

Height and Distance Formulas

Real-Life Applications of Height & Distance

Real-life examples where the concepts of Height & Distance is involved are:

Application 1: It can help us find the Slope and Height During Construction. If we know the angle of elevation and the distance between the building and the ground, we can calculate the height. Likewise, if the additional pieces of information are provided, the angle of elevation, angle of depression, and distance of the object can be determined.

Application 2: Using trigonometric ratios, the height of a peak or building can be measured.

Application 3: Depending on the situation, the angle of depression and angle of elevation are utilised in flight. It allows us to calculate the required angle of elevation or depression to reach the target.

Understanding the Concepts of Height & Distance

Trigonometric formulas may be used to calculate the height and distance of everything.

1. Calculation of Height Using Trigonometry

If you know the distance between your position and the building and the angle at which you can view the top, you can calculate the height.

We use trigonometry to calculate the height because the surface of the ground, the height of the building, and the line of elevation all form a right angle triangle with 90 degrees between the building and the ground.

Base

The 'base' of the right-angled triangle is created by the height of the building and line of sight is distance. The horizontal level's length also indicates the distance since it forms the base of the triangle, which is at a specific height but parallel to the ground.

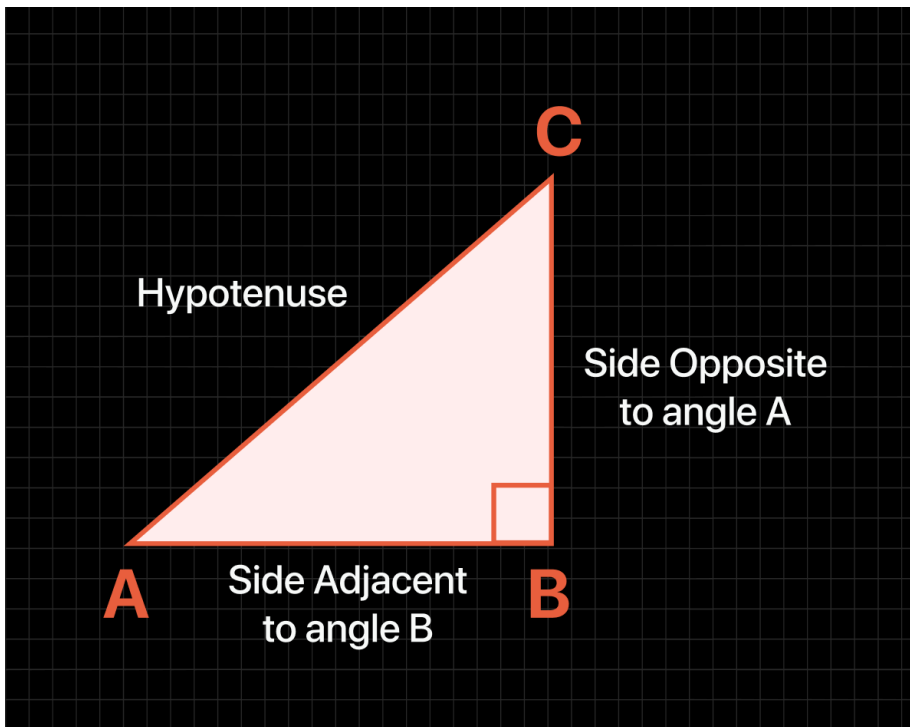
Hypotenuse

The 'hypotenuse' of a right-angled triangle is the line of sight. The 'height' of the triangle is formed by a line perpendicular to the ground.

We can use the following formula to calculate the angle of elevation or depression:

$$\sin \theta = \frac{\text{height}}{\text{hypotenuse}} \quad \cos \theta = \frac{\text{base}}{\text{hypotenuse}} \quad \tan \theta = \frac{\text{height}}{\text{base}}$$

Here, θ is either the angle of elevation or depression.



Here AB is the base and BC is the height.

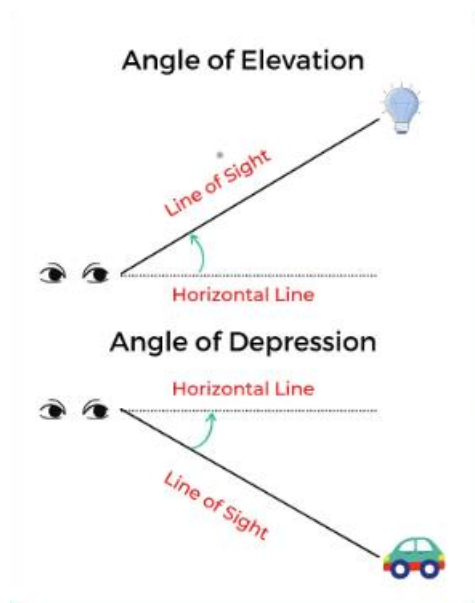
- There are basically two terms associated with heights and distances which are as follows :
 - Angle of Elevation.
 - Angle of Depression.

Formulas for Angle of Elevation

- **The Angle of elevation** of the point viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level, i.e., the case when we raise our head to look at the object

Formulas for Angle of Depression

- **The angle of depression** of a point on the object being viewed is the angle formed by the line of sight with the horizontal when the point is below the horizontal level, i.e., the case when we lower our head to look at the point being viewed.



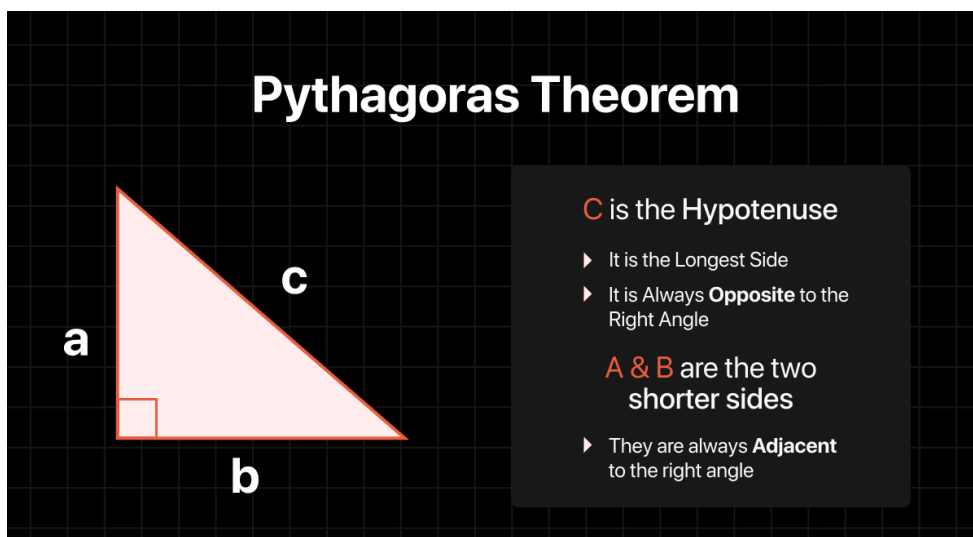
2. Pythagorean Theorem

Pythagoras' theorem states that the square of a right-angled triangle's hypotenuse equals the sum of the squares of its base and height.

Pythagoras' theorem can be used to calculate the length of the sides of a right-angled triangle when height and distance are involved.

$$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{height})^2$$

If the length of the base, height and hypotenuse of a right-angle triangle is a, b and c respectively.



Then, $a^2 + b^2 = c^2$

Thus, if the length of any two sides is known then the length of the third side can be found by using the Pythagoras theorem which is also called the Pythagorean triplets.

Height and Distance Formulas for Trigonometric ratio

- ΔABC is a right angled triangle where AB is the perpendicular, AC is the hypotenuse, and BC is the base.

Then,

- $\sin \theta = \frac{AB}{AC}$
- $\cos \theta = \frac{BC}{AC}$
- $\tan \theta = \frac{AB}{BC}$
- $\operatorname{cosec} \theta = \frac{AC}{AB}$
- $\sec \theta = \frac{AC}{BC}$
- $\cot \theta = \frac{BC}{AB}$
- **Trigonometrical Identities:**
 - $\sin^2 \theta + \cos^2 \theta = 1$
 - $1 + \tan^2 \theta = \sec^2 \theta$
 - $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

Degrees/ Ratio	0°	30°	45°	60°	90°
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined

Question 1. A boy is flying a kite in the evening. The thread of the kite was 120 m long and the angle of elevation with the boy's eyes was 30°. Find the height of the kite?

Question 2. The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 4.6 m away from the wall. What is the length of the ladder?

Question 3. Find the angle of elevation of the sun when the shadow of a pole of 18 m height is $6\sqrt{3}$ m long?

Question 4: Determine the angle of elevation of the sun, when the length of the shadow of a tree is 3 times the height of the tree.

Question 5: A 80 m long ladder is leaning on a wall. If the ladder makes an angle of 45° with the ground, find the distance of the ladder from the wall.

Question 6: The angle of elevation of the top of a tower from point A on the ground is 45° . On moving 20 meters toward the tower, the angle of elevation of the top of the tower becomes 60° . Find the height of the tower (in meters).

Question 7: From a point 375 meters away from the foot of a tower, the top of the tower is observed at an angle of elevation of 45° , then the height (in meters) of the tower is?

Question 8: The top of a 25 meter high tower makes an angle of elevation of 45° with the bottom of an electric pole and angle of elevation of 30° with the top of pole. Find the height of the electric pole.

Question 9: A boy is standing on a riverbank. His angle of elevation from the treetop, which is on the opposite bank is 45° . When he walks 20 meters away from his place, his angle of elevation becomes 30° . What is the height of the tree?

Question 10: From the top of a building, which is 75m high, the angles of depression of the top and the bottom of a pole which is present on the same plane as the building are 30° and 45° , What is the pole's height?

Question 11: A girl is standing on the ground at a point P. If she looks at the topmost point of the building, the angle of elevation is 60° . If the point P is 32 m from the building, what is its height?

Question 12: A lighthouse is 80m high, a person observes two ships. The angles of elevation of both these ships as observed from the lighthouse are 45° and 30° , What is the distance between the two ships?

Question 13: A building which is 20 meters high, a man is flying a kite. He is using a thread that is 75m long to keep the kite in the air. If the thread is making an angle of 45° with the horizontal, then calculate the height of the kite from the ground.

Question 14: The angles of depression of the top and bottom of a tree, as observed from the top of a building, which is 90mtrs high are 30° and 60° . What is the height of the tree?

Question 15: The upper part of a pole is broken. The top of the pole touches the ground making an angle of 30° . The distance of the point where the top touches the ground to the base of the pole is 72m. What is the height of the pole?