$0, 1$, 10, 100

345.19

$\times 100 + 4\times10 + 5\times1 + 1\times\frac{1}{10} + 9\times\frac{1}{100}$

Number Systems.

Numbers

Real No.

can be represented in or number line.

Used in daily life

Complex no.

Cannot be plotted in number line

eg:- $a + ib$

imaginary n

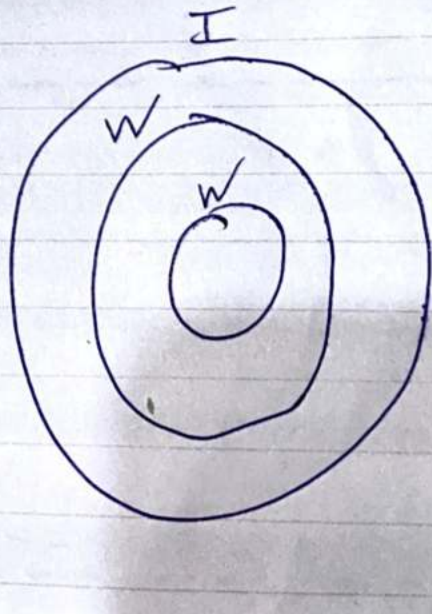
Z or I

Integers - -ve, 0, +ve

No decimals.

Decimals - 1.3, 7.59, etc

2 conc. even or odd nos = 2



hich of the foll. are integers

-31, $-\frac{2}{3}$, 0.7, 0, 46

Natural Nos. $\{0, 1, 2, 3 \ldots n\}$
↳ Any No. which starts from 0

whole No $= \{1, 2, 3 \ldots\ldots\}$
↳ Any No. which starts from 1

rational No
↳ Any No that can be written in the form of $\frac{p}{q}$ where $q \neq 0$

$x^0 = 1$ (anything to the power of $0 = 1$)

Non terminating decimal

egs:- $\sqrt{2}$, π
$\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$,
$\sqrt{11}$

eg:- $\frac{1}{3} = 0.333\ldots = 0.\overline{3}$
$2.1456\,1456\,1456\ldots = 2.\overline{1456}$
$3.2555\ldots = 3.2\overline{5}$

Identify rational & irrational nos. from the fo

Rational.

40.52, 3.1111, −1.232323…, 3.1415…

0.12736…, 6.1720428

Irrational.

Even & Odd Nos. → Which are not divisible by 2

Nos divisible by 2

Even = 2, 4, 6, 8, 10, …

odd = 1, 3, 5, 7, 8 …

Imp. properties of even & odd.

Add

a) Even + Even = Even.

2 + 4 = 6

b) Even + odd = odd

2 + 3 = 5

c) odd + odd = even

3 + 5 = 8

b

4 - 2 = 2
Even - even = Even.

Even - odd = odd
6 - 1 = 5

odd - odd = even.
5 - 3 = 2

mult :-

Even × Even = Even.
2 × 4 = 8

Even × odd = Even.
2 × 3 = 6

odd × odd = odd
3 × 5 = 15.

prime nos :- divisible by 1 & itself.

eg :- 2, 3, 5, 7, 11, 13, 17, . . ~

Composite no :- which have more than 2 fac.

eg 6 = 1, 2, 3, 6

9 = 1, 3, 9

★ 1 is neither a prime nor a composite Number

Smallest prime no - 2

Smallest composite - 4

Smallest and even prime no - 2

-11- odd prime no = 3

Smallest even composite - 4

-11- odd composite - 9

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ab is a 2 digit no b/w 11 - 20
where $a \neq b$ find ba such that
it is also a prime no.

$ab = 13, 17, 19$

$ba = 31, 71, 91$

HCF → Highest common Factor.

Factor = 12, 18

$12 = 2 \times 2 \times 3$
$18 = 2 \times 3 \times 3$
HCF $= 2 \times 3 = 6$

25, 30, 50

$25 = 5 \times 5$
$30 = 5 \times 2 \times 3$
$50 = 5 \times 5 \times 2$
HCF $= 5$

P.T.O

HCF of 12, 18, 36
division method.

$$\begin{array}{c|c}
2 & 12, 18, 36 \\ \hline
3 & 6, 9, 18 \\ \hline
 & 2, 3, 6
\end{array}$$

HCF $\Rightarrow 2 \times 3 = \underline{\underline{6}}$

$$\begin{array}{c|c}
5 & 30, 40, 50 \\ \hline
2 & 6, 8, 10 \\ \hline
 & 3, 4, 5
\end{array}$$

HCF $= 5 \times 2 = \underline{\underline{10}}$

LCM = lowest common Multiple

2, 4, 6

P.F

$2 = 1 \times 2$

$4 = 1 \times 2 \times 2$

$6 = 1 \times 2 \times 3$

$= 1 \times 2 \times 2 \times 3$

$= 12$

division

$$\begin{array}{r|l} 2 & 2, 4, 6, \\ \hline 2 & 1, 2, 3 \\ \hline 3 & 1, 1, 3 \\ \hline & 1, 1, 1 \end{array}$$

LCM $= 2 \times 2 \times 3$

$= 12$

Find LCM

12, 15, 18

12 = 2 × 2 × 3

15 = 5 × 3

18 = 2 × 3 × 3

= 3 × 2 × 2 × 3 × 5

= 180

2	12, 15, 18
3	6, 15, 9
2	2, 5, 3
5	1, 5, 3
3	1, 1, 3
	1, 1, 1

3
12
15
180

2	12, 15, 18
2	6, 15, 9
3	3, 15, 9
3	1, 5, 3
5	1, 5, 1

Find smallest no divisible by

3	21, 28, 45
7	7, 28, 15
	1, 4, 15

3	21, 28, 45
3	7, 28, 15
7	7, 28, 5
4	1, 4 ~~28~~, 5
5	1, 1, 5
	1, 1, 1

$LCM = 3 \times 3 \times 4 \times 5 \times 7$

$= 1260$

Find 3 digit largest Number divisible by (LCM)

5	15	25	35
	3	5	7

3	15	25	35
5	5	25	35
5	1	5	7
7	1	1	7
	1	1	1

$$\begin{array}{r} 75 \\ \times 7 \\ \hline 525 \end{array}$$

LCM = 525

★ LCM × K (for 4 digit)

where K = constant

K = 2

∴ 525 × 2 = 1050 (smallest 4 digit no)

Find largest no. which divides 720 & 1680 completely. (HCF)

3	720	1680
2	240	560
2	120	280
2	60	140
2	30	70
5	15	35
	3	7

don't start with 3

(start with lowest)

$$\begin{array}{r} 80 \\ \times 3 \\ \hline 240 \end{array}$$

24

21

For fractions

$$\text{LCM of fraction} = \frac{\text{LCM of Numerator}}{\text{HCF of denominator}}$$

$$\text{HCF of fraction} = \frac{\text{HCF of Numerator}}{\text{LCM of denominator}}$$

Q) Find the HCF & LCM of $\frac{4}{3}$ & $\frac{3}{7}$

$$LCM = \frac{\text{LCM of }(4,3)}{\text{HCF of }(3,7)} = \frac{12}{1} = 12$$

$$HCF = \frac{\text{HCF of }(4,3)}{\text{LCM of }(3,7)} = \frac{1}{21}$$

Q2 HCF & LCM of $4\frac{1}{2} = \frac{9}{2}$, $\frac{6}{2}$, $10\frac{1}{2} = \frac{21}{2}$

$$LCM = \frac{\text{LCM of }(9,6,21)}{\text{HCF of }(2,2,2)} = \frac{126}{2} = 63$$

LCM

2	9	6	21
3	9	3	21
	3	1	7

$$HCF = \frac{\text{HCF of }(9,6,21)}{\text{LCM of }(2,2,2)} = \frac{3}{2}$$

HCF

3	9	6	21
	3	2	7

Q3 LCM & HCF of $0.3, 0.7, 0.9$

$\frac{3}{10}, \frac{7}{10}, \frac{9}{10}$

$LCM = \frac{LCM(3,7,9)}{HCF(10,10,10)} = \frac{63}{10}$

$= \frac{63}{10} = 6.3$

$HCF = \frac{HCF(3,7,9)}{LCM(10,10,10)} = \frac{1}{10} = 0.1$

$HCF \times LCM = 1^{st} no \times 2^{nd} no$

→ HCF × LCM = product of 2 nos.

$HCF = 15$

3	60, 75
5	20, 25
	4, 5

2	60, 75
3	30, 75
5	10, 25
	2, 5

LCM = 300

$15 \times 300 = 4500$

$60 \times 75 = 4500$

LCM & HCF of 2 Nos. are 225 & 5 respectively if first no. is 25, then find the other no.

$225 \times 5 = 1125$

$LCM \times HCF = a \times b$

$225 \times 5 = 25 \times b$

$b = \frac{225 \times 5}{25}$

$\frac{1125}{25} = 45$

$b = 45$

LCM & HCF of 2 no's are 200 & 15 resp. If first no is 60 find sec.

$200 \times 15 = 60 \times b$

$b = \frac{200 \times 15}{60}$

$b = 50$

Find 4 digit smallest no which is divisible by 5, 8, 9, 15

2	5, 8, 9, 15
2	5, 4, 9, 15
2	5, 2, 9, 15
3	5, 1, 9, 15
3	5, 1, 3, 5
5	5, 1, 1, 5
	1, 1, 1, 1

$24 \times 3 = 72$

$72 \times 5 = 360$

constant

$360 \times K = 360 \times 3$

$2, 3, 5$

$= 1080$

Find 3 digit small no divisible by

2 | 4, 5, 12
2 | 2, 5, 6
3 | 1, 5, 3
5 | 1, 5, 1
| 1, 1, 1

LCM = 60

60 × K = 60 × 2
= 120

∴ The required 3 digit smallest no is 120.

Find 3 digit largest no which when divided by 8, 12, 15 leaves remainder 7 in each case.

2 | 8, 12, 15
2 | 4, 6, 15
2 | 2, 3, 15
3 | 1, 3, 15
5 | 1, 1, 5
| 1, 1, 1

24
× 5

LCM = 120

∴ The reqd no = ~~120+7~~ 120K + 7
let K = 8
120 × 8 + 7
960 + 7 = 967

find the no when divided by 7 & 8 leaves remainder 1

LCM = 56

The reqd no = 56 + 1
= 57

Find the no when divided by 7 & 8 leaves remainder 1 & 2 respectively

LCM = 7 × 8
= 56

6 (7, 8) 6
1, 2

Reqd = LCM (7, 8) − 6
= 56 − 6
= 50

Find 4 digit smallest which when divided by 7, 8, 9 leaves remainder 4, 5 & 6 respectively.

3 (7, 8, 9) 3
4, 5, 6

2 | 7, 8, 9
2 | 7, 4, 9
2 | 7, 2, 9
3 | 7, 1, 9
3 | 7, 1, 3
7 | 7, 1, 1
| 1, 1, 1

Reqd = LCM − 3
= 504 − 3
= 501

LCM = 504K
= 504 × 2
= 1008
= 1008 − 3
= 1005

24
× 3
72
× 7
504

What is the smallest Number when divided by by 4, 5, 6 & 8 leaves remainder of 3 in each case but leaves no remainder when divided by 9.

2	4, 5, 6, 8
2	2, 5, 3, 4
2	1, 5, 3, 2
3	1, 5, 3, 1
5	1, 5, 1, 1
	1, 1, 1, 1

$Lcm = 120 + 3$
$= \underline{123}$

divisibility test of 9:-
Add the digits = sum should be divisible by 9

$\therefore 120K + 3$
let $K = 2$
$120 \times 2 + 3 = 240 + 3$
$= 243$

$243 = 2+4+3 = 9$ is divisible by 9.

3 bells ring together at 11 am. They ring at every 20 mins, 30 mins & 40 mins of interval resp. When will 3 bells ring together again.

Sol^n

2	20, 30, 40
2	10, 15, 20
2	5, 15, 10
3	5, 15, 5
5	5, 5, 5
	1, 1, 1

LCM = $\underline{120}$

$\therefore$ 120 mins
= 2 hrs
$\therefore$ 11:00 + 2:00 = 1:00 PM.

$\therefore$ All the 3 bells ring again at 1:00 PM.

Linear Equations:-

$3x + 5 = 15$ → variable

$3x = 10$

$x = 5$

coeff of variable

x^1 — linear — str. line

x^2 — quadratic — curve

x^3 — polynomial

$2x + 5 \geq 15$

$2x + 5 \leq 15$

Inequalities

= → Equal to

≥ → greater than or equal to

≤ → less than " "

> → greater than

< → less than

Solve the foll:-

1) $3(x+4) = 21$

$3x + 12 = 21$

$3x = 21 - 12$

$3x = 9$

$x = 3$

$3(x+4) = 21$

$x + 4 = \frac{21}{3} = 7$

$x + 4 = 7$

$x = 3$

better when left & right fully divisible.

2) $\frac{2x}{3} - \frac{3x}{5} = 8$

$\frac{10x}{15} - \frac{9x}{15} = 8$

$\frac{x}{15} = 8$

$x = 8 \times 15$

$x = 120$

3) $3x - 9 = 5x - 3$

$~~-9-3 = 5x+3x~~$

$~~6 = 2x~~$

$~~x = 3~~$

$3x - 5x = 9 - 3$

$-2x = 6$

$x = -3$

4) $3(x-3) = 4(2x+1)$

$3x - 9 = 8x + 4$

$3x - 8x = 9 + 4$

$-5x = 13$

$x = -\frac{13}{5}$

5) $\frac{(x-2)}{(x-3)} = \frac{(x-1)}{(x+1)}$

$(x-2)(x+1) = (x-1)(x-3)$

$x^2 + x - 2x - 2 = x^2 - 3x - x + 3$

$-x - 2 = -4x + 3$

$4x - x = 3 + 2$

$3x = 5$

$x = \frac{5}{3}$

6) $\frac{5-7x}{2+4x} = \frac{-8}{7}$

$7(5-7x) = -8(2+4x)$

$35-49x = -16-32x$

$35+16 = 49x-32x$

$51 = 17x$

$x = \frac{51}{17} = 3$

$x = 3$

7) $\frac{8x-5}{7x+1} = \frac{-4}{5}$

$5(8x-5) = -4(7x+1)$

$40x-25 = -28x-4$

$40x+28x = 25-4$

$68x = 21$

$x = \frac{21}{68}$

8) $2(x-2) - 5(x-5) = 4(x-8) - 2(x-2)$

$2x-4-5x+25 = 4x-32-2x+4$

$-3x+21 = 2x-28$

$-3x-2x = -28-21$

$-5x = -49$

$x = \frac{49}{5}$

9) $\frac{2}{3x-1} + \frac{3}{3x+1} = \frac{5}{3x}$

$\cancel{\frac{2(3x+1)}{(3x-1)(3x+1)} + \frac{3(3x-1)}{(3x+1)(3x-1)} = \frac{5}{3x}}$

$\frac{2(3x+1)+3(3x-1)}{(3x-1)(3x+1)} = \frac{5}{3x}$

$\frac{6x+2+9x-3}{(3x)^2-(1)^2} = \frac{5}{3x}$

$\frac{15x-1}{9x^2-1} = \frac{5}{3x}$

$3x(5x-1) = 5(9x^2-1)$

$45x^2-3x = 45x^2-5$

$+3x = +5$

$x = \frac{5}{3}$

10) $\frac{2x+5}{x+4} = 1$

$2x+5 = x+4$

$x = -1$

(i) $5-2(x+1) = 4(3-x) - 2x$

~~$5x - 2x - 5 + 2$~~

$5 - 2x + 2 = 12 - 4x - 2x$

$7 - 2x = 12 - 6x$

$6x - 2x = 12 - 7$

$4x = 5$

$x = \frac{5}{4}$

(2) $\frac{2}{x+3} = \frac{3}{5-x}$

$2(5-x) = 3(x+3)$

$10 - 2x = 3x + 9$

$10 - 9 = 3x + 2x$

$1 = 5x$

$x = \frac{1}{5}$

(3) $2(x-2) - 5(x-5) = 4(x-8) - 2(x-2)$

$2x - 4 - 5x + 25 = 4x - 32 - 2x + 4$

$-3x + 21 = 2x - 28$

$28 + 21 = 3x + 2x$

$49 = 5x$

$x = \frac{49}{5}$

HW

1) $\frac{3-7x}{15+2x} = 0$

2) $\frac{(0.4y-3)}{(1.5y+9)} = \frac{-7}{5}$

3) $2(3x-1) + 3(3x+1) = \frac{5}{3x}$

$2x-6 \quad +x-1 = 5x-5 - 2x + 4$

4) $2(x-3) + 1(x-1) = 5(x-1) - 2(x-2)$

5) $\frac{[17(2-y) - 5(y+12)]}{(1-7y)} = 8$

Ans: 1) $\frac{3-7x}{15+2x} = 0$

$3 - 7x = 0(15+2x)$

$3 - 7x = 0$

$+7x = +3$

$x = \frac{3}{7}$

Ans 2) $\frac{(0.4y-3)}{(1.5y+9)} = \frac{-7}{5}$

$5(0.4y-3) = -7(1.5y+9)$

$2y - 15 = -10.5y - 63$

$10.5y + 2y = -63 + 15$

$12.5y = -48$

$y = -3.84$

Ans 3) $2(3x-1) + 3(3x+1) =$

$6x - 2 + 9x + 3 = \frac{5}{3x}$

$15x + 1 = \frac{5}{3x}$

$3x(15x+1) = 5$

$45x^2 + 3x = 5$

$45x^2 + 3x - 5 = 0$

4) $2(x-3)+1(x-1) = 5(x-1) - 2(x-2)$
$2x-6+x-1 = 5x-5-2x+4$
$3x-7 = 3x-1$

5) $\frac{[17(2-y)-5(y+12)]}{(1-7y)} = 8$

$[17(2-y)-5(y+12)] = 8(1-7y)$

$34-17y-5y\cancel{+60}-60 = 8-56y$
$-22y-26 = 8-56y$
$56-22y = 26+8$
$34y = 34$
$y = 1$

1) The length of a rectangle is 4 times its width. If the perimeter of the rectangle is 80m, find the length & breadth of the rectangle.

Let width be x
So the $L = 4x$

$\therefore$ Perimeter $= 2(l+b)$
$80 = 2(4x+x)$
$80 = 2(5x)$
$40 = 5x$
$x = 8m$
$\therefore$ breadth $= 8m$
length $= 4x$
$= 4\times 8$
$= 32m$

2) The present ages of Andy & Mike are in the ration 5:3. If Andy had been 7 years older & Mike 7 years younger, the age of Andy would have been 3 times the age of Mike. Find their present ages.

$\rightarrow$ Let present age of
Andy $= 5x$
Mike $= 3x$

Given.
$5x+7 = 3(3x-7)$
$5x+7 = 9x-21$
$21+7 = 9x-5x$
$28 = 4x$
$4x = 28$
$x = 7$

$\therefore$ Present age
Andy $= 5\times 7 = 35$ years
Mike $= 3\times 7 = 21$ years.

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3) 3 nos are in ratio 2:3:4 & their sum is 54.

let the nos be $2x$, $3x$, $4x$

given;

$$2x+3x+4x = 54$$
$$9x = 54$$
$$x = 6$$

nos are $2x = 2\times6 = 12$

$3x = 3\times6 = 18$

$4x = 4\times6 = 24$

Maths

4) The sum of 3 consecutive even nos is 54. find the nos.

let the 1st even no. be x

2nd even no be $x+2$

3rd even no be $x+4$

$$\therefore x + x+2 + x+4 = 54$$
$$\therefore 3x + 6 = 54$$
$$3x = 48$$
$$x = \frac{48}{3}$$
$$x = 16$$

$\therefore$ the nos are 16, 18, 20

4) Divide 22 into 2 parts in such a way that $\frac{2}{3}$ of 1 part $= \frac{1}{4}$ of other part.

$= x + y = 22 \quad \therefore x = 22 - y$

$$\overbrace{x \quad | \quad 22-x}^{22}$$

let first part $= x$

2nd part $= (22-x)$

also $\frac{2}{3}x = \frac{1}{4}y$

$$\therefore \frac{2}{3}(22-y) = \frac{1}{4}y$$
$$2(22-y) = \frac{3}{4}y$$
$$44 - 2y = \frac{3}{4}y$$
$$176 - 8y = 3y$$
$$176 = 11y$$
$$y = 16$$

$$\therefore x + y = 22$$
$$\therefore x + 16 = 22$$
$$\therefore x = 6$$

The length of a ▭ is 4m less than 3 times its breadth. if perimeter of ▭ is 32 m. find its l & b.

let breadth be x

$l = \cancel{4-3x}\ 3x-4$

perimeter $= 2(l+b)$

$32 = 2(3x-4+x)$

$16 = 4x-4$

$\cancel{12=4x}$ $\quad 20 = 4x$

$\cancel{x=3}$ $\quad x = 5$

$\therefore l = (3\times5)-4$

$= 15-4$

$l = 11$

$\therefore l = 11m$ & $b = 5m$.

2 nos. are such that the ratio b/w them is 5:2. if each is increased by 1 the ratio b/w new nos so formed is 7:3 find the orig. nos.

let the nos be x

$\therefore$ 1st no $= 5x$ $\quad$ new nos :- 1st $= 5x+1$

2nd no $= 2x$ $\quad$ 2nd $= 2x+1$

$\therefore$ new nos.

$$\frac{5x+1}{2x+1} = \frac{7}{3}$$

$3(5x+1) = 7(2x+1)$

$15x+3 = 14x+7$

$1x = 4$

$x = 4$

Q If $\frac{3}{5}^{th}$ of a number is 4 more than $\frac{1}{2}$ the no then what is the no?

let the no be 'x'

$\frac{3}{5}x = 4 + \frac{1}{2}x$

$\frac{3x}{5} = 4 + \frac{x}{2}$

$3x = 5\left(4 + \frac{x}{2}\right)$

$3x = 20 + \frac{5x}{2}$

$3x - \frac{5x}{2} = 20$

$\frac{6x - 5x}{2} = 20$

$x = 40$

Supplimentary is 180°

Q The diff. in the measure of 2 complementary angles is 12°. find the measure of the angles

90°

→ Let the 1st ∠ be x

∴ complement of $x = 90 - x$

∴ $x - (90 - x) = 12$

∴ $x - 90 + x = 12$

$2x - 90 = 12$

$2x = 90 + 12$

$2x = 102$

$x = 51°$

∴ other angle

$B = 90 - x$

∴ $B = 90 - 51$

$B = 39°$

∴ the req^d ∠s are 51° & 39°

Q The cost of 2 tables & 3 chairs is ₹ 705. If the table costs ₹ 40 more than the chair find the cost of each chair & table.

let the cost of 1 chair be x.

let the cost of 1 table be x+40

∴ $2(x+40) + 3x = 705$

$2x + 80 + 3x = 705$

$5x + 80 = 705$

$5x = 785$ ~~$x = ₹157$~~

$5x = 625$

$x = ₹125$

∴ cost of 1 chair is ₹ 125.

∴ cost of 1 table $= x + 40$

$= 125 + 40$

$= ₹165$

Q The diff b/w 2 nos is 48. The ratio of 2 nos is 7:3 what are 2 nos.

let the nos be "7x & 3x"

$7x - 3x = 48$

$4x = 48$

$x = 12$

∴ the nos are $7x = 7 \times 12$

$= 84$

$3x = 36$.

Q 3 nos are in the ratio 3:4:5 & their LCM is 2400 then find their HCF.

Let the 3 nos be 3x, 4x & 5x.

$3x \times 4x \times 5x = 2400$

$60x^3 = 2400$

$x^2 = 40$

$\therefore$ HCF = 40

2 | 2400
2 | 1200
2 | 600
2 | 300
150

LCM = All the prime nos with Highest power.
HCF = common no with lowest.

2 5
2 2 3
2 7
$2 \times \cancel{2} \times 5 \times 3 \times 7$
420

Q Find LCM & HCF of 24, 36, 40.

2	24	36	40
2	12	18	20
2	6	9	10
3	3	9	10
3	1	3	10
2	1	1	10
5	1	1	5
	1	1	1

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

$40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5$

$LCM = 2^3 \times 3^2 \times 5^1$

$= 360$

What is the greatest no which divides 639, 1065 & 1491 exactly.

3 | 639 1065 1491
| 213 355 497

639) 1065 (1
639
426) 639 (1
426
213) 426 (2
426
xxx

$\therefore$ HCF of 1065 & 639 is 213

213) 1491 (7
1491
xxx

Linear eq. by Elimination method.

Soln:-

i) $2x+y=-4$ —(i)
$5x-3y=1$ —(ii)

$y=-2x-4$

$\therefore 5x-3y=1$
$5x-3(-2x-4)=1$
$5x+6x+12=1$
$11x=-11$
(substitution) $x=-1$

(elimination)
Multiplying eq i by 5
eq ii by 2

$\therefore 5(2x+y)=-4\times5$
$10x+5y=-20$

$2(5x-3y)=1\times2$
$10x-6y=2$

$\therefore 10x+5y=-20$
$-10x+6y=-2$
$11y=-22$
$y=-2$

put subs. $y=-2$ in eq i

$\therefore 2x+(-2)=-4$
$2x-2=-4$
$\therefore 2x=-2$
$x=-1$

$2x+y=-4$ -(i) } $\times3$
$5x-3y=1$ -(ii) } $\times1$

Multiplying eq (i) by 3 & eq 2 by 1

$3(2x+y)=-4\times3$
$6x+3y=-12$
$5x-3y=1$
$11x=-11$
$\therefore x=-1$

Subs $x=-1$ in eq (i)

$\therefore 2(-1)+y=-4$
$-2+y=-4$
$y=-4+2$
$y=-2$

$\therefore x=-1,\ y=-2$

ii) $2x+3y=13$ — (i) } $\times5$
$5x-2y=4$ — (ii) } $\times2$

Xing eq i by 5 & eq ii by 2

$\therefore 5(2x+3y)=13\times5$
$10x+15y=65$
$2(5x-2y)=4\times2$
$10x-4y=8$

$\therefore 10x+15y=65$
$-10x+4y=8$
$19y=57$
$y=3$

Puting $y=3$ in eq i
$\therefore 2x+3(3)=13$
$2x+9=13$
$2x=4$
$x=2$

$\therefore x=2,\ y=3.$

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iii) $15x - 3y = -6 \quad -(i)$ } $\times 10$
$14x - 10y = -56 \quad -(ii)$ } $\times 3$

x ing eq (i) by 10 & eq (ii) by 3

$\therefore 150x - 30y = -60$
$-42x + 30y = +168$
$108x = 108$

$\therefore x = 1$

Putting $x = 1$ in ii
$\therefore 14(1) - 10y = -56$
$14 - 10y = -56$
$-10y = -56 - 14$
$-10y = -70$
$y = 7$

iv) $2x + y = 56 \quad -(i)$ } $\times 1$
$x + 2y = 82 \quad -(ii)$ } $\times 2$

x ing eq (ii) by 2 & eq i by 1

$\therefore 2x + y = 56$
$2(x + 2y) = 82 \times 2$
$2x + 4y = 164$

$\therefore 2x + y = 56$
$-2x - 4y = -164$
$-3y = -108$
$y = 36$

$\therefore$ Subs ~~eq~~ $y = 36$ in eq ii

$x + 2y = 82$
$x + 2(36) = 82$
$x + 72 = 82$
$x = 10$

v) $x + y = 12 \quad -(i)$ } $\times 6$
$\frac{x}{y+1} = \frac{7}{6}$
$6x = 7y + 7$ ~~(ii)~~
$6x - 7y = 7 \quad -(ii)$ } $\times 1$

x ing eq (i) by 6 & eq (ii) by 1

$\therefore 6x + 6y = 72$
$-6x + 7y = -7$
$13y = 65$
$y = 5$

Putting $y = 5$ in eq(i)
$\therefore x + y = 12$
$x + 5 = 12$
$x = 12 - 5$
$x = 7$

Cross multiplication method.

2) $2x + 3y = 13$
$5x - 2y = 4$

3

A fraction is such that if numerator is Xed by 3 & the denominator is reduced by 2 we get $\frac{3}{5}$ but if the numerator is Ned by 4 & the denominator is doubled we get $\frac{5}{14}$ find the frac.

→ let the Numerator be x & denominator be y

$\therefore \frac{3x}{(y-2)} = \frac{3}{5}$ —(i) | $\frac{x+4}{2y} = \frac{5}{14}$ —(ii)

$15x = 3y - 6$
$15x - 3y = -6$
$5x - y = -2$ (÷ by 3) (iii)

$\cancel{14x} + 14x + 56 = 10y$
$14x - 10y = -56$
$7x - 5y = -28$ — (iv)

Xing eq iii by 5
$25x - 5y = -10$
$-7x + 5y = +28$
$18x + 0 = 18$

$\cancel{x} =$
$\boxed{x = 1}$

putting x=1 in eq 3

$5 - y = -2$
$-y = -7$
$\boxed{y = 7}$

$\therefore x = 1, \ y = 7$

$\therefore$ the reqd eq $= \frac{x}{y} = \frac{1}{7}$

The sum of the numerator & denominator of a frac. is 12. If the denominator is increased by 1, the fract becomes $\frac{7}{6}$. Find fract.ⁿ

let num. be x & denom by y

$x + y = 12$ —(i) | $\frac{x}{(y+1)} = \frac{7}{6}$
Xing (i) by 7 | $6x = 7y + 7$
| $6x - 7y = 7$ (ii)

$\therefore 7x + 7y = 84$
$6x - 7y = 7$
$13x = 91$

$\boxed{x = 7}$

putting x=7 in eq(i)

$7 + y = 12$
$\boxed{y = 5}$

$\therefore$ reqd ~~eq~~ frac $= \frac{7}{5}$

If twice the age of a son is added to the age of the father, the sum is 56.
But if twice the age of the father is added to the age of the son the sum is 82. find their ages

→ let the son's age be x & father's age be y

$\therefore 2x + y = 56$ (i)

$x + 2y = 82$ (ii)

Xing (i) by 2

$4x + 2y = 112$ - (iii)

$\underline{-x + 2y = 82}$

$3x = 30$

$\boxed{x = 10}$

putting $x = 10$ in eq (i)

$20 + y = 56$

$\boxed{y = 36}$

son's age is 10, father's age is 36.

A No consists of 2 digits whose sum is 5. When the digits are reversed, the number becomes greater by 9.

→ let the digit in units place be x & ten's place be y

$\therefore$ the no formed = $10y + x$

when digits are reversed = $10x + y$

by the given condition.

$x + y = 5$ — (i)

$10x + y = 10y + x + 9$

$10x - x + y - 10y = 9$

$9x - 9y = 9$

$x - y = 1$ - (ii)

$x + y = 5$

$x - y = 1$

$2x = 6$

$\boxed{x = 3}$

$\therefore \cancel{6 + y = 5}$

$\therefore \cancel{y = -1}$

$\therefore 3 + y = 5$

$\boxed{y = 2}$

Solve the foll by using the method mentioned below.

1) $3x + 4y = 11$
$7x + 3y = 7$
By the method of ~~substitution~~/Elimination.

2) $2x + y = -4$
$5x - 3y = 1$ } by cross multiplication.

Ans 1) $3x + 4y = 11$ — (i) $\times 3$
$7x + 3y = 7$ — (ii) $\times 4$
Xing eq (i) by 3
$3(3x + 4y) = 11 \times 3$
$9x + 12y = 33$ — (iii)

Xing eq (ii) by 4
$\therefore 4(7x + 3y) = 7 \times 4$
$28x + 12y = 28$ — (iv)

$\therefore 9x + 12y = 33$
$\underset{(-)}{}28x \underset{(-)}{+} 12y = \underset{(-)}{}28$
$-19x = 5$
$\therefore x = \frac{-5}{19}$

putting $x = \frac{-5}{19}$ in eq (i)

$\therefore 3\left(\frac{-5}{19}\right) + 4y = 11$

$-\frac{15}{19} + 4y = 11$

$-\frac{15}{19} + \frac{76y}{19} = 11$
$\therefore -15 + 76y = 11 \times 19$
$-15 + 76y = 209$
$76y = 224$
$y = \frac{224}{76}$
$y = \frac{56}{19}$

$\frac{56}{19}$ (rough work: $+12$, 224, 76, 38, 19)

$\frac{-15}{19} + \frac{224}{19}$

Roots of quadratic equation.

$ax^2 + bx + c = 0$

If α & β are the two roots of the eqⁿ then

$\alpha + \beta = \frac{-b}{a}$ | $\alpha\beta = \frac{c}{a}$

$$x^2 - (\text{sum of roots})x + (\text{products of roots}) = 0$$

$\therefore x^2 - (\alpha+\beta)x + (\alpha\beta) = 0$

$\therefore x^2 - \left(\frac{-b}{a}\right)x + \left(\frac{c}{a}\right) = 0$

$\therefore \frac{ax^2 - bx + c}{a} = 0$

$\therefore ax^2 - bx + c = 0$

Form a quad. eq. whose roots are $\alpha = 3+\sqrt{2}$ $\beta = 3-\sqrt{2}$

$\cancel{\alpha + \beta = \frac{-b}{a}}$ let $\alpha = 3+\sqrt{2}$ & $\beta = 3-\sqrt{2}$

$\cancel{(3+\sqrt{2}) + (3 ...}$

we know,

$$x^2 - (\alpha+\beta)x + (\alpha\beta) = 0$$

here, $\alpha + \beta = (3+\sqrt{2}) + (3-\sqrt{2}) = 6$

$\alpha\beta = (3+\sqrt{2})(3-\sqrt{2}) = 3^2 - (\sqrt{2})^2$ $\left[(a+b)(a-b) = a^2 - b^2\right]$

$= 9 - 2 = 7$

$\therefore x^2 - 6x + 7 = 0.$

If 1 of the root of eq $4x^2-2x+(p-4)=0$ is the reciprocal of the other find the value of p.

Let the 1st root be α

$\therefore$ the 2nd root $= \frac{1}{\alpha}$

$\therefore$ product of 2 roots $= \frac{c}{a}$

$\alpha \times \frac{1}{\alpha} = \frac{(p-4)}{4}$

$1 = \frac{(p-4)}{4}$

$4 = p-4$

$\therefore p = 4+4$

$\boxed{p = 8}$

If $p^2x^2 - q^2 = 0$, then $x = ?$

$p^2x^2 - q^2 = 0$

$\therefore p^2x^2 = q^2$

$x^2 = \frac{q^2}{p^2}$

$\therefore x = \pm\sqrt{\frac{q^2}{p^2}}$

$= \pm\frac{q}{p}$

$\therefore x = \frac{q}{p}$ or $x = -\frac{q}{p}$

A natural no when inc. by 12, equals 160 times its reciprocal. find x.

Let the no. be x.

given:

$x + 12 = \frac{160}{x}$

$\therefore x^2 + 12x = 160$

$\therefore x^2 + 12x - 160 = 0$

$\therefore x^2 + 20x - 8x - 160 = 0$

$\therefore x(x+20) - 8(x+20) = 0$

$(x+20)(x-8) = 0$

$\therefore x = -20$ or $x = 8$.

as x is a natural no.

so $x = 8$

160

Functions & Derivatives

Functions

Application of func^n in economics

1) Total cost — C

2) Avg. cost (AC) - $\frac{C}{x}$

3) Profit (P) = R - C

4) Revenue — R

5) Demand - D

6) Supply - S

eg. find TC & AC for the foll. where,

$C = 2x^2 + 3x - 2$ at $x = 1$ & $x = 5$.

$\therefore C_{x_1} = 2(1)^2 + 3(1) - 2$

$= 2 + 3 - 2$

$= 5 - 2$

$\underline{\underline{C_{x_1} = 3}}$

$C_{x_5} = 2(5)^2 + 3(5) - 2$

$= 2(25) + 15 - 2$

$= 50 + 15 - 2$

$= 65 - 2$

$\underline{\underline{C_{x_5} = 63}}$

$A \cdot C = \frac{C}{x}$

$= \frac{2x^2 + 3x - 2}{x}$

$= \frac{2x^2}{x} + \frac{3x}{x} - \frac{2}{x}$

$= 2x + 3 - \frac{2}{x}$

$A \cdot C_{x_1} = 2(1) + 3 - \frac{2}{1}$

$= 2 + 3 - 2$

$= 5 - 2$

$\underline{\underline{A \cdot C_{x_1} = 3}}$

$A C_{x_5} = 2(5) + 3 - \frac{2}{5}$

$= 10 + 3 - 0.4$

$= 13 - 0.4$

$\underline{\underline{A \cdot C_{x_5} = 12.6}}$

2) If tot. revenue func.

$R = 2x^2 + 3x + 25$

& tot. cost func. $C = x^2 + 10$

find tot. prof. & tot revenue.

$x = 2$

$\therefore P = R - C$

$= (2x^2 + 3x + 25) - (x^2 + 10)$

$= 2x^2 + 3x + 25 - x^2 - 10$

$P = x^2 + 3x + 15$

$\therefore P = (2)^2 + 3(2)$

$= 4 + 6 + 15$

$= 10 + 15$

$\underline{\underline{P = 25}}$

$R = 2x^2 + 3x + 2$

$R_{x_2} = 2(2)^2 + 3(2) + 2$

$= 2(4) + 6 + 25$

$= 8 + 6 + 25$

$= 14 + 25$

$\underline{\underline{R_{x_2} = 39}}$

3) If $R = 3x^2 + 10x - 2$
$C = 2x^2 - 8x - 10$
find tot. profit ~~for~~ total profit.
$x = 5$

$P = R - C$
$\therefore P = (3x^2 + 10x - 2) - (2x^2 - 8x - 10)$
$= (3x^2 + 10x - 2) - 2x^2 + 8x + 10$
$= 3x^2 - 2x^2 + 10x + 8x - 2 + 10$
$P = x^2 + 18x + 8$

$\therefore P_{x_5} = (5)^2 + 18(5) + 8$
$= 25 + 90 + 8$
$= 115 + 8$
$P_{x_5} = 123$

Functions

$2x + 3 = 5 \rightarrow$ linear.
$4x^2 + 3x + 7 = 0 \rightarrow$ quadratic.
$\left.\begin{array}{l} 3x + 2y = 12 \\ 4x - y = 3 \end{array}\right\}$ simultaneous.

$y = 3x^2 + 5x + 7$
$y = ax^2 + bx + c$
$y = ax + b$

y is a function of x
$\therefore y = f(x)$
where $f(x) = ax + b / ax^2 + bx + c$

here, y is dependent var., x is independent

$x = 5y^2 + 3y + 7$

☆ RHS is always independent
LHS is always dependent.

Types of functions:-

1) Constant function - Funcⁿ remains constant ~~for~~ for diff values of x.
$f(x) = K$ where K is constant.

eg $f(x) = 5$
$f(x) = -10$ $\quad f(x) = 0$
$f(x) = 4$

2) Linear func^n $f(x) = ax+b$ where a & b are constants

eg $f(x) = 2x+5$ $f(x) = -3x+2$

$f(x) = 3x$

$f(x) = 5x+7$

$f(x) = x-3$

3) Quadratic func^n: $f(x) = ax^2+bx+c$ where a, b & c are constants

eg:- $f(x) = 2x^2+5x+3$

$f(x) = 3x^2+4$

$f(x) = 5x^2+7x+1$

$f(x) = x^2-3x+4$

$f(x) = -3x^2+2x+1$

4) ~~Power~~ Power func^n

$f(x) = x^n$, n is any number

eg:- $f(x) = x^5$

$f(x) = x^{-7}$

$f(x) = x^{\frac{1}{3}}$

$f(x) = x^{-\frac{1}{2}}$

5) Exponential func^n

$f(x) = e^x$

where $e = 2.71$ - irrational no.

$f(x) = a^x$, a is any no.

eg: $f(x) = 4^x$

$f(x) = (-2)^x$

6) Step function

$f(x) = 3x+5$ for $0 \le x < 1$

$= 2x+3$ for $1 \le x < 3$

$= 6$ for $3 \le x < 5$

$= 0$ for > 5 find $f(1), f(0), f(4)$

sol^n:

$f(1) = 2x+3 \Big]_{x=1} = 2(1)+3 = 5$

$f_0 = 3x+5 \Big]_{x=0} = 3(0)+5 = 5$

$f_4 = 6$

find the values at $x = 0, 2, -3, 1, 6, -1$
for $f(x) = 7x+9$

Sol:- Given $f_x = 7x+9$

At $x=0$, $f_{(0)} = 7(0)+9$
$= 9$

At $x=2$ $f_{(2)} = 7(2)+9$
$= 14+9$
$= 23$

At $x=-3$ $f_{(-3)} = 7(-3)+9$
$= -21+9$
$= -12$

At $x=1$ $f_{(1)} = 7(1)+9$
$= 16$

At $x=6$ $f_{(6)} = 7(6)+9$
$= 42+9$
$f_6 = 51$

At $x=-1$ $f_{(-1)} = 7(-1)+9$
$= -7+9$
$f_{(-1)} = 2$

$f(x) = x^2-2x+7$
find the value at $x = \frac{1}{4}$

Sol Given $f(x) = x^2-2x+7$
At $x = \frac{1}{4}$, $f(\frac{1}{4}) = (\frac{1}{4})^2 - 2\times\frac{1}{4}+7$

$= \frac{1}{16} - \frac{1}{2} + 7$

$= \frac{1}{16} - \frac{8}{16} + 112$

$= \frac{112-7}{16}$

$f(\frac{1}{4}) = \frac{105}{16}$.

If $f(x) = \sqrt{x}$, find $f(4)$, $f(\frac{1}{4})$, $f(49)$, $f(\frac{1}{100})$

given $f(x) = \sqrt{x}$
at $x=4$, $f(4) = \sqrt{4} = 2$

at $x = \frac{1}{4}$ $f(\frac{1}{4}) = \sqrt{\frac{1}{4}} = \frac{1}{2}$

at $x = 49$ $f(49) = \sqrt{49} = 7$

at $x = \frac{1}{100}$ $f(\frac{1}{100}) = \sqrt{\frac{1}{100}} = \frac{1}{10}$

$2^3 = 2 \times 2 \times 2$

If $f(x) = x \cdot e^{x}$ find $f(0)$ & $f(1)$

1) At $x=0$, $f(0) = 0 \cdot e^0 = 0$

2) At $x=1$ $f(1) = 1 \cdot 2.71 = \underline{\underline{2.71}}$

$f(x) = x \cdot e^{2x}$

$f(2) = 2 \cdot e^4$
$= 2 \times (2.71)^4$
$= 2 \times (\cancel{40.84})$ $2 \times (53.93)$
$= \cancel{81.68}$ 107.87

$f(x) = 5 + e^{x}$ $f(0)$ & $f(2)$

at $x = 0$, $f(0) = 5 + e^0$
$= 5 + 1$
$= 6$

at $x = 2$, $f(2) = 5 + e^2$
$= 5 + (2.71)^2$
$= 5 + 7.3441$
$= \underline{\underline{12.3441}}$

If $f(x) = 3x + 2$ for $1 \le x \le 2$
$= 4x - 1$ for $2 < x \le 3$
$= 6$ for $3 < x \le 5$

find $f(2)$, $f(3)$, $f(4)$

At $x = 2$, $f(2) = 3x + 2$
$f(2) = 3(2) + 2$
$\underline{\underline{f(2) = 8}}$

At $x = 3$, $f(3) = 4(3) - 1$
$= 12 - 1$
$\underline{\underline{f(3) = 11}}$

At $x = 4$, $f(4) = 6$

If $f(x) = 3x+5$ for $-3 \le x \le -1$
$= 2x+1$ for $-1 \le x < 2$
$= 2-x$ for $2 \le x \le 4$

$f(-2)$, $f(2)$, $f(-1)$, $f(3)$, $f(1)$

At $x = -2$, $fx = 3x+5$
$f(2) = -6+5$
$f(2) = \cancel{11} -1$

At $x=-1$, $[fx = 2x+1]_{x=-1}$
$f_2 = -2+1 = \cancel{3} -1$

At $x = 2$, $fx = 2-x]_{x=2}$
$f(2) = 2-2$
$= 0$

At $x = 3$, $fx = 2-x$
$f(3) = 2-3$
$f(3) = -1$

At $x=1$, $fx = 2x+1$
$f(1) = 2(1)+1$
$= 2+1$
$f(1) = 3$

$fx = 3$ for $-3 \le x \le -1$
$= 6x-3$ for $-1 \le x \le 0$
$= 3x-3$ for $0 \le x \le 1$

find $f(-2)$, $f(\frac{1}{2})$, $f(0)$, $f(-1)$

at $x=-2$, $fx = 3$
$\therefore f(-2) = 3$

at $x = \frac{1}{2}$, $fx = 3x-3]_{x=\frac{1}{2}}$
$f\frac{1}{2} = 3\frac{1}{2} - 3$
$= \frac{3}{2} - 3$
$f(\frac{1}{2}) = -1.5$

at $x = 0$, $fx = 3x-3$
$f(0) = 3(0)-3$
$= 0-3$
$f(0) = -3$

at $x=-1$, $f(x) = 6x-3$
$f(-1) = 6(-1)-3$
$= -6-3$
$f(-1) = -9$

$f(x) = 3$ for $-3 \le x < -1$
$= 6x-3$ for $-1 \le x < 0$
$= 3x-3$ for $0 \le x \le 1$

find $f(-2)$, $f(\frac{1}{2})$, $f(-1)$, $f(0)$

$f(-2) = 3$

$f(\frac{1}{2}) = 3x-3$
$= \frac{3 \cdot 1}{2} - 3$
$= \frac{3}{2} - 3$
$= \frac{-3}{2}$

$f(-1) = 6x-3$
$= -6-3$
$= -9$

$f(0) = 3x-3$
$= 0-3$
$f(0) = -3$

$f(x) = x-2$ for $1 < x \le 2$
$= 2x-1$ for $2 < x < 4$
$= 25$ for $4 \le x \le 5$

find $f(2)$, $f(4)$ & $f(4.5)$

$f(2) = x-2$
$f(2) = 2-2$
$f(2) = 0$

$f(4) = 25$

$f(4.5) = 25$

$f(x) = 2x+a$ & $f(\frac{1}{2}) = 2$ find a.

$f(\frac{1}{2}) = \frac{2 \cdot 1}{2} + a$
$2 = 1 + a$
$a = 1$

Logarithmic func / log fn

$f(x) = a^x$

$y = a^x$

$\log_a y = x \rightarrow$ log func

$\hookrightarrow$ log of y with base a.

$\log_2 x \rightarrow$ log fn with base 2

$\log_e y \rightarrow$ log fn with $\boxed{\text{base } e}$
$\downarrow$
Natural Base.

properties:

1) $\log_e (xy) = \log_e x + \log_e y$

2) $\log_e \left(\frac{x}{y}\right) = \log_e x - \log_e y$

3) $\log_e (x^r) = r \log_e x$

Note $\log 1 = 0$

1) $f(x) = x^2 + \log x$

Find $f(1)$

$\therefore f(1) = 1^2 + \log 1$ $[\log 1 = 0]$

$= 1 + 0$

$f(1) = 1$

If $f(x) = \log x^5$

find $f(1)$

~~$f(1) = \log 1^5$~~

$f(x) = 5 \log x$

$f(1) = 5 \log 1$ $[\log 1 = 0]$

$= 5 \times 0$

$= 0$

If $f(x) = \log_e x$

Show that.

$f(xy) = f(x) + f(y)$

$f(x/y) = f(x) - f(y)$

$f(x^r) = r f(x)$

proof:

1) $f(xy) = \log_a (xy)$

$= \log_a x + \log_a y$.

$= f(x) + f(y)$ - proved

2) $f(x/y) = \log_a \left(\frac{x}{y}\right)$

$= \log_a x$ [cut off]

$= f(x)$ [cut off]

3) $f(x^r) = \log_a (x$ [cut off]

$= r \log_a$ [cut off]

$= r f(x)$ [cut off]

Demand fn (D)
p = price
D = Demand.

price ↓ → demand ↑

Supply fn (S)

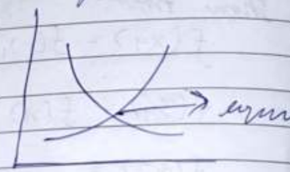
S↑ = P↑

Demand = Supply.

equilibrium price

1) The pt. at which demand & supply of goods are equal.

The quantity at equi. price is called equilibrium quantity



Total Revenue fun (R)

R = P D where P = price
D = demand.

Total cost fn (C)

C = fixed cost + variable.

$C = a + bx$
where a = fixed cost.
b = variable cost.

profit fn (P)

$P = R - C$

Q. The demand curve is given by $q = 2 - \left(\frac{1}{5}\right)p$ where q is quantity & p is price. If the corresponding supply curve is given $q = 0.2 + \left(\frac{7}{10}\right)p$, find the equilib. price & quantity offered at equilibrium price.

→ Demand = supply.

$2 - \left(\frac{1}{5}\right)p = 0.2 + \left(\frac{7}{10}\right)p$

$2 - 0.2 = \left(\frac{7}{10}\right)p + \left(\frac{1}{5}\right)p$

$1.8 = p\left(\frac{7+2}{10}\right)$

$1.8 = p\left(\frac{9}{10}\right)$

$18 = 9p$

$p = 2$

$q = 2 - \left(\frac{1}{5}\right)p$
$= 2 - \left(\frac{1}{5}\right)2$
$= 2 - \left(\frac{2}{5}\right)$
$= \frac{8}{5} = 1.6$

If $p \log D = 24$, find the total revenue b

Total Revenue $(R) = pD$

Given $p \log D = 24$

$$\therefore p = \frac{24}{\log D}$$

$$\therefore R = pD$$

$$\therefore p = \frac{24}{\log D} \times D$$

$$= \frac{24D}{\log D}.$$

Q $y = q$ & x is price of a commodity demand curve is given by $2x + y - 600 = 0$ supply curve is given by $5x - y - 100 = 0$ find eq price & quantity offered at eq price

Demand curve = supply curve.

$$2x + y = 600 \qquad 5x - y = 100$$

$$y = 600 - 2x \qquad y = 5x - 100$$

$$\therefore 600 - 2x = 5x - 100$$

$$\cancel{5x - 2x =}$$

$$600 + 100 = 5x + 2x$$

$$700 = 7x$$

$$\therefore x = 100$$

Subs $x = 100$ in

$$y = 600 - 2x$$

$$y = 600 - 200$$

$$\boxed{y = 400}$$

quantity offered is 400

Q) A workshop produces toy cars. The total cost fn is given by $C = 2000 + 50x$, where C = total cost of producing x cars. The total Rev. (R) is given by $R = 100x$ find the breakeven point at which the workshop will have BEP.

at BEP, $P = 0$

$$R - C = 0$$

$$R = C$$

$$100x = 2000 + 50x$$

$$100x - 50x = 2000$$

$$50x = 2000$$

$$\boxed{x = 40}$$

The BEP of workshop is 40

At this point, no profit, no loss.

A company produces torches. The weekly total cost fn is given by $C = 20.5x + 5500$

a) If each torch is sold at ₹30, what is the min quantity that needs to be produced to assure no loss not prof. (BEP)

Imp

Q1 If $P = 3D^2 + 4D + 7$

Find i) Marginal Rev. fn

ii) Avg Rev. fn

iii) Revenue Fn

Also find their values when $D = 4$ units.

Q2 If cost f^n $C = 4x^3 + 3x^2 - 5x + 7$

then find i) Avg cost ii) Marginal cost.

iii) Marginal Avg cost f^n

Also find AC, MC, MAC when $x = 3$.

Q3 If demand f^n is given as $D = \frac{P+2}{P-2}$

where P is price then find price elasticity of demand (η_d) when $P = 6$.

Ans Q 2 Avg cost $= \frac{C}{x} = \frac{4x^3 + 3x^2 - 5x + 7}{x}$

$\underline{AC = 4x^2 + 3x - 5 + \frac{7}{x}}$

Marginal cost $= \frac{d(C)}{dx}$

$= \frac{d}{dx}(4x^3 + 3x^2 - 5x + 7)$

$= 4 \times 3 x^{3-1} + 3 \times 2 x^{2-1} - 5 + 0$

$\underline{MC = 12x^2 + 6x - 5}$

$MAC = \frac{d}{dx}(4x^2 + 3x - 5 + 7x^{-1})$

$= 4 \times 2 x^{2-1} + 3 - 0 + 7(-1)x^{-1-1}$

$= 4 \times 2x + 3 - 7x^{-2}$

$MAC = 8x + 3 - \frac{7}{x^2}$

when $x = 3$

$AC = 4(3)^2 + 3(3) - 5 + \frac{7}{3}$

$= 4 \times 9 + 9 - 5 + 2.34$

$= 36 + 9 - 5 + 2.34$

$= 42.34$

$\underline{MC} = 12x^2 + 6x - 5$

$= 12 \times 9 + 18 - 5$

$= 108 + 54 - 5$

$= \cancel{127}\ 121$

Q1 $P = 3D^2 + 4D + 7$

Rev $F^n = P \cdot D$

$= (3D^2 + 4D + 7) \times D$

Rev $F^n = 3D^3 + 4D^2 + 7D$

Avg Rev $= \frac{\text{Rev } F^n}{D}$

Avg Rev $= 3D^2 + 4D + 7$

Marg Rev $F^n = \frac{d}{dx}(\text{Rev } F^n)$

$= \frac{d}{dx}(3D^3 + 4D^2 + 7D)$

$= 3 \times 3 D^{3-1} + 4 \times 2 D^{2-1} + 7$

Marg Rev $F^n = 9D^2 + 8D + 7$

Application of derivatives

1) ~~Pre~~

2nd order derivatives.

$$f'(x) = \frac{dy}{dx}$$

$$f''(x) = \frac{d^2y}{dx^2}$$

$$f(x) = x^7 + e^x + \log x$$

$$f'(x) = \frac{dy}{dx}$$

$$= \frac{d}{dx}(x^7 + e^x + \log x)$$

$$f'(x) = \cancel{\frac{d}{dx}}\ 7x^6 + e^x + \frac{1}{x}$$

$$f''(x) = \frac{d^2y}{dx^2}$$

$$= \frac{d^2}{dx^2}\left(7x^6 + e^x + \frac{1}{x}\right)$$

$$= \cancel{42x^5 + e^x +}$$

$$= 7(x^5) + e^x + x^{-1}$$

$$= 7(6x^5) + e^x + (-1)x^{-1-1}$$

$$= 42x^5 + e^x - x^{-2}$$

$$= 42x^5 + e^x - \frac{1}{x^2}$$

The demand f^n is given by $P = 30 + 6D - D^2$ where P = Price, D = demand find Total Revenue, Avg R & MR when $D = 4$

$$TR = P \times D$$

$$= (30 + 6D - D^2)D$$

$$= 30D + 6D^2 - D^3$$

$$AvgR = 30 + 6D - D^2 \quad \left(AvgR = \frac{\cancel{P} \times \cancel{D}}{\cancel{D}}\right)$$

$$MR = \frac{d(TR)}{dx} \qquad \frac{d(TR)}{dx}$$

$$\cancel{= \frac{d}{dx}(30 + 6D - D^2)}$$

$$= \frac{d}{dx}(30D + 6D^2 - D^3)$$

$$= 30 + 12D - 3D^2$$

$$= 12D - 3D^2$$

When $D = 4$

$$TR = 30D + 6D^2 - D^3$$

$$= 30(4) + 6(4)^2 - (4)^3$$

$$= 120 + 6 \times 16 - 64$$

$$= 120 + 96 - 64$$

$$= 216 - 64$$

$$= 152$$

$AR = 30 + 6D - D^2$

The cost of a com. given by $C = x^2 + 4x + 4$ find AC & MC also find out put for which MC = AC.

$$AC = \frac{C}{x}$$
$$= \frac{x^2 + 4x + 4}{x}$$
$$= x + 4 + \frac{4}{x}$$

$$MC = \frac{d}{dx}(C)$$
$$= \frac{d}{dx}(\cancel{x + 4 + 4x^{-1}}\ x^2 + 4x + 4)$$
$$= 2x + 4$$

when AC = MC

$$\therefore x + 4 + \frac{4}{x} = 2x + 4$$
$$x - 2x + \frac{4}{x} = 0$$
$$-x + \frac{4}{x} = 0$$
$$\frac{-x^2 + 4}{x} = 0$$
$$\therefore -x^2 + 4 = 0$$
$$\therefore -x^2 = -4$$
$$x = 2$$

Minima & Maxima

$$f'(x) = \frac{dy}{dx}$$
$$f''(x) = \frac{d^2y}{dx^2}$$

If $f''(x) > 0$, f^n is minima.

$f''(x) < 0$, f^n is maxima.

Increasing & decreasing f^n

If $f'(x) > 0$, f^n is ↑ing.

$f'(x) < 0$, f^n is ↓ing

If $y = x^3$ show that y is ↑ing for all non 0 x.

$$y = x^3$$
$$\therefore f'(x) = \frac{dy}{dx}$$
$$= \frac{d(x^3)}{dx}$$
$$= 3x^2$$
$$x \neq 0 = x^2 > 0$$
$$\therefore 3x^2 > 0$$

$\therefore \frac{dy}{dx} > 0$ hence f^n is increasing.

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find the value of x for which the f^n

$f(x) = x^3 - 2x^2 + x + 10$ is
(i) Decreasing ii) Increasing.

$f'(x) = \frac{dy}{dx}$

$= \frac{d}{dx}(x^3 - 2x^2 + x + 10)$

$= 3x^2 - 4x + 1$

$= 3x^2 - 3x - x + 1$

$= 3x(x-1) - 1(x-1)$

$= (x-1)(3x-1)$

If $f'(x) < 0$ (Decreasing)

$3x - 1 < 0$ or $x - 1 < 0$

$3x < 1$ $x < 1$

$x = \frac{1}{3}$

If $f'(x) > 0$ (Increasing)

$3x - 1 > 0$

$3x > 1$ or $x - 1 > 0$

$x > \frac{1}{3}$ $\therefore x > 1$

Elasticity of demand

If D is the demand & P is the price of a commodity, the elasticity of demand (η) is given by:

$\eta = \left(\frac{-P}{D}\right) \times \frac{d}{dP}(D)$

$\eta \Rightarrow$ neu

If:-

i) $\eta = 0 \rightarrow$ Demand is constant, i.e perfectly ela

ii) $\eta = 1 \rightarrow$ " " Proportional to price

iii) $\eta > 1$ " " elastic.

iv) $0 < \eta < 1$ - " " inelastic.

Relation b/w Avg Revenue (AR) & MR

$MR = AR\left(1 - \frac{1}{\eta}\right)$

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1) If $MR = 25$ & elasticity η demand w.r.t price is 2, find AR

Soln = $\eta = 2$

$MR = AR\left(1 - \frac{1}{\eta}\right)$

$MR = 25$

$\therefore 25 = AR\left(1 - \frac{1}{2}\right)$

$25 = \frac{AR}{2}$

$AR = 50$

2) If $MR = 40$ & $AR = 60$ find elasticity of demand w.r.t price.

$MR = 40$

$AR = 60$

$MR = AR\left(1 - \frac{1}{\eta}\right)$

$\therefore 40 = \cancel{60 - \frac{60}{\eta}}\ 60\left(1 - \frac{1}{\eta}\right)$

$\frac{40}{60} = 1 - \frac{1}{\eta}$

$\frac{40}{60} \cancel{-1} = -\frac{1}{\eta}$

$\frac{40-60}{60} = \frac{-1}{\eta}$

$\frac{-20}{60} = \frac{-1}{\eta}$

$\frac{1}{3} = \frac{1}{\eta}$

$\eta = 3$

3) The demand Fn given by $D = \frac{2P+5}{P-2}$ where D is demand & P is the price, find price elasticity of demand when price = 8.

$\eta = \left(\frac{-P}{D}\right) \times \frac{d}{dP}(D)$

$D = \frac{2P+5}{P-2}$

$\eta = \frac{-P}{D} \times \frac{d}{dP}(D)$

$\left[\frac{d}{dx}\left(\frac{u}{v}\right)\right]$

$\frac{d}{dP}(D) = \frac{d}{dP}\left(\frac{2P+5}{P-2}\right)$

$= \frac{(P-2)\frac{d}{dP}(2P+5) - (2P+5)\frac{d}{dP}(P-2)}{(P-2)^2}$

$= \frac{(P-2)^2 - (2P+5)}{(P-2)^2}$

$= \frac{2P-4-2P-5}{(P-2)^2}$

$= \frac{-9}{(P-2)^2}$

$= \frac{-9}{(P-2)^2}$

$\eta = \left[\frac{-P}{\frac{2P+5}{P-2}}\right] \times \left[\frac{-9}{(P-2)^2}\right]$

$= \frac{9P(P-2)}{(2P+5)(P-2)^2}$

$= \frac{9P}{(2P+5)(P-2)}$

when $P = 8$

$\eta = \frac{9 \times 8}{(2 \times 8 + 5)(8-2)}$

$= \frac{\cancel{72}^{24}}{\cancel{21}_7 \times 6}$

$= \frac{\cancel{24}^{4}}{\cancel{42}_{7}}$

$= \frac{4}{7}$