Measures of Dispersion

R C↑ 28 29 (30) 31 32 ↓D

S C↓ 10 20 (30) 40 50 ↑D

Mean = Median = 30

Dispersion → variation or scatteredness of data

Using Dispersion we compare the two or more than two sets of data.

More Dispersion = Less Consistent

1) Absolute Measures of Disp.
   a) Range
   b) Quartile Deviation
   c) Standard "
   d) Mean "

2) Relative Measures of Dispersion
   a) Coefficient of Range
   b) " " Quartile Deviation
   c) " " Variation
   d) " " Mean Deviation

1) A.M.D & R.M.D

Range & Coefficient of Range

Range = Max − Min

Coff. of Range = (Max − Min) / (Max + Min)

Coeff. of ... Max + Min

(1) Find Range & Coeff. of Range

17, 21, 18, 25, 17, 21, 20, 22

Range = 25 − 17 = 8

$$CR = \frac{25-17}{25+17} = 0.19$$

(2) Following are the temp. of two cities. Find CR & compare.

A → 30 29.5 34 31 33

B → 31 36 37 35 36

Range (A) = 34 − 29.5 = 4.5

$$CR(A) = \frac{34-29.5}{34+29.5} = 0.07$$

Range (B) = 37 − 31 = 6

$$CR(B) = \frac{6}{37+31} = 0.09$$

City B has more dispersion than City A.

(3) Max = 38.5, CR = 0.1

Min = ?

$$CR = \frac{Max - Min}{Max + Min}$$

$$\Rightarrow 0.1 = \frac{38.5 - a}{38.5 + a}$$

$$\Rightarrow 0.1(38.5 + a) = 38.5 - a$$

$\Rightarrow \quad 3.85 + 0.1a = 38.5 - a$

$\Rightarrow \quad 0.1a + a = 38.5 - 3.85$

$\Rightarrow \quad 1.1a = 34.65$

$\Rightarrow \quad a = 31.5$

$\text{Min} = 31.5$

(4) Find Range & CR

x	1	2	3	4	5	6
f	10	12	25	6	18	4

$\text{Range} = 6 - 1 = 5$

$CR = \frac{5}{1+6} = \frac{5}{7} = 0.71$

(2) Quartile Deviation & Coeff. of Q.D.

[QD is also called Semi Inter Quartile Range]

$QD = \frac{Q_3 - Q_1}{2}$

$\text{Coeff. of } QD = \frac{Q_3 - Q_1}{Q_3 + Q_1}$

(1) Find QD & Coeff.

x	1	2	3	4	5
F	2	5	10	8	5
≤ cf	2	7	17	25	30

Q_1 & Q_3 $\quad N = 30, \frac{N}{4} = 7.5, \frac{3N}{4} = 22.5$

$Q_1 = 3, \quad Q_3 = 4$

$Q_1 = 3$, $Q_3 = 4$

$$QD = \frac{Q_3 - Q_1}{2} = \frac{4-3}{2} = 0.5$$

$$\text{Coeff of } QD = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{4-3}{4+3}$$

$$= 0.14$$

(2) Find QD & Coeff. of QD

152, 154, 169, 160, 159, 160.
162, 165

Sol.

152 (154) 159 160
160 (162) 165 169

$\therefore N = 8$, $Q_1 = \frac{N}{4}$ th obs.

$= 2$ nd obs.

$= 154$

$Q_3 = \frac{3N}{4}$ th obs $= 6$ th obs.

$= 162$

$$QD = \frac{Q_3 - Q_1}{2} = 4,$$

$$C(QD) = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{162 - 154}{162 + 154}$$

$$= 0.025$$

(3) Find QD & Coeff.

CI	f	New CI	≤Cf
20-24	7	19.5 - 24.5	7
	(12)	(24.5 - 29.5) Q_1 class	19

20-24	7	17.5 – 24.5	19
25-29	12	24.5 – 29.5 (Q1 class)	43
30-34	24	29.5 – 34.5	61
35-39	18	34.5 – 39.5 (Q3 class)	71
40-44	10	39.5 – 44.5	

$N = 71, \quad \frac{N}{4} = 17.75 \quad \frac{3N}{4} = 53.25$

$$Q_1 = l_1 + \frac{(l_2 - l_1)\left(\frac{N}{4} - cf\right)}{f}$$

$$= 24.5 + \frac{5(17.75 - 7)}{12}$$

$$Q_1 = 24.5 + 4.48 = 28.98$$

$$Q_3 = l_1 + \frac{(l_2 - l_1)\left(\frac{3N}{4} - cf\right)}{f}$$

$$= 34.5 + \frac{5(53.25 - 43)}{18}$$

$$Q_3 = 37.35$$

~~(3) St~~

$$QD = \frac{Q_3 - Q_1}{2} = \frac{37.35 - 28.98}{2}$$

$$= 4.185$$

$$CQD = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{37.35 - 28.98}{37.35 + 28.98}$$

$$= 0.126$$

(3) Standard Deviation & Coefficient of Variation

S.D. (σ sigma)

(i) ungrouped data

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$$

(ii) grouped data (discrete & continuous)

$$\therefore \sigma = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - (\bar{x})^2}$$

Coefficient of variation (CV)

$$CV = \frac{\sigma}{\bar{x}} \times 100\ \%$$

(1) Find SD & CV of

2, 3, 4

x	x^2
2	4
3	9
4	16
9	29

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2}$$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{9}{3} = 3$$

$$\sigma = \sqrt{\frac{29}{3} - 9}$$

$$= \sqrt{9.67 - 9} = \sqrt{0.67} = 0.82$$

$$CV = \frac{\sigma}{\bar{x}} \times 100\ \% = \frac{0.82}{3} \times 100\ \% = 27.33\ \%$$

(2) Find S.D. & CV. and compare the two data

(x) Marks in Maths : 18 17 19 12 15

(y) Marks in BC : 20 15 17 10 12

(y) Marks in BC : 20 15 17 10 12

Sol.

x	x^2	y	y^2
18	324	20	400
17	289	15	225
19	361	17	289
12	144	10	100
15	225	12	144
81	1343	74	1158

$\bar{x} = \frac{81}{5} = 16.2$ $\quad \bar{y} = \frac{74}{5} = 14.8$

$$\sigma_x = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2} = \sqrt{\frac{1343}{5} - (16.2)^2}$$

$$= \sqrt{6.16}$$

$$= 2.48$$

$$\sigma_y = \sqrt{\frac{\Sigma y^2}{n} - (\bar{y})^2} = \sqrt{\frac{1158}{5} - (14.8)^2}$$

$$= \sqrt{231.6 - 219.04}$$

$$= \sqrt{12.56}$$

$$= 3.54$$

CV for x (maths) $= \frac{\sigma_x}{\bar{x}} \times 100\%$

$$= \frac{2.48}{16.2} \times 100\%$$

$$= 15.31\%$$

CV for y (BC) $= \frac{\sigma_y}{\bar{y}} \times 100\%$

$= \frac{3.54}{14.8} \times 100\%$

$= 23.91\%$

CV for Maths $= 15.31\%$.

" BC $= 23.91\%$.

BC more var. & less consistency

Maths less " & more "

(3) Find SD & CV

x	f	fx	fx^2
1	10	10	10
2	18	36	72
3	12	36	108
4	9	36	144
5	5	25	125
	Σf	Σfx	Σfx^2
	54	143	459

$\sigma = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2}$

$\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$[fx^2 = fx \times x$
$= fx \times x$

$\bar{x} = \frac{143}{54}$
$= 2.65$

$\sigma = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2}$

$= \sqrt{\frac{459}{54} - (2.65)^2}$

$= \sqrt{8.5 - 7.02}$

$= \sqrt{1.48} = 1.22$

$CV = \frac{\sigma}{\bar{x}} \times 100\% = \frac{1.22}{2.65} \times 100\%$

= 46.04 %

(4)

C I	f (freq)	x	fx	fx^2
0 – 10	5	5	25	125
10 – 20	8	15	120	1800
20 – 30	12	25	300	7500
30 – 40	11	35	385	13475
40 – 50	6	45	270	12150
	$\Sigma f = 42$		$\Sigma fx = 1100$	$\Sigma fx^2 = 35050$

S.D & CV ??

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{1100}{42} = 26.19$$

$$\sigma = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2} = \sqrt{\frac{35050}{42} - (26.19)^2}$$

$$= \sqrt{834.52 - 685.92}$$

$$= \sqrt{148.60}$$

$$= 12.19$$

$$CV = \frac{\sigma}{\bar{x}} \times 100\% = \frac{12.19}{26.19} \times 100\%$$

$$= 46.54\%$$

Find A.M., S.D. & CV

$x \rightarrow -2, -1, 4, 3, 2, 5$

Sol: $x^2 \rightarrow 4, 1, 16, 9, 4, 25$

$\Sigma x = 11 \qquad \Sigma x^2 = 59$

$$AM = \bar{x} = \frac{\Sigma x}{n} = \frac{11}{6} = 1.83$$

$$\sigma = \sqrt{\frac{\Sigma x^2}{} - (\bar{x})^2} = \sqrt{\frac{59}{} - (1.83)^2}$$

$$SD = \sigma = \sqrt{\frac{\Sigma x^2}{n} - (\bar{x})^2} = \sqrt{\frac{59}{6} - (1.83)^2}$$

$$= \sqrt{9.8 - 3.35}$$

$$= 2.55$$

$$CV = \frac{\sigma}{\bar{x}} \times 100\% = \frac{2.55}{1.83} \times 100\%$$

$$= 139.34\%$$

(4) Mean Deviation & Coefficient of Mean Deviation

(a) For ungrouped data

(i) MD from mean

$$MD(\bar{x}) = \frac{\Sigma |x - \bar{x}|}{n}$$

(ii) MD from Median

$$MD(Me) = \frac{\Sigma |x - Me|}{n}$$

(iii) MD from Mode

$$MD(Mo) = \frac{\Sigma |x - Mo|}{n}$$

(b) For grouped data

(i) $$MD(\bar{x}) = \frac{\Sigma f |x - \bar{x}|}{\Sigma f}$$

(ii) $$MD(Me) = \frac{\Sigma f |x - Me|}{\Sigma f}$$

(iii) $$MD(Mo) = \frac{\Sigma f |x - Mo|}{\Sigma f}$$

Coeff. of $MD(\bar{x}) = \frac{MD(\bar{x})}{\bar{x}}$

$MD(Me) = \frac{MD(Me)}{Me}$

$MD(Mo) = \frac{MD(Mo)}{Mo}$

① Find MD from $\bar{x}$, Me, Mo & coeff.

x
6
5
10
4
25

$\bar{x} = \frac{\Sigma x}{n}$ $\quad \bar{x} = \frac{25}{4} = 6.25$ ✓

4 5 ↓ 6 10 $\quad Me = \left(\frac{n+1}{2}\right)$th obs.

$= 2.5$ th obs.

$= \frac{5+6}{2}$

$= 5.5$

$\bar{x} = 6.25$, $Me = 5.5$

x	$\lvert x - \bar{x} \rvert$	$\lvert x - Me \rvert$
6	0.25	0.5
5	1.25	0.5
10	3.75	4.5
4	2.25	1.5
		7.0

4 | 2.25 | 1

7.5 | 7.0

$$MD(\bar{x}) = \frac{\Sigma|x-\bar{x}|}{n} = \frac{7.5}{4} = 1.88$$

$$MD(Me) = \frac{\Sigma|x-Me|}{n} = \frac{7.0}{4} = 1.75$$

$$\text{Coeff. of } MD(\bar{x}) = \frac{MD(\bar{x})}{\bar{x}} = \frac{1.88}{6.25} = 0.30$$

$$\text{Coeff. of } MD(Me) = \frac{MD(Me)}{Me} = \frac{1.75}{5.5} = 0.32$$

(2) Find $MD(Me)$, $MD(M_o)$ & coeff.

$x \rightarrow 2, 3, 10, 6, 3, 4, 1$

Sol. Me

1 2 3 (3) 4 6 10

$$Me = \left(\frac{n+1}{2}\right) \text{th obs}$$

$$= 4 \text{ th obs.} = 3$$

$M_o = 3$

X

$M_0 = 3$

x	$\lvert x - M_e \rvert = \lvert x - M_0 \rvert$
2	1
3	0
10	7
6	3
3	0
4	1
1	2
	$\sum \lvert x - M_e \rvert = \sum \lvert x - M_0 \rvert = 14$

$$MD(M_e) = MD(M_0) = \frac{\sum |x - M_e|}{n} = \frac{\sum |x - M_0|}{n}$$

$$= \frac{14}{7} = 2$$

Coeff. of $MD(M_e)$ = Coeff of $MD(M_0)$

$$= \frac{2}{3} = 0.67$$

[Combined A.M (Practice Sums)

$n_1 = 100$ $\quad n_2 = 50$

$\bar{x}_1 = 60$ $\quad \bar{x}_2 = 90$

$$\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$= \frac{100 \times 60 + 50 \times 90}{100 + 50}$$

$$= \frac{10500}{150} = 70$$

(2) The avg. daily wages for 90 workers is Rs. 59. The avg. wages of 50 male workers is 63. Find avg. wage for female workers?

Sol. $n_1 + n_2 = 90$

$n_1 = 50, \quad n_2 = 40$

$\bar{x}_1 = 63 \quad \bar{x}_2 = ?$

$\bar{x}_c = 59$

$$\bar{x}_c = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$\Rightarrow 59 = \frac{50 \times 63 + 40\bar{x}_2}{90}$$

$$\Rightarrow 5310 = 3150 + 40\bar{x}_2$$

$$\Rightarrow 40\bar{x}_2 = 2160$$

$\Rightarrow \bar{x}_2 = \frac{2160}{40} = 54$

Ans. Avg. wage of females = Rs. 54/-

* Find MD(Mean) & its coefficient

CI	f	x	$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$
0 – 5	8	2.5	7.5	60
5 – 10	10	7.5	2.5	25
10 – 15	15	12.5	2.5	37.5
15 – 20	6	17.5	7.5	45
	39			167.5

$\bar{x} = \frac{\Sigma fx}{\Sigma f} = 9.93 = 10$

Mean Dev. $(\bar{x}) = \frac{\Sigma f\lvert x-\bar{x}\rvert}{\Sigma f} = \frac{167.5}{39}$

$= 4.3$

Coeff. of Mean Dev$(\bar{x}) = \frac{\text{Mean Dev}(\bar{x})}{\bar{x}}$

$= \frac{4.3}{10} = 0.43$

* Find MD(Median) & its coefficient.

CI	f	x	≤ Cf	$\lvert x-Me\rvert$	$f\lvert x-Me\rvert$
5 – 15	5	10	5 (cf)	18.25	93.75

5–15	5	10	5	18.75	93.75
15–25	10	20	15 (cf)	8.75	87.5
25–35 (l_1)	20 (f)	30	35	1.25	2.5
35–45	10	40	45	11.25	112.5
	45				318.75

$N = 45, \quad \frac{N}{2} = 22.5$

$$Me = l_1 + \frac{(l_2 - l_1)\left(\frac{N}{2} - cf\right)}{f}$$

$$= 25 + \frac{10(22.5 - 15)}{20}$$

$$= 28.75$$

$$MD(\text{median}) = \frac{\Sigma f|x - Me|}{\Sigma f} = \frac{318.75}{45}$$

$$= 7.08$$

$$\text{Coeff of } MD(Me) = \frac{MD(\text{median})}{\text{Median}}$$

$$= \frac{7.08}{28.75} = 0.25$$