel wires: C=$\frac{\varepsilon_0 \ell}{\log_e (d/a)}$ where d is the separation between wires and a radius of each wire (d>>a)

19. Series Combination of Capacitors

(a) $q_1 = q_2 = q_3 = q$ (Charge remains same)

(b) $V_1 = \frac{q}{C_1}, V_2 = \frac{q}{C_2}, V_3 = \frac{q}{C_3}$ (Potential difference is different)

Spectrum

(c) $\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$

(d) For two capacitors in series: $C = C_1C_2/(C_1 + C_2)$

(e) Energy stored: $U = U_1 + U_2 + U_3$

20. Parallel Combination of Capacitors

(a) $V_1 = V_2 = V_3 = V$ (Potential difference remains same)

(b) $q_1 = C_1V$, $q_2 = C_2V$, $q_3 = C_3V$ (Charges are different)

(c) $C = C_1 + C_2 + C_3$

(d) $U = U_1 + U_2 + U_3$

21. Effect of dielectric

(a) Field inside dielectric, $E_d = E_0/K$

(b) Polarization charges on surface of dielectric:

(i) $Q_p = Q\left(1-\frac{1}{K}\right)$; (ii) $\sigma_p = \frac{Q_p}{A} = \sigma\left(1-\frac{1}{K}\right)$

(c) Polarization vector: (i) $|\vec{P}| = Q_p/A$; (ii) $|\vec{P}| = \varepsilon_0\chi E_d$

22. Capacitance formulae with dielectric

(a) $C - \frac{\varepsilon_0 A}{d-t\left(1-\frac{1}{K}\right)} = \frac{K\varepsilon_0 A}{K\,d-t\,(K-1)}$ (For a dielectric slab of thickness t)

(b) $C=\frac{\varepsilon_0 A}{d-t}$ (For a metallic slab of thickness t)

(c) $C=\frac{\varepsilon_0 A}{d}\left(\frac{K_1+K_2}{2}\right)$

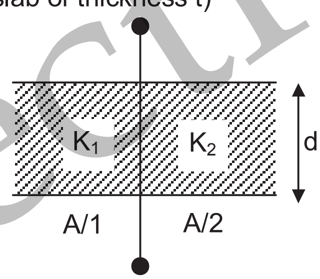


Fig. 11

(d) $C=\frac{2\varepsilon_0 A}{d}\left(\frac{K_1K_2}{K_1+K_2}\right)$

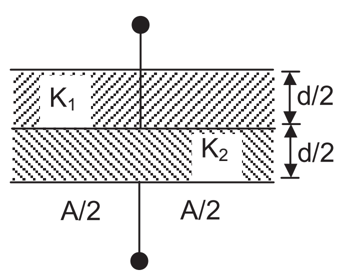
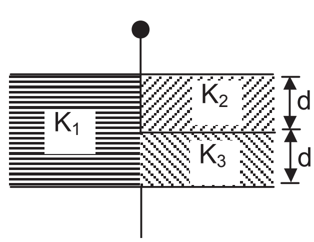


Fig. 12

(e) $C=\frac{\varepsilon_0 A}{4\,d}\left(K_1+\frac{2K_2\,K_3}{K_2+K_3}\right)$



Spectrum

(f) For *n* plates with alternate plates connected: $C = (n-1)\,\varepsilon_0 A/d$

(g) $C = \dfrac{\varepsilon_0 A}{\left(\dfrac{t_1}{K_1} + \dfrac{t_2}{K_2} + \dfrac{t_3}{K_3}\right)}$

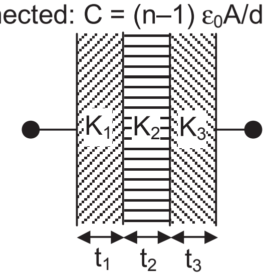


Fig. 14

23. Spherical capacitor with inner sphere grounded

(a) $C = \dfrac{4\pi\varepsilon_0 r_1 r_2}{(r_2 - r_1)} + 4\pi\varepsilon_0 r_2$

(b) Charge on inner sphere = $-q_1$, while charge on outer sphere = $+q_2$

(c) Magnitude of charge on inner sphere: $q_1 = \left(\dfrac{r_1}{r_2}\right) q_2$

24. Insertion of dielectric slab

(a) Battery remains connected when slab is introduced
(i) $V' = V$; (ii) $C' = KC$; (iii) $Q' = KQ$; (iv) $E' = E$; (v) $U' = KU$

(b) Battery is disconnected after charging the capacitor and slab is introduced
(i) $Q' = Q$; (ii) $C' = KC$; (iii) $E' = E/K$; (iv) $V' = V/K$; (v) $U' = U/K$

25. Charge transfer, Common potential and energy loss when two capacitors are connected

(a) Common potential: $V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2} = \dfrac{q_1 + q_2}{C_1 + C_2}$

(b) Charge transfer: $\Delta q = \dfrac{C_1C_2}{C_1 + C_2}(V_1 - V_2)$

(c) Energy loss: $\Delta U = \dfrac{1}{2}\dfrac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$

26. Charging and discharging of a capacitor

(a) Charging: (i) $q = q_0(1 - e^{-t/RC})$; (ii) $V = V_0(1 - e^{-t/RC})$; (iii) $I = I_0 e^{-t/RC}$;
(iv) $I_0 = V_0/R$

(b) Discharge: (i) $q = q_0 e^{-t/RC}$; (ii) $V = V_0 e^{-t/RC}$; (iii) $I = -I_0 e^{-t/RC}$
(b) Time constant: $\tau = RC$

Spectrum

CURRENT ELECTRICITY

1. Electric Current

(a) $I = (q/t)$; (b) $I = (dq/dt)$; (c) $I = (ne/t)$; (d) $q = \int I dT$

2. Ohm's law, Resistivity and Conductivity

(a) $V = IR$; (b) $R = \rho(\ell/A)$; (c) $\sigma = (1/\rho)$ (d) $v_d = (eE\tau/m)$; (e) $I = neAv_d$;

(f) $R = \left(\frac{m}{ne^2\tau}\right)\left(\frac{\ell}{A}\right)$; (g) $\rho = \frac{m}{ne^2\tau}$; (h) $\sigma = \frac{ne^2\tau}{m}$

3. Current density

(a) $J = (I/A)$; (b) $J = nev_d$; (c) $J = \sigma E$; (d) $\mu = (v_d/E)$; (e) $\sigma = ne\mu$

4. Temperature Coefficient of Resistance

(a) $R = R_0[1 + \alpha(T - T_0)]$; (b) $\alpha = \frac{R - R_0}{R_0(T - T_0)}$; (c) $\rho = \rho_0 [1 + \alpha(T - T_0)]$;

(d) $\alpha = \frac{\rho - \rho_0}{\rho_0(T - T_0)}$

5. Cell: (a) $E = \frac{W}{Q}$; **(b)** $I = \frac{E}{r + R}$; **(c)** $V = E - Ir$ **(where** $V = IR$**)**

6. Series Combination of Resistances

(a) $R = R_1 + R_2 + R_3$; (b) $V = V_1 + V_2 + V_3$;

(c) $I = \text{constant} = I_1 = I_2 = I_3$; (d) $V_1 = IR_1, V_2 = IR_2, V_3 = IR_3$

7. Parallel Combination of resistances

(a) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$;

(b) $I = I_1 + I_2 + I_3$;

(c) $V = \text{constant} = V_1 = V_2 = V_3$;

(d) $I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3}$

(e) For a parallel combination of two resistances:

(i) $R = \frac{R_1R_2}{R_1 + R_2}$; (ii) $I_1 = \frac{R_2}{R_1 + R_2}I$; (iii) $I_2 = \frac{R_1}{R_1 + R_2}I$

8. Heating effect of current

(a) $W = VIt$;

(b) $P = VI$;

(c) $P = I^2R = V^2/R$;

(d) $H = I^2 Rt$ Joule $= \frac{I^2Rt}{J}$ Calorie

9. Electric bulb: (a) Resistance of filament $R = V^2/P$;

(b) Maximum current that can be allowed to pass through bulb, $I_{max} = (P/V)$

10. Total Power Consumed

(a) Parallel combination: $P = P_1 + P_2 + P_3$

(b) Series combination: $\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3}$

Spectrum

11. Effect of stretching a resistance wire

$$\frac{R_2}{R_1} = \frac{\ell_1}{\ell_2} \text{ x } \frac{A_1}{A_2} = \left(\frac{\ell_2}{\ell_1}\right)^2 = \left(\frac{A_1}{A_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^4 \qquad [\because \ \ell_1 A_1 = \ell_2 A_2]$$

12. Cells in series: $I = \frac{nE}{nr + R} = \frac{E}{R}$ (if n r <<R)

13. Cells in parallel: $I = \frac{E}{(r/n) + R} = \frac{E}{R}$ (if r << R)

$= \frac{nE}{r}$ (if r >>R)

14. Mixed Combination (m rows with each containing n cells in series)

(a) $I = \frac{nE}{(nr/m) + R} = \frac{m\,n\,E}{nr + m\,R}$;

(b) I is maximum when n r = m R ;

(c) $I_{max} = \frac{m\,n\,E}{2\sqrt{m\,n\,r\,R}}$

15. Chemical effect of current:

(a) Faraday's first law of electrolysis: m = Zq = ZIt

(b) Faraday's second law of electrolysis:

(i) $m \propto W$ (W = ECE) or m/W = constant (where W = atomic weight/valency)

(ii) As $\frac{m_1}{m_2} = \frac{Z_1}{Z_2}$ and $\frac{m_1}{m_2} = \frac{W_1}{W_2}$; so $\frac{Z_1}{Z_2} = \frac{W_1}{W_2}$

(c) Faraday : 1 Faraday = 96,500 Coulomb

(d) $\frac{W}{Z} = F =$ Faraday's constant

16. Thermo e.m.f. : $e = \alpha\theta + \frac{\beta\theta^2}{2}$ (where $\theta = \theta_H = \theta_C$)

17. Neutral temperature: $\theta_N = -\left(\frac{\alpha}{\beta}\right)$ $\left[\left(\frac{de}{d\theta}\right)_{\theta_N} = 0\right]$

18. Temperature of inversion: $\theta_N = \frac{\theta_1 + \theta_C}{2}$ $[\because \theta_I - \theta_N = \theta_N - \theta_C]$

19. Thermoelectric power or Seebeck Coefficient: $S = \frac{de}{d\theta} = \alpha + \beta\theta$

20. Peltier effect:

(i) Heat absorbed per second at a junction when a current I flows = πI (where π = Peltier coefficient)

(ii) Peltier coefficient, $\pi = S\theta_H$

21. Thomson Coefficient:

(i) Heat absorbed/ sec. = $I \int_{\theta_X}^{\theta_H} \sigma d\theta$

Spectrum

(ii) Thomson coefficient, $\sigma = \frac{\Delta Q/\text{time}}{I\,\Delta\theta}$

Spectrum

MAGNETIC EFFECTS OF CURRENT

22. **Biot–Savart law** : dB = $\frac{\mu_0}{4\pi}\frac{I\,d\ell\sin\theta}{r^2}$

23. **Field due to a long straight wire**: $B=\frac{\mu_0 I}{2\pi r}$

24. **Field due to a circular coil**:

(a) at centre: B = $\frac{\mu_0 N I}{2a}$;

(b) at an axial point: B = $\frac{\mu_0 N I a^2}{2\left(a^2+x^2\right)^{3/2}}$

(c) on axis when x >> a : B = $\frac{\mu_0\, N I a^2}{2x^3}$

(d) point of inflexion: It occurs at x = a/2

Field at the point of inflexion: B = $\left(\frac{4\mu_0 N I}{5\sqrt{5}\,a}\right)$ = 0.716 B_{centre}

25. **Magnetic moment of circular coil**: (a) M = NIA ; (b) Field: B= $\frac{\mu_0}{4\pi}\frac{2M}{x^3}$

26. **Field due to an arc of current**:

(a) B = $(\mu_0 I\ell/4\pi R^2)$;

(b) B = $(\mu_0 I\theta/4\pi R)$

(c) At the centre of a semicircular coil: B = $(\mu_0 I/4R)$

27. **Field due to finite length of wire**: B = $\frac{\mu_0 I}{4\pi a}(\sin\phi_1+\sin\phi_2)$

28. **Field at the centre of a square loop**: B = $\left(\frac{2\sqrt{2}\,\mu_0 I}{\pi\ell}\right)$

29. **Ampere's law**: (a) $\oint \vec{B}.d\vec{\ell}=\mu_0 I$; (b) $\oint \vec{H}.d\vec{\ell}=1$

30. **Field due to a current in cylindrical rod**:

(a) outside: B = $(\mu_0\, I/2\pi r)$; (b) surface: B = $(\mu_0\, I/2\pi R)$; (c) inside: B= $\frac{\mu_0 I r}{2\pi R^2}$

31. **Field due to a current carrying solenoid**:

(a) inside: B = $\mu_0 n\, I$; (b) at one end : B = $(\mu_0 n\, I/2)$

(c) at an axial point: B = $\frac{\mu_0 n I}{2}(\cos\alpha_2-\cos\alpha_1)$

32. **Field due to a toroid**: (a) inside: B=$\mu_0 nI$ – $\mu_0 NI/2\pi R$; (b) outside: B = 0

33. **Force on electric current**: $\vec{F}=\vec{H}\times\vec{B}$

34. **Force between two parallel conductors**: $\frac{F}{\ell}=\frac{\mu_0 I_1 I_2}{2\pi d}$

35. **Comparison of magnetic and electric forces between two moving charges**: $(F_{magnetic}/F_{electric})$ = (v^2/c^2)

Spectrum

36. Force on a current loop in a magnetic field: $\vec{F} = 0$ (any shape)

37. Torque on a current loop in a magnetic field: $\vec{\tau} = \vec{M} \times \vec{B}$ or τ M B sin θ

38. Moving coil galvanometer:

(a) τ – N I A B ;

(b) τ =Kθ ;

(c) $I = \left(\frac{K}{N\,AB}\right)\theta$;

(d) Current sensitivity = (θ/I) = (NaB/K) ; (e) Voltage sensitivity = (θ/V) = (θ/IR) = (NAB/KR)

39. Ammeter:

(a) Shunt resistance S = (I_gG/ I– I_g);

(b) Length of shunt wire, ℓ = S $\pi r^2/\rho$;

(c) Effective resistance of ammeter, R_A = GS/(G+S);

(d) For an ideal ammeter, R_A = 0

40. Voltmeter:

(a) High resistance in series, $R = \left(\frac{V}{I_g} - G\right)$;

(b) For converted Voltmeter, R_V = R + G;

(c) For an ideal Voltmeter, $R_V = \infty$

41. Force on a moving charge:

(a) $\vec{F} = q\left(\vec{v} \times \vec{B}\right)$; (b) F = q v B sin θ

42. Path of a moving charge in a magnetic field

(a) When $\vec{v}$ is $\perp$ to $\vec{B}$:

(i) path = circular; (ii) r = (mv/qB) ; (iii) ν = (qB/2πm);
(iv) T = (2πm/qB) ; (v) ω = qB/m)

(b) When angle between $\vec{v}$ and $\vec{B}$ is θ:

(i) path=helical ; (ii) r = (m$v_\perp$/qB) = (mv sin θ/qB);

(iii) ν = (qB/2πm); (iv) $T = \frac{2\pi m}{q\,B}$; (v) ω = (qB/m);

(vi) pitch p = 2πr/tan θ (where tan θ = ($v_\perp/v_{||}$)

43. Cyclotron:

(i) T = (2πm/qB) ;

(ii) ν = (qB/2πm) ;

(iii) ω = (θB/m) ;

(iv) radius of particle acquiring energy E, r = [$\sqrt{}$(2mE)/qB];

(v) velocity of particle at radius r, v = qBr/m;

(vi) the maximum kinetic energy (with upper limit of radius = R)

$$K_{max} = \frac{1}{2}\frac{q^2B^2R^2}{m}$$

Spectrum

44. Magnetic field produced by a moving charge:

(a) $\vec{B} = \frac{\mu_0}{4\pi} \frac{q\,(\vec{v} \times \vec{r})}{r^3}$;

(b) $B = \frac{\mu_0}{4\pi} \frac{q\,v \sin\theta}{r^2}$

Spectrum

MAGNETIC PROPERTIES OF CURRENT

45. Magnetic field:

(a) $B = \frac{F_{max}}{q\,v}$; (b) $B = \frac{1}{I}\left(\frac{dF}{d\ell}\right)_{max}$

46. Atomic magnetic moments:

(a) $\mu_L = -\frac{eL}{2m}$; (b) $\mu_S = -\frac{eS}{m}$;

(c) $\mu_J = -g\frac{eJ}{2m}$; (d) $\mu_B = \frac{e\,h}{4\,\pi\,m} = 0.927 \times 10^{-23}$ J/T

47. Intensity of magnetization: $I = (M/V)$

48. Magnetizing field:

(a) $H = \frac{B}{\mu_0} - I$;

(b) For vacuum, $H = \frac{B}{\mu_0}$;

(c) For medium, $h = B/\mu$;

(d) $H = nI$ (solenoid);

(e) $H = I/2\pi r$ (straight wire);

(f) $H = \frac{I\,d\ell \sin\theta}{r^2}$ (Biot-Savart law) ;

(g) $\oint \vec{H}\cdot d\vec{\ell} = I_{free}$

49. Magnetic susceptibility: $\chi = (I/H)$

50. Magnetic permeability:

(a) $\mu = (B/H)$; (b) $\mu_r = (\mu/\mu_0)$; (c) $\mu_r = (B/B_0)$

51. Other relations:

(a) $\mu = \mu_0\,(1+\chi)$; (b) $\mu_r = 1 + \chi$ or $\chi = \mu_r - 1$;

(c) $B = B_0\,(1+\chi)$: (d) $B = \mu_0\,(H + I)$

52. Pole strength: $m = F/B$

53. Magnetic moment of dipole : $M = m \times 2\ell$

54. Field due to a pole: $B = \frac{\mu_0}{4\pi}\left(\frac{m}{r^2}\right)$

55. Field due to a bar magnet:

(a) Axial point: $B = \frac{\mu_0}{4\pi}\frac{2\,Mr}{\left(r^2 - \ell^2\right)^2} = \frac{\mu_0}{4\pi}\left(\frac{2M}{r^3}\right)$ (if $r >> \ell$)

(b) Equatorial point: $B = \frac{\mu_0}{4\pi}\frac{M}{\left(r^2 + \ell^2\right)^{3/2}} = \frac{\mu_0}{4\pi}\left(\frac{M}{r^3}\right)$

(c) At arbitrary point: $B = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{1 + 3\cos^2\theta}$

Spectrum

56. Force and torque on a dipole in uniform magnetic field

(a) $\vec{F} = 0$; (b) $\vec{\tau} = \vec{M} \times \vec{B}$; (c) τ MB sin θ

57. Potential energy of a dipole in magnetic field: $U = -\vec{M}.\vec{B} = -MB\cos\theta$

58. Tangent galvanometer:

(a) $B = B_H \tan\theta$;

(b) $I = K \tan\theta$, where $K = \dfrac{2r\,B_H}{\mu_0 n}$

59. Vibration magnetometer: $T = 2\pi\sqrt{\dfrac{I}{M\,B_H}}$

Spectrum

ELECTROMAGNETIC INDUCTION

60. Magnetic flux:

(a) $d\phi = \vec{B}.d\vec{A} = BdA\cos\theta$; (b) $\phi = \int \vec{B}.d\vec{A}$;

(c) $\phi = BA\cos\theta$; (d) $\oint \vec{B}.d\vec{S} = 0$; (e) $\overline{\nabla}.\vec{B} = 0$

61. Faraday's laws of e.m. induction:

(a) Induced e.m.f., $e = -(d\phi/dt)$;

(b) Induced current, $I = \frac{e}{R} = -\frac{1}{R}\frac{d\phi}{dt}$;

(c) Induced charge, $q = (\phi_1 - \phi_2)/R$

62. Motion of a conducting rod:

(a) $\vec{F} = -e(\vec{v} \times \vec{B})$;

(b) Induced e.m.f., e = B/v

(c) For a rod rotating with angular frequency ω or rotating disc, induced e.m.f.,

$e = \frac{1}{2}B\ell^2\omega = B\pi f\ell^2 = Baf$

63. Motion of conducting loop in a magnetic field:

(a) Induced e.m.f. $e = B\ell v$; (b) Induced current, $I = (e/R) = (B\ell v/R)$

(c) $F = I\ell B = B^2\ell^2 v/R$; (d) $P = Fv = I\ell Bv = B^2\ell^2v^2R$;

(e) $H = I^2R = (B^2\ell^2v^2/R)$;

(f) In non uniform magnetic field, $e = (B_1 - B_2)\ell v$ and $I = (B_1 - B_2)\ell v/R$

64. Rotating loop:

(a) $\phi = NAB\cos\omega t = \phi_0\cos\omega t$, with $\phi_0 = NAB$;

(b) $e = e_0\sin\omega t$, where $e_0 = NaB\omega$; (c) $I = (e_0\sin\omega t/R) = I_0\sin\omega t$, with $I_0 = e_0/R$

65. Induced electric field: Induced e.m.f. = $\int \vec{E}.d\vec{\ell}$

66. Self Inductance:

(a) $L = \phi/I$;

(b) $e = -(LdI/dt)$;

(c) $L = \mu_0N^2A/\ell = \mu_0n^2A\ell$ (For a solenoid with air core);

(d) $L = \mu_r\mu_0N^2A/\ell$ (For a solenoid with a material core);

(e) $L = \mu_0N^2\pi R/2$ (For a plane circular coil)

67. Mutual inductance:

(a) $M = (\phi_2/I_1)$; (b) $e_2 = -M(dI_1/dt)$; (c) $M = \mu_0N_sN_p A/\ell p$

68. Series and parallel combination

(a) $L = L_1 + L_2$ (if inductors are kept far apart and joined in series)

(b) $L = L_1 + L_2 \pm 2M$ (if inductors are connected in series and they have mutual inductance M)

(c) $L = \frac{L_1L_2}{L_1 + L_2}$ or $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$

(if two inductors are connected in parallel and are kept for apart)

Spectrum

(d) $M = K\sqrt{(L_1 L_2)}$

(if two coils of self inductances, L_1 and L_2 are over each other)

69. Energy stored in an inductor:

(a) $U = \frac{1}{2} L I^2$; (b) $u_B = (B^2/2\mu_0)$

70. Growth and decay of current in LR circuit

(a) $I = I_0 (1 - e^{-t/\tau})$ (for growth), where $\tau = L/R$

(b) $I = I_0 e^{-t/\tau}$ (for decay), where $\tau = L/R$

Spectrum

ALTERNATING CURRENT

71. A.C. Currrent and e.m.f. :

(a) $I = I_0 \sin(\omega t \pm \phi)$;

(b) $e = e_0 \sin(\omega t \pm \phi)$;

(c) $< I > = 0$, $< I >_{1/2} = \frac{2I_0}{\pi} = 0.637\ I_0$;

(d) $< I^2 > I_0^2 / 2$;

(e) $I_{rms} = (I0/\sqrt{2}) = 0.707\ I_0$;

(f) form factor = $\pi/2\sqrt{2}$

72. A.C. response of R, L, C and their series combinations

(a) Resistance only:

- (i) $e = e_0 \sin \omega t$;
- (ii) $I = I_0 \sin \omega t$;
- (iii) phase difference $\phi = 0$;
- (iv) $e_0 = I_0 R$;
- (v) $e_{rms} = I_{rms} R$

(b) Inductance only:

- (i) $e = e_0 \sin \omega t$;
- (ii) $I = I_0 \sin(\omega t - \pi/2)$;
- (iii) current lags the voltage or voltage leads the current by a phase $\pi/2$; (iv) $e_0 = I_0 X_L$;
- (iv) $e_{rms} = I_{rms} X_L$; (vi) $X_L = \omega L$

(c) Capacitance only:

- (i) $e = e_0 \sin \omega t$;
- (ii) $I = I_0 \sin(\omega t + \pi/2)$;
- (iii) current leads the voltage or voltage lags the current by a phase $\pi/2$; (iv) $e_0 = I_0 X_C$;
- (v) $e_{rms} = I_{rms} X_C$;
- (vi) $X_C = (1/\omega C)$

(d) Series LR circuit:

- (i) $e = e_0 \sin \omega t$;
- (ii) $I = I_0 \sin(\omega t + \phi)$;
- (iii) the current lags the voltage or voltage leads the current by a phase $\phi = \tan^{-1}(X_L/R)$;
- (iv) $\cos\phi = (R/Z)$ and $\sin\theta = (XL/Z)$;
- (v) Impedance, $Z = \sqrt{[R^2 + (\omega L)^2)]}$;
- (vi) $e_0 = I_0 Z$;
- (vii) $e_{rms} = I_{rms} Z$

(e) Series RC circuit:

- (i) $e = e_0 \sin \omega t$;
- (ii) $I = I_0 \sin(\omega t + \phi)$;
- (iii) The current leads the voltage or voltage lags behind the current by a phase $\phi = \tan^{-1}(X_C/R)$
- (iv) $\cos\phi = (R/Z)$;
- (v) Impedance, $Z = \sqrt{[R^2 + (1+\omega C)^2)]}$;
- (vi) $e_0 = I_0 Z$;
- (vii) $e_{rms} = I_{rms} Z$

(f) Series LCR circuit:

Spectrum

(i) $e = e_0 \sin \omega t$;
(ii) $I = I_0 \sin (\omega t - \phi)$;
(iii) $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$, ϕ is positive for $X_L > X_C$, ϕ is negative for $X_L < X_C$;
(iv) current lags and circuit is inductive if $X_L < X_C$;
(v) current leads and circuit is capacitive if $X_L < X_C$; (vi) $e_0 = I_0 Z$;
(vi) Impedance, $Z = \sqrt{[R^2 + (X_L - X_C)^2]}$;
(viii) $\cos \phi = (R/Z)$ and $\sin \phi = \left(\frac{X_L - X_C}{Z}\right)$

73. Resonance

(a) Resonance frequency, $f_r = \left(\frac{1}{2\pi\sqrt{LC}}\right)$

(b) At resonance, $X_L = X_C$, $\phi = 0$, $Z = R$ (minimum), $\cos \phi = 1$, $\sin \phi = 0$ and current is maximum ($=E_0/R$)

74. Half power frequencies

(a) lower, $f_1 = f_r - \frac{R}{4\pi L}$ or $\omega_1 = \omega_r - \frac{R}{2L}$

(b) upper, $f_2 = f_r + \frac{R}{4\pi L}$ or $\omega_2 = \omega_r + \frac{R}{2L}$

75. Band width: $\Delta f = \frac{R}{2\pi L}$ or $\Delta\omega = \frac{R}{L}$

76. Quality factor

(a) $Q = \frac{\omega_r}{\Delta\omega} = \frac{\omega_r L}{R}$;

(b) As $\omega_r = \frac{1}{\sqrt{LC}}$, hence $Q \alpha \sqrt{L}$, $Q \alpha \frac{1}{R}$ and $Q \alpha \frac{1}{\sqrt{C}}$;

(c) $Q = \frac{1}{\omega_r CR}$;

(d) $Q = \frac{(X_L)_{res}}{R}$ or $\frac{(X_C)_{res}}{R}$;

(e) $Q = \left(\frac{f_r}{\Delta f}\right)$ or $\Delta f = \frac{f_r}{Q}$

77. At resonance, peak voltages are

(a) $(V_L)_{res} = e_0 Q$; (b) $(V_C)_{res} = e_0 Q$; (c) $(V_R)_{res} = e_0$

78. Conductance, susceptance and admittance

(a) Conductance, $G = (1/R)$;
(b) Susceptance, $S = (1/X)$;
(c) $S_L = (1/X_L)$ and $S_C = (1/X_C) = \omega C$;
(d) admittance $Y = (1/Z)$;
(e) Impedance add in series while admittance add in parallel

79. Power in AC circuits

(a) $P_{av} = \frac{1}{2} E_0 I_0 \cos \phi = E_{rms} I_{rms} \cos \phi$;

(b) Power factor, $\cos \phi = \frac{\text{Real power}}{\text{Virtual power}} = \frac{P_{av}}{E_{rms} I_{rms}}$

Spectrum

(c) Cos ϕ = (R/Z)

(d) (i) R only : $\phi = 0$, $\cos\phi = 1$, $P_{av} = I^2_{rms} R = \frac{e^2_{rms}}{R}$

(ii) C only : $\phi = -90^0 = -\pi/2$, $\cos\phi = 0$, $P_{av} = 0$

(iii) L only : $\phi = 90^0 = \pi/2$, $\cos\phi = 0$, $P_{av} = 0$

(iv) Series RL or RC: $\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$ or $\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$

$P_{av} = E_{rms} I_{rms} \cos\phi = \frac{E^2_{rms} R}{Z^2} = I^2_{rms} R$

(iv) Series LCR: $\phi = \tan^{-1}\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$, $P_{AV} = \frac{E^2_{rms} R}{Z^2} = I^2{}_{rms} R$,

At resonance, $\phi = 0$, $\cos\phi = 1$ and $P_{av} = I^2_{rms} R = E^2_{rms}/R$

80. Parallel LCR circuit

(a) $\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$;

(b) $Y = \sqrt{G^2 + (S_L - S_C)^2}$;

(c) $I_0 = E_0 Y$;

(d) $\tan\phi = \frac{S_L - S_C}{G}$;

(e) $\omega_r = \frac{1}{\sqrt{LC}}$ or $\omega_r = \sqrt{\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)}$;

(f) in parallel resonance circuit, impedance is maximum, admittance is minimum and current is minimum.

81. Transformer:

(a) $C_p = N_p\left(\frac{d\phi}{dt}\right)$ and $e_s = N_s\left(\frac{d\phi}{dt}\right)$

(b) $\left(\frac{e_p}{e_s}\right) = \left(\frac{N_p}{N_s}\right)$

(c) $\because$ $e_p I_p = e_s I_s$, so $\left(\frac{I_s}{I_p}\right) = \left(\frac{e_p}{e_s}\right) = \left(\frac{N_p}{N_s}\right)$

(d) Step down: $e_s < e_p$, $N_s < N_p$ and $I_s > I_p$

(e) Step up : $e_s > e_p$, $N_s > N_p$ and $I_s < I_p$

(f) Efficiency, $\eta = \left(\frac{e_s I_s}{e_p I_p}\right)$

82. AC generator: $e = e_0 \sin(2\pi f t)$, (where $e_0 = NBA\omega$)

83. DC motor:

(a) $I = \left(\frac{E - e}{R}\right)$

(b) $IE = Ie = I^2R$

(c) efficiency, $\eta\left(\frac{e}{E}\right) = \frac{\text{Back emf}}{\text{Applied emf}}$

Spectrum

LIGHT

1. Intensity of light

(a) Spherical wave front: (i) $I = \dfrac{P}{4\pi r^2}$, (ii) amplitude $\propto \dfrac{1}{r}$

(b) Cylindrical wave front: (i) $I \propto \dfrac{1}{r}$, (ii) amplitude $\propto \dfrac{1}{\sqrt{r}}$

(c) Plane wave front: (i) $I \propto r_0$, (i) $A \propto r_0$ (i.e. I and A are both constants)

2. **Law of reflection**: Angle of incidence (i) = Angle of reflection (r)

3. **Law of reflection**: Snell's law: $\eta = \dfrac{\sin i}{\sin r}$

4. Other relations

(a) $2\eta_1 = \dfrac{v_1}{v_2}$ and $\eta = \dfrac{c}{v}$

(b) $\lambda_{medium} = \dfrac{\lambda_{air}}{\eta}$ or $v_{medium} = \dfrac{v_{air}}{\eta}$ ($\because \nu_{medium} = \nu_{air}$)

(c) $\eta_1 \sin i = \eta_2 \sin r$

5. Electromagnetic nature of light

(a) The magnitude of $\vec{E}$ and $\vec{B}$ are related in vacuum by: $B = \dfrac{E}{C}$

(b) $\vec{E}$ and $\vec{B}$ are such that $\vec{E} \times \vec{B}$ is always in the direction of propagation of wave

(c) $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$ and $v = \dfrac{1}{\sqrt{\mu\varepsilon}}$

(d) Refractive index, $\eta = \sqrt{(\mu_r \varepsilon_r)}$ ($\mu_r = \mu/\mu_0$ and $\varepsilon_r = \varepsilon/\varepsilon_0$)
For non–magnetic material, $\mu_r \approx 1$ and $\eta = \sqrt{(\varepsilon_r)}$

(e) The *EM* wave propagating in the positive x–direction may be represented by:
$E_y = E_0 \sin(kx - \omega t)$ and $B_z = B_0 \sin(kx - \omega t)$

6. Energy transmitted by an electromagnetic wave

(a) Energy density of electromagnetic wave is: $u = u_e + u_m = \dfrac{1}{2}\varepsilon_0 E^2 + \dfrac{1}{2}\dfrac{B^2}{\mu_0}$

(b) As for *EM* wave, $B = \dfrac{E}{C}$ and $\dfrac{1}{c} = \sqrt{(\mu_0 \varepsilon_0)}$, hence

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}\frac{E^2}{c^2} = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\varepsilon_0 E^2 = \varepsilon_0 E^2$$

(c) Time averaged value of energy density is: $\bar{u} = \dfrac{1}{2}\varepsilon_0 E_0^{\,2}$

7. Intensity of an electromagnetic wave

(a) In a medium: $I = \left(\dfrac{1}{2}\varepsilon_0 E_0^2\right)v$

(b) In free space: $I = \left(\dfrac{1}{2}\varepsilon_0 E_0^2\right)c$

8. Pointing vector

Spectrum

(a) $\vec{S} = \vec{E} \times \vec{H} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) = c^2\varepsilon_0 (\vec{E} \times \vec{B})$

(b) $S = c\varepsilon_0 E^2 = \sqrt{(\varepsilon_0/\mu_0)}E^2$

(c) $I = \overline{S}$ and $\overline{S} = c\overline{u}$

(d) Impedance of free space, $Z = \sqrt{(\mu_0/\varepsilon_0)} \cong 377$ ohm

9. Pressure of EM Radiation

(a) Change in momentum (normal incidence)

$$\Delta p = \frac{U}{c} = \frac{\overline{S}\,A\Delta t}{c} \quad \text{(absorber)}$$

$$\Delta p = \frac{2U}{c} = \frac{\overline{S}\,A\Delta t}{c} \quad \text{(reflector)}$$

(b) Pressure (normal incidence)

$$P = \frac{\overline{S}}{c} = \overline{u} \quad \text{(absorber)}$$

$$P = \frac{2\overline{S}}{c} = 2\overline{u} \quad \text{(reflector)}$$

(c) Pressure for diffused radiation

$$P = \frac{1}{3}\frac{\overline{S}}{c} = \frac{1}{3}\overline{u} \quad \text{(absorber)}$$

$$P = \frac{2}{3}\frac{\overline{S}}{c} = \frac{2}{3}\overline{u} \quad \text{(reflector)}$$

10. **Quantum theory of light**:

(a) Energy of photon, $E = h\nu = hc/\lambda$

(b) Momentum, $p = \frac{E}{c} = \frac{h}{\lambda}$

(c) Rest mass of photon = 0

(d) Mass equivalent of energy, $m = (E/c^2)$

11. **Inclined mirrors**: number of images

(a) When 360^0 is exactly divisible by θ^0 and $360^0/\theta^0$ is an even integer then the number of images formed is

$$n = \frac{360}{\theta} - 1 \quad \text{(whatever may be location of the object)}$$

(b) When 360^0 is exactly divisible by θ^0 and $360/\theta)$ is an odd integer, then the number of images formed is

$$n = \frac{360}{\theta} - 1 \quad \text{(for symmetrical placement)}$$

$$= \frac{360}{\theta} \quad \text{(for unsymmetrical placement)}$$

Spectrum

(c) When 360^0 is not exactly divisible by θ, then the number of images formed is
= integer value of n (where $n = 360/\theta$)

12. Reflection amplitude and intensity

(a) When a ray of light is incident (with angle of incidence $i \approx 0$) from a medium 1 of refractive index η_1 to the plane surface of medium 2 of refractive index η_2, then reflection amplitude is

$$R = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$$

(b) The ratio of the reflected intensity and the incident intensity is: $\frac{I_r}{I_i} = \left(\frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}\right)^2$.

13. Refraction of light

(a) $\eta = \frac{\sin i}{\sin r}$; (b) ${}^1\eta_2 = \frac{\sin \theta_1}{\sin \theta_2}$;

(c) ${}^1\eta_2 = \frac{1}{2\eta_1}$; (d) Cauchy's relation: $\eta = A + \frac{B}{\lambda^2}$

14. Parallel slab

(a) Angle of incidence, i = Angle of emergence, e

(b) Lateral shift = [t sin (i – r)/cos r]

15. **Composite block**: $\eta_1 \sin \theta_1 = \eta_2 \sin \theta_2 = \eta_3 \sin \theta_3$ = constant

16. Apparent depth

(a) $a = \frac{R}{\eta} = \frac{t}{\eta}$ (where R = Real depth)

(b) If there is an ink spot at the bottom of a glass slab, if appears to be raised by a distance

$x = t - a = t - \frac{t}{\eta} = t\left(1 - \frac{1}{\eta}\right)$, where t is the thickness of the glass slab

(c) If a beaker is filled with immissible transparent liquids of refractive indices η_1, η_2, η_3 and individual depths t_1, t_2, t_3 respectively, then the apparent depth of the beaker is:

$$a = \frac{t_1}{\eta_1} + \frac{t_2}{\eta_2} + \frac{t_3}{\eta_3}$$

17. **Total internal reflection: Critical angle i_C is given by**: $\sin i_C = \frac{1}{\eta}$

18. **For a luminous body at a depth d inside a liquid**: Radius of bright circular patch at the surface

$$r = d \tan i_C = \frac{d}{\sqrt{\eta^2 - 1}}$$

19. **For optical fibre**: $\sin i \le \sqrt{\left[(n_2/n_1)^2 - 1\right]}$

20. **Prism**:

(a) $i + e = A + \delta$

(b) $r_1 + r_2 = A$;

(c) At minimum deviation: i = e and $r_1 = r_2$. Hence, $\eta = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

(d) For small angle prism: $\delta = (\eta - 1) A$

Spectrum

21. **Dispersion**:

(a) $\delta_{red} < \delta_{violet}$ because $\eta_{red} < \eta_{violet}$

(b) Angular dispersion: $\theta = \delta_V - \delta_R = (\eta_V - \eta_R)A$

(c) Dispersive power: $\omega = \frac{\delta_V - \delta_R}{\delta_Y} = \frac{\eta_V - \eta_R}{\eta_Y - 1} = \frac{\eta_B - \eta_R}{\eta_Y - 1}$ (In practice)

(d) Dispersion without deviation: (i) $\delta_C + \delta_F = 0$ or $\frac{A_F}{A_C} = -\frac{(\eta_C - 1)}{(\eta_F - 1)}$

(ii) Also, angular dispersion, $\theta = A_C (\eta_C - 1)(\omega_C - \omega_F)$

(e) Deviation without dispersion: (i) $\theta_C + \theta_F = 0$ or, $\frac{A_F}{A_C} = -\frac{\eta_{CV} - \eta_{CR}}{\eta_{FV} - \eta_{FR}}$

(ii) Also, $\frac{\omega_F}{\omega_C} = -\frac{\delta_{CY}}{\delta_{FY}}$

22. **Principle of superposition**: $y = y_1 + y_2$

23. Superposition of waves of equal frequency and constant phase difference

(a) Resultant wave amplitude, $a = \sqrt{(a_1^2 + a_2^2 + 2a_1a_2 \cos\phi)}$

(b) Resultant wave intensity, $I = I_1 + I_2 + 2\sqrt{(I_1I_2)} \cos\phi$

(c) If $a_1 = a_2 = a_0$, and $I_1 = I_2 = I_0$, then $a = 2a_0 \cos(\phi/2)$ and $I = 4I_0 \cos^2(\phi/2)$

24. Constructive interference

(a) conditions: $\phi = 2n\pi \equiv 0, 2\pi, 4\pi, 6\pi, \ldots..$
or, $\Delta = n\lambda \equiv 0, \lambda, 2\lambda, 3\lambda, \ldots\ldots$

(b) $a_{max} = a_1 + a_2$

(c) $I_{max} \propto (a_1 + a_2)^2$

(d) $I_{max} = I_1 + I_2 + 2\sqrt{(I_1I_2)} = (\sqrt{I_1} + \sqrt{I_2})^2$

(e) $I_{max} = 4I_0$; If $I_1 = I_2 = I_0$

25. Destructive interference

(a) conditions: $\phi = (2n-1)\pi \equiv \pi, 3\pi, 5\pi, \ldots\ldots$ or, $\Delta = (2n-1)\frac{\lambda}{2} = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots\ldots\ldots$

(b) $a_{min} = a_1 - a_2$

(c) $I_{min} \propto (a_1 - a_2)^2$

(d) $I_{min} = I_1 + I_2 - 2\sqrt{(I_1I_2)} = (\sqrt{I_1} - \sqrt{I_2})^2$

(e) $I_{min} = 0$ if $I_1 - I_2 = I_0$

26. Young's double slit experiment

(a) Phase difference, $\phi = \frac{2\pi}{\lambda}(S_2P - S_1P) = \frac{2\pi}{\lambda}$ x path difference

(b) $A = 2a_0 \cos(\phi/2)$ and $I \; 4I_0 \cos^2(\phi/2)$

(c) Position of nth fringe on the screen:

(i) for bright fringe, $x_n = \frac{nD\lambda}{d}$

(ii) for dark fringe, $x_n = \frac{(2n-1)D\lambda}{2d}$

27. **Fringe width**:

(a) Linear fringe width, $\beta = \frac{D\lambda}{d}$

Spectrum

(b) Angular fringe width, $\alpha = \frac{\lambda}{d}$

(c) $\beta_{liquid} = \frac{\beta_{air}}{{}^{air}\eta_{liquid}}$ or $\lambda_{liquid} = \frac{\lambda_{air}}{{}^{air}\eta_{liquid}}$

(d) $\beta_{water} = \frac{3}{4}\beta_{air}$

28. **When a thin shit is introduced in the path of one of the interfering waves**:
(a) $(\eta-1)\, t = n\lambda$
(b) Shift of the central fringe = $\frac{(\eta-1)\, t\, \beta}{\lambda}$

29. **Fringe visibility**: $V = \frac{I_{max} - I_{min}}{I_{max} - I_{min}}$

30. **Frensel's biprism**:
(a) $d = 2a\,(\eta-1)\,\alpha$; (b) $d = \sqrt{(d_1 d_2)}$
(c) $\beta = (D\lambda/d)$; (d) $d_{liquid} < d_{air}$, for example, $d_{water} = d_{air}/4$
(e) $\beta_{liquid} > \beta_{air}$; $\beta_{liquid} = \beta_{air}\left(\frac{\eta_g - 1}{\eta_g - \eta t}\right)$

31. **Newton's rings**:
(a) Diameter of nth dark fringe, $D_n = \sqrt{(4n\lambda R)}$
(b) $\lambda = \frac{D_{n+p}^2 - D_n^2}{4\,pR}$ and $\eta = \frac{D_{n+p}^2 - D_n^2}{D'^2_{n+p} - D'^2_n}$

32. **Thin films**: For reflected light

$2\eta t \cos r = n\lambda$ (Dark fringe)
$2\eta t \cos r = (n - \frac{1}{2})\lambda$ (Bright fringe)

33. **Diffraction**:
(a) $a \sin\theta = n\lambda$ (a = width of slit)
(b) Half angular width of central maxima, $\theta = \sin^{-1}(\lambda/a)$
(c) Intensity distribution of the screen $I = I_0\left(\frac{\sin\phi}{\phi}\right)^2$

where, $\phi = \frac{\pi a\, y}{\lambda D}$ and I_0 = Intensity at central point of screen

(d) Limit of resolution of telescope: $\theta = \frac{1.22\,\lambda}{a}$
(e) Resolving power of telescope = $\frac{1}{\theta} = \frac{a}{1.22\lambda}$

34. **Spherical mirrors**:
(a) Focal length: $f = (R/2)$
(b) Mirror formula: $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$
(c) Newton's formula: $f^2 = xy$ (x and y are the distances of the object and image from the principal focus respectively)

Spectrum

(d) Linear magnification: $m = \frac{I}{o} = -\frac{v}{u} = \frac{f}{f-u} = \frac{f-v}{f}$

(e) Longitudinal magnification: $m = -\frac{v^2}{u^2}$

35. **Spherical lenses**:

(c) A single spherical surface:

(i) $\frac{\eta_2}{v} - \frac{\eta_1}{u} = \frac{(\eta_2 - \eta_1)}{R}$ [For an object placed in a medium of refractive index η_1]

(ii) $\frac{\eta_1}{v} - \frac{\eta_2}{u} = \frac{(\eta_1 - \eta_2)}{R}$ [For an object placed in a medium of refractive index η_2]

(iii) First principal focus: $f_1 = \frac{R}{(\eta - 1)}$ where $\eta = \eta_2/\eta_1$

(iv) Second principal focus: $f_2 = \frac{\eta R}{(\eta - 1)}$

(v) Magnification: $m = \frac{v/\eta_2}{u/\eta_1}$

(d) Lens Maker's formula:

(i) $\frac{1}{f} = \left(\frac{\eta_2}{\eta_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ or, $\frac{\eta_1}{v} - \frac{\eta_1}{u} = (\eta_2 - \eta_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

[When medium is same on both sides of the lens]

(ii) $\frac{\eta_3}{v} - \frac{\eta_1}{u} = \left(\frac{\eta_2 - \eta_1}{R_1}\right) + \left(\frac{\eta_3 - \eta_2}{R_2}\right)$

[When different medium exist on two sides of the lens]

(e) Biconvex or biconcave lens of the same radii for two surfaces: $\frac{1}{f} = \frac{2(\eta - 1)}{R}$

(f) Linear magnification: $m = \frac{I}{O} = \frac{v}{u} = \frac{f-v}{f} = \frac{f}{f+u}$

(g) Power of lens: $P = \frac{1}{f}$

(h) Lenses in contact:

(i) $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$; (ii) $P = P_1 + P_2$

(iii) For lenses separated by a distance d = $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

(i) Achromatic lens combinations: Condition of achromatism, $\frac{\omega}{f_y} = -\frac{\omega'}{f'_y}$

36. **Silvering at one surface**:

(a) $\frac{1}{F} = \frac{1}{f_\ell} + \frac{1}{f_m} + \frac{1}{f_\ell} = \frac{2}{f_\ell} = \frac{2(\eta - 1)}{R}$

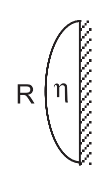


Fig. 1

Spectrum

(b) $\frac{1}{F} = \frac{2}{f_\ell} + \frac{1}{f_m} = 2\left[\frac{(\eta - 1)}{R}\right] + \frac{2}{R} = \frac{2n}{R}$

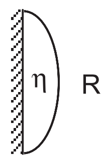


Fig. 2

(c) $\frac{1}{F} = \frac{2}{f_\ell} + \frac{1}{f_m} = 2(\eta - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{2}{R_2}$

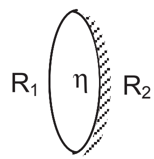


Fig. 3

37. Optical Instruments

(a) Astronomical Telescope:

(i) For normal adjustment: m = $\frac{f_0}{f_e}$

(ii) For near–point adjustment: m = $\frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$

(b) Simple Microscope:

(i) For normal adjustment: m = $\frac{D}{f}$

(ii) For near–point adjustment = m = $1 + \frac{D}{f}$

(c) Compound Microscope:

(i) For normal adjustment: m = $\frac{v_0}{u_0}\left[\frac{D}{f_e}\right]$

(ii) For near–point adjustment: m = $\frac{v_0}{u_0}\left[1 + \frac{D}{f_e}\right]$

Spectrum

MODERN PHYSICS
CATHODE RAYS AND POSITIVE RAYS

1. Cathode rays

(a) Thomson identified cathode rays as an electron beam.

(b) Specific charge q/m as measured by Thomson is: $(q/m) = 1.759 \times 10^{11}$ Coulomb/Kg

2. Positive rays

(a) Positive rays were discovered by Goldstein.

(b) (q/m) for positive rays is much less than that of electrons.

3. Motion of charge particle through electric field (Field ⊥ to initial velocity)

(a) The path is parabolic: $y = (qE/2mu^2)x^2$

(b) The time spent in the electric field: $t = (L/u)$

(c) The y–component of velocity acquired: $v_y = (qEL/mu)$

(d) The angle at which particle emerges out $\tan\theta = qEL/mu^2$

(e) The displacement in y-direction, when the particle emerges out of the field: $y_1=(qEL^2/2mu^2)$

(f) The displacement on the screen = $Y = (qELD/mu^2)$

4. Motion of charged particle through magnetic field (Field ⊥ to initial velocity)

(a) The path is circular with radius: $r = (mu/qB)$

(b) Momentum of the particle: $p = qBr$

(c) The deflection on the screen: $X = (qBLD/mu)$

5. Mass spectrographs

(a) Thomson's mass spectrograph

- (i) Traces on the screen are parabolic in nature
- (ii) Inner parabola corresponding to heavy M white outer parabola to light M.
- (iii) The upper portion of parabola is due to small v ions, while lower portion is due to high v ions.
- (iv) Only $v = \infty$ ions can reach vertex of parabola.
- (v) Equation of parabola: $X^2 = (B^2LD/E)\,(q/M)\,Y = K\,(q/M)\,Y$

(b) Brain bridge mass spectrograph

- (i) Velocity selector: $v = (E/B)$
- (ii) Other relations: $r = (Mv/qB') = (ME/qBB')$ (whre B' is the magnetic field in dome); $d = 2r$; $(d_2 - d_1) \propto (M_2 - M_1)$; $M_1 : M_2 = d_1 : d_2$ [where d_1 and d_2 are the distances of traces 1 and 2 from the slit S_2 of velocity selector].

Spectrum

PHOTOELECTRIC EFFECT

6. **Threshold frequency**: $v_0 = \frac{\text{Work function}}{h} = \frac{W}{h}$

7. **Threshold wavelength**: $\lambda_0 = \frac{c}{v_0} = \frac{hc}{hv_0} = \frac{hc}{W}$

(To calculate λ_0, use hc = 1240 (eV) (nm) = 1.24 x 10^{-6} eV) (m)

8. Maximum kinetic energy of emitted photoelectrons

(a) $K_{max} = \frac{1}{2} mv^2_{max} = eV_0$

(b) $K_{max} = hv - W = h(v - v_0) = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$

9. **Slope of (V_0 v) graph** = $\frac{h}{e}$

10. Energy, momentum and mass of a photon

(a) Rest mass of photon = 0 (b) $E = hv = \frac{hc}{\lambda}$

(c) $p = \frac{E}{c} = \frac{h}{\lambda}$ (d) $m = \frac{E}{c^2} = \frac{h}{c\lambda}$

11. **Number of photons**:

(a) number of photons per sec per m^2, $n_p = \frac{\text{Intensity}\left(\text{Watt/m}^2\right)}{h\,v}$

(b) number of photons incident per second, $n_p = \frac{\text{Power}\left(\text{Watt}\right)}{hv}$

(c) number of electrons emitted per second = (efficiency of surface) x number of photons incident per second.

12. **Compton wavelength**:

(a) $\lambda_c = \frac{h}{m_0 C}$ =2.426 pm

(b) Change in wavelenth, $(\lambda' - \lambda) = \lambda_c (1 - \cos\phi)$

Spectrum

ATOMIC STRUCTURE

13. Rutherford's α–particle scattering

(a) $N(\theta) \propto \text{cosec}^4(\theta/2)$

(b) Impact parameter, $b = \frac{(Ze^2)\cot(\theta/2)}{(4\pi\varepsilon_0)E}$, (where $E = \frac{1}{2}mu^2$ = KE of the α–particle)

14. **Distance of closest approach**: $r_0 = \frac{2Ze^2}{(4\pi\varepsilon_0)E}$ (where $E = \frac{1}{2}mu^2$ = KE of the α–particle)

15. Bohr's atomic model

(a) $L = mvr = \frac{nh}{2\pi}$

(b) $h\nu = E_i = E_f = \frac{hc}{\lambda}$

(c) Radius of nth orbit:

(i) $r_n \propto \frac{n^2}{Z}$, (ii) $r_n = \frac{n^2}{Z}\left(\frac{h^2}{4\pi^2 m ke^2}\right)$

(iii) Bohr's radius: $a_0 = (h^2/4\pi^2mke^2) = 0.529$ Å

(iv) Ratio of radii: $r_1 : r_2 : r_3 = 1 : 4 : 9$; $r_N : r_{He^+} : r_{Li^{++}} = 1 : \frac{1}{2} : \frac{1}{3} = 6 : 3 : 1$

(d) Velcotiy of electron in nth orbit:

(i) $v_n = \frac{Z}{n}\left(\frac{c}{137}\right) = \frac{Z}{n}\alpha c$ (where $\alpha = \frac{2\pi Ke^2}{ch} = \frac{1}{137}$ = fine structures constant)

(ii) $v_1 : v_2 ; v_3 = 1 : \frac{1}{2} : \frac{1}{3} = 6 : 3 : 2$

(iii) v_1 = velocity of electron is 1st orbit of H–atom = (c/137)

(e) Total energy of electron:

(i) Potential energy, $U = -(kZe^2/r)$

(ii) $K = \frac{1}{2}mv^2 = (kZe^2/2r)$

(iii) $E = K + U = -(kZe^2/2r) = (U/2) = -K$

(iv) $K = -(U/2)$ or $U = 2K = 2E$

(v) $E_n = -\frac{13.6\,Z^2}{n^2}$ eV $= -\frac{Z^2}{n^2}\left(\frac{2\pi^2 m k^2 e^4}{h^2}\right) = -\frac{2.18 \times 10^{-18} Z^2}{n^2}$ J

(f) Ionization energy $= -E_1 = +(13.6Z^2)$eV

(i) For H–atom, I.E. = 13.6 eV

(ii) For He^+ – ion, I.E = 54.4 eV

(iii) For Li^{++}–ion, I.E. = 122.4 eV

(g) Ionization potential:

(i) For H–atom, I.P. = 13.6 V

(ii) For He^+ ion, I.P. = 54.42

(h) Series formula (wave number $\bar{\nu} = 1/\lambda$

Spectrum

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad \text{where } R = \frac{2\pi^2\, m\, k^2 e^4}{c\, h^3} = 1.097 \times 10^7 \text{ m}^{-1}$$

(i) Series formula for H–atom

(i) Lyman series: $\frac{1}{\lambda} = R\left(1 - \frac{1}{n^2}\right), n = 2, 3, 4, ...\infty$

(ii) Balmer series: $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), n = 3, 4, 5....\infty$

(iii) Paschen series: $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right), n = 4, 5, 6....\infty$

(iv) Brackett series: $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$, n = 5, 6, 7….∞

(v) P–fund series: $\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right), n = 6, 7, 8 ....\infty$

(j) Series limits (λ_{min})

(i) Lyman: λ_{min} = 912 Å

(ii) Balmer: λ_{min} = 3645 Å

(iii) Paschen: λ_{min} = 8201 Å

16. **Number of emission lines from excited state n** = n(n–1)/2

17. Time period of revolution

(a) $T_n \propto (n^3/Z^2)$; (b) $T_1 = 1.5 \times 10^{-16}$ sec ; (c) $T_1 : T_2 : T_3 = 1 : 8 : 27$

18. Frequency of revolution

(a) $v_n \propto (Z^2/n^3)$; (b) $v_1 = 6.6 \times 10^{15}$ Hz ; (c) $v_1 : v_2 : v_3 = 1 : \frac{1}{8} : \frac{1}{27}$

19. Current due to orbital motion

(a) $I_n \propto (Z^2/n^3)$; (b) I_1 = 1 mA

20. Magnetic field at nucleus due to orbital motion of electron

(a) $B_n \propto (Z^3/n^5)$; (b) B_1 = 12.5 Tesla

21. **Magnetic moment**:

(a) $M_n = (eL/2m) = (nhe/4\pi m)$;

(b) $M_1 = (eh/4\pi m) = \mu_B$ = Bohr Magneton = 9.27×10^{-24} Am^2

22. **Magnitude of angular momentum**: $L = \sqrt{[\ell(\ell+1)]}\,(h/2\pi)$

23. Angle of angular momentum vector from z–axis

(a) $\cos\theta = [m_\ell/\sqrt{\{\ell(\ell+1)\}}]$; (b) the least angle is for $m\ell = \ell$ i.e. $\cos\theta_{min} = [\ell/\sqrt{\{\ell(\ell+1)\}}]$

24. Magnitude of spin angular momentum

$S = \sqrt{[s(s+1)]}\,(h/2\pi) = \frac{\sqrt{3}}{2}(h/2\pi)$

X–RAYS

25. **Continuous X–rays**:

(a) $v_{max} = (eV/h)$; (b) $\lambda_{min} = (hc/eV) = (12400/V)$ Å

Spectrum

26. **Characterictic X–rays**:

(a) $\lambda_{K\alpha} < \lambda_{L\alpha} < \lambda_{M\alpha}$; (b) $v_{K\alpha} > v_{L\alpha} > v_{M\alpha}$

27. **Frequency of K_α line** : $v(K_\alpha) = \frac{3cR}{4}(Z-1)^2 = 2.47 \times 10^{15}(Z-1)^2$

28. **Wavelength of K_α line**: $\lambda(K_\alpha) = [4/3R(Z-1)^2] = [1216/(Z-1)^2]$Å

29. **Energy of K_α X–ray photon**: $E(K_\alpha) = 10.2\,(Z-1)^2$ eV

30. **Mosley's law**:

(a) $v = a\,(Z-b)^2$, where $a = (3cR/4) = 2.47 \times 10^5$ Hz

(b) For K_α line, b = 1; (c) $\sqrt{v} \propto Z$

31. **Bragg's law**: $2d\sin\theta = n\lambda$

32. **Absorption formula**: $I = I_0 e^{-\mu x}$

33. **Half–value thickness**: $x_{1/2} = (0.693/\mu)$

MATTER WAVES

34. **For photons**:

(a) $E = hv = (hc/\lambda)$;

(b) $p = (hv/c) = (E/c) = (h/\lambda)$;

(c) $m = (E/c^2) = (hv/c^2) = h/c\lambda$

(d) rest mass = 0, charge = 0, spin = 1 $(h/2\pi)$

35. **Matter waves**:

(a) de Broglie wavelength, $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$ $\left[\because E = \frac{1}{2}mv^2 = qV\right]$

(b) (i) For electron $\lambda_e = \frac{12.27}{\sqrt{V}}$ Å

(ii) For proton, $\lambda_p = \frac{0.286}{\sqrt{V}}$ Å

(iii) For alpha particle $\lambda_\alpha = \frac{0.101}{\sqrt{V}}$ Å

(c) For particle at temperature T : $\lambda = \frac{h}{\sqrt{3mKT}}$ $\left(E = \frac{3}{2}kT\right)$

(i) For neutron or proton: $\lambda = (25.2/\sqrt{T})$ Å [if $E = (3/2)\,kT$, average energy]

but $\lambda = \frac{30.8}{\sqrt{T}}$ Å [if E = kT, most probable energy]

(d) The wavelength of electron accelerated by potential difference of V volts is: $\lambda_e = \frac{12.27}{\sqrt{V}}$ Å

Hence, accelerating potential required for obtaining de Broglie wavelength for as electron is:

$$V = \frac{150.6}{\lambda_e^2}\,\text{volt}$$

(e) Condition for obtaining stable orbit: $2\pi r_n = n\lambda$

(f) The phase velocity of a de Broglie wave of wavelength λ and frequency v is

$$v_p = v\lambda = \frac{E}{h} \times \frac{h}{mv} = \frac{mc^2}{h} \times \frac{h}{mv} = \frac{c^2}{v} \text{ i.e. } v_p > c.$$

(g) Group velocity, $v_g = (d\omega/dk)$. It is found that group velocity is equal to particle velocity i.e., $v_g = v$

Spectrum

RADIOACTIVITY

36. **Decay law**: (a) $(dN/dt) = -\lambda N$; (b) $N = N_0 e^{-\lambda t}$; (c) $(N/N_0) = (1/2)^{t/T}$

37. **Half life and decay constant**:

(a) $\lambda = -\frac{(dN/dt)}{N}$; (b) $\lambda T = \log_e 2$ or $T = (0.693/\lambda)$ or $\lambda = (0.693/T)$

38. **Mean life**:

(a) $\tau = (1/\lambda)$ or $\lambda = (1/\tau)$; (b) $T = 0.693\tau$ or $\tau = 1.443\ T$

39. **Activity**:

(a) $R = |dN/dt|$; (b) $R = \lambda N$; (c) $R = R_0 e^{-\lambda t}$; (d) $(R/R_0) = (1/2)^{t/T}$;
(e) 1 Becquerel = 1 dps ; (f) 1 curie = 1 ci = 3.7×10^{10} dps;
(g) 1 Rutherford = 1Rd = 10^6 Rd = 10^6 dps

40. **Decay of active mass**:

(a) $m = m_0 e^{-\lambda t}$; (b) $(m/m_0) = (1/2)^{t/T}$; (c) $N = \frac{6.023 \times 10^{23} \times m}{A}$

41. **Radioactive equilibrium**: $N_A\lambda_A = N_B\lambda_B$

42. **Decay constant for two channels**: (a) $\lambda = \lambda_1 + \lambda_2$; (b) $T = \frac{T_1 T_2}{T_1 + T_2}$

43. **Gamma intensity absorption**: (a) $I = I_0 e^{-\mu x}$; (b) Half value thickness, $x_{1/2} = (0.693/\mu)$

NUCLEAR PHYSICS

44. **Atomic mass unit**: (a) 1 amu = 1.66×10^{-27} kg ; (b) 1 amu $\equiv$ 1u $\equiv$ 931.5 MeV

45. Properties of nucleus

(a) Radius: $R = R_0 A^{1/3}$ where R_0 = 1.2 fermi
(b) Volume: $V \propto A$ $\left[\because V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3\ A\right]$
(c) Density: $\rho = 2.4 \times 10^{17}$ Kg/m^3 (ρ is independent of A)

46. **Mass defect**: $\Delta M = Zm_p + (A-Z)\,m_n - M$

47. **Packing fraction**: $f = \Delta/A$ = mass excess per nucleon [$\Delta = -\Delta M$ = mass excess]

48. **Binding energy**: $\Delta E = BE = (\Delta M)c^2$

49. **Binding energy per ncuelon**:

(a) BEN = (BE/A);
(b) BEN for Helium = 7.1 MeV/nucleon
(c) BEN for Deuterium = 1.1 MeV/nucleon

ELECTRONICS

50. Richarson equation

(a) $J = AT^2 e^{-W/KT}$ where $A = 60 \times 10^4$ A/K^{2}m^2
(b) $J = AT^2 e^{11600\,W/T}$ [$\therefore$ K = Boltzmann's constant = 1.38×10^{-23} J/K = 8.62×10^{-5} eV/K
Hence, 1/K = 11600 kelvin/eV]
(c) $I = AT^2 S e^{-W/KT}$

Spectrum

51. **Child's law**: $I_p = KV_p^{3/2}$ [K = constant of proportionality]

52. Diode resistance

(a) Static plate resistance: (i) $R_p = (V_p / I_p)$; (ii) $R_p \propto V_p^{-1/2}$ (iii) $R_p \propto I_p^{-1/3}$

(b) Dynamic plate resistance: (i) $r_p = (\Delta V_p / \Delta I_p)$; (ii) $r_p \propto v_p^{-1/2}$; (iii) $r_p \propto I_p^{-1/3}$.

53. **Triode Constants**:

(a) $r_p = \left(\frac{\Delta V_p}{\Delta I_p}\right)_{V_g = \text{constant}}$; (b) $g_m = \left(\frac{\Delta I_p}{\Delta V_g}\right)_{V_p = \text{constant}}$;

(c) $\mu = \left(\frac{\Delta V_p}{\Delta V_g}\right)_{I_p = \text{consant}}$; (d) $\mu = r_p \times g_m$; (e) $r_p \propto I_p^{-1/3}$; (f) $g_m \propto I_p^{1/3}$

54. **Plate current equation**: $I_p = K\left(V_g + \frac{V_p}{\mu}\right)^{3/2}$

55. **Cut off voltage**: $V_g = -(V_p/\mu)$

56. **Triode as an amplifier**:

(a) $I_p = (\mu V_g / R_L + r_p)$; (b) $A = (\mu R_L / R_L + r_p)$

(c) $A_{max} = \mu$; (d) $\mu = A\left(1 + \frac{r_p}{R_L}\right)$; (e) $A = \mu/2$ if $R_L = r_p$

57. Conductivity of semi conductors

(a) Intrinsic: (i) $\sigma = e(n_e \mu_e + n_h \mu_h)$; (ii) $\sigma = \sigma_0 e^{-E_g/2KT}$

(b) Extrinsic: (i) n–type : $\sigma = e n_e \mu_e$; (ii) p–type : $\sigma = e n_h \mu_h$

58. **Transistor**:

(a) $I_E = I_C + I_B$ $(I_B << I_E, I_B << I_C)$

(b) Current gains: (i) $\alpha = \frac{I_C}{I_E}$, $\alpha_{ac} = \frac{\Delta I_C}{\Delta I_E}$

(ii) $\beta = \frac{I_C}{I_B}$, $\beta_{ac} = \frac{\Delta I_C}{\Delta I_B}$

(c) Relation between α and β: $\alpha = \frac{\beta}{1+\beta}$ or $\beta = \frac{\alpha}{1-\alpha}$

Spectrum