

## 4

# Ratio, Proportion And Percentage

## RATIO

Ratio is the relation which one quantity bears to another with respect to magnitude, the comparison being made by considering what multiple, part or parts the first quantity is of the second. This relation may be expressed in two ways. For example, if Abhigyan walks at a speed of 4 Km/hr to go to school and Ishaan walks at a speed of 8 Km/hr to go to school, then we may say that :

- i) Abhigyan walks at a speed of 4 Km/hr less than that of B or
- ii) Abhigyan's speed is half that of B.

The first result is obtained by subtracting Abhigyan's speed from Ishaan's speed. The second result is obtained by dividing Abhigyan's speed by Ishaan's speed i.e. by forming a fraction whose numerator is speed of Abhigyan and whose denominator is the speed of Ishaan.

Hence, in comparing two quantities of the same kind with respect to magnitude of quantities, for example, two lengths, two heights, two weights etc., a fraction is formed whose numerator and denominator show the number of unit of two quantities compared. Such a fraction is called the ratio of two quantities.

The ratio can also be expressed as a fractions. For example, if there are 20 males and 8 females in an office, this can be written as the males and females in that office are in the ratio  $\frac{20}{8}$  or  $\frac{5}{2}$  or 5 : 2.

Similarly, female and male are in the ratio 2 : 5. The first term of the two quantities in a ratio is called the antecedent and the second term is called consequent.

In general, if  $a$  and  $b$  ( $b \neq 0$ ) i.e. the magnitudes of two quantities (or numbers), then  $\frac{a}{b}$  is the ratio of  $a$  to  $b$  and is generally written as  $a : b$  (read as  $a$  is to  $b$ ). Here,  $a$  is called antecedent and  $b$  the consequent. Each  $a$  and  $b$  is called a **term** of the ratio.

## PROPERTIES OF RATIO

1. Ratio is an abstract number. It has no unit attached to it.

For example, ratio of ₹ 40 to ₹ 60 is  $\frac{₹ 40}{₹ 60} = \frac{2}{3}$  which is an abstract number.

2. Ratio can be expressed in lowest term

For example  $18 : 27 = \frac{18}{27} = \frac{18 \div 9}{27 \div 9} = \frac{2}{3} = 2 : 3$

3. Ratio exists only between two quantities of the same kind.

For example, there is no ratio between 5 kg and ₹ 25.

4. Quantities to be compared must be in the same units.

For example, 65 paise : ₹ 1.45 =  $\frac{65}{145} = \frac{13}{29} = 13 : 29$

5. Since, ratio is a fraction, therefore, its value remains same if its numerator and denominator are multiplied or divided by the same non-zero number.

For example,  $4 : 5 = \frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15} = 12 : 15$

$\therefore 4 : 5 = 12 : 15$

6. If the terms of the given ratio are fractions, then convert them in whole numbers by multiplying each term by the LCM of their denominators.

For example,  $\frac{1}{3} : \frac{3}{7} = \frac{1}{3} \times 21 : \frac{3}{7} \times 21 = 7 : 9$

7. To compare two ratios, make the denominator of both the ratios equal.

For example, which ratio is greater  $13 : 15$  or  $22 : 25$ ?

LCM of 15 and 25 = 75

$\therefore \frac{13}{15} = \frac{13 \times 5}{15 \times 5} = \frac{65}{75}$  and  $\frac{22}{25} = \frac{22 \times 3}{25 \times 3} = \frac{66}{75}$

Since  $65 < 66$

8. If a quantity increases or decreases in the ratio  $a : b$ , then new quantity =  $\frac{b}{a}$  of the original quantity.

For example, if the price of an article is increased from ₹ 6 to ₹ 10, the ratio of the new value to the old value is  $10 : 6$ , i.e.  $5 : 3$  and we say that the price has increased in the ratio  $5 : 3$ .

### TYPES OF RATIOS

1. A ratio  $\frac{a}{b}$  is said to be of greater inequality if  $a > b$ .
2. A ratio  $\frac{a}{b}$  is said to be of lesser inequality if  $a < b$ .
3. A ratio  $\frac{a}{b}$  is said to be of equality if  $a = b$ .
4. **Inverse ratio or Reciprocal ratio** : one ratio is said to be inverse of the other if their product is one.

If  $\frac{a}{b} \times \frac{b}{a} = 1$ , then  $\frac{b}{a}$  is the inverse of  $\frac{a}{b}$  and  $\frac{a}{b}$  is inverse of  $\frac{b}{a}$

5. **Compound ratio** : Let  $a : b$  and  $c : d$  be two ratios. Then the ratio  $ac : bd$  is said to be compound ratio of the two ratios  $a : b$  and  $c : d$ . The ratio of the product of the first terms and the product of second terms of two or more ratios is called their compound ratio.
6. **Continued ratio** : A relation (or comparison) by division between the magnitude of three or more quantities of the same kind is called the continued ratio.

For example, the continued ratio of three similar quantities  $a, b, c$  is written as  $a : b : c$ . The continued ratio of four similar quantities  $a, b, c, d$  is written as  $a : b : c : d$ .

7. **Duplicate ratio** : When a ratio is compounded with itself, then it is called duplicate ratio. Thus compounding  $a : b$  and  $a : b$  we get  $a^2 : b^2$

$\therefore a^2 : b^2$  is the duplicate Ratio of  $a : b$ .

8. **Triplicate ratio** : When a ratio is compounded with its duplicate ratio, then it is called triplicate ratio. If  $a : b$  is compounded with its duplicate  $a^2 : b^2$  we get  $a^3 : b^3$ . Then  $a^3 : b^3$  is called the triplicate ratio of  $a : b$ .

9. **Sub-duplicate ratio** : The ratio  $\sqrt{a} : \sqrt{b}$  is said to be the sub-duplicate ratio of  $a : b$ .

**10. Sub-triplicate ratio :** The ratio  $\sqrt[3]{a} : \sqrt[3]{b}$  is said to be sub-triplicate ratio of  $a : b$ .

**Examples 1 :**

In a ratio which is equal to 5 : 6, if the antecedent is 40, find the consequent.

**Solution :**

Let the consequent be X.

$$\therefore 40 : X = 5 : 6$$

$$\frac{40}{X} = \frac{5}{6} \Rightarrow X = \frac{40 \times 6}{5} = 48$$

Hence, the required consequent is 48.

**Example 2 :**

Divide ₹ 1,875 into three parts in the ratio 4 : 5 : 6.

**Solution :**

Sum of the terms of the ratio = 4 + 5 + 6 = 15

$$\text{First part} = \frac{4}{15} \text{ of ₹ } 1,875 = ₹ \left( \frac{4}{15} \times 1,875 \right) = ₹ 500$$

$$\text{Second part} = \frac{5}{15} \text{ of ₹ } 1,875 = ₹ \left( \frac{5}{15} \times 1,875 \right) = ₹ 625$$

$$\text{Third part} = \frac{6}{15} \text{ of ₹ } 1,875 = ₹ \left( \frac{6}{15} \times 1,875 \right) = ₹ 750$$

**Example 3 :**

If  $A : B = 2 : 3$  and  $B : C = 4 : 5$ , find : (i)  $A : C$ , (ii)  $A : B : C$

**Solution :**

$$\text{i) Given } A : B = 2 : 3 \Rightarrow \frac{A}{B} = \frac{2}{3}$$

$$B : C = 4 : 5 \Rightarrow \frac{B}{C} = \frac{4}{5}$$

$$\text{Now } \frac{A}{B} \times \frac{B}{C} = \frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$$

$$\therefore A : C = 8 : 15$$

ii) LCM of two values of B i.e. 3 and 4 is 12.

$$\text{Thus } A : B = 2 : 3 = 2 \times 4 : 3 \times 4 = 8 : 12$$

$$B : C = 4 : 5 = 4 \times 3 : 5 \times 3 = 12 : 15$$

$$\therefore A : B : C = 8 : 12 : 15. \text{ Which is in its simplest form}$$

**Example 4 :**

Find three positive numbers in the ratio 3 : 5 : 7 such that the sum of their squares is equal to 1,328.

**Solution :**

Let the three numbers be  $3x$ ,  $5x$  and  $7x$  then

$$(3x)^2 + (5x)^2 + (7x)^2 = 1,328$$

$$9x^2 + 25x^2 + 49x^2 = 1,328$$

$$83x^2 = 1,328$$

$$x^2 = 1,328/83$$

$$x^2 = 16$$

$$\therefore x = 4$$

$$\therefore 3x = 12, \quad 5x = 20, \quad 7x = 28$$

Thus the three numbers are 12, 20 and 28.

**Example 5 :**

Two numbers are in the ratio 2 : 3. If 5 is added to each number, the ratio becomes 5 : 7, find the numbers.

**Solution :**

Let the numbers be  $2X$  and  $3X$ . Then

$$\frac{2X + 5}{3X + 5} = \frac{5}{7}$$

$$\Rightarrow 7(2X + 5) = 5(3X + 5)$$

$$\Rightarrow 14X + 35 = 15X + 25$$

$$\Rightarrow 35 - 25 = 15X - 14X$$

$$\Rightarrow 10 = X$$

Hence the numbers are  $2 \times 10 = 20$  and  $3 \times 10 = 30$ .

**Example 6 :**

A certain sum of money is divided into three parts in the ratio 4 : 7 : 9. If the second part is ₹ 420, find the total amount and the other two parts.

**Solution :**

Since the money has been divided into three parts in the ratio 4 : 7 : 9.

Let the three parts be ₹  $4x$ , ₹  $7x$  and ₹  $9x$  respectively.

$$\text{Then total amount} = ₹ (4x + 7x + 9x) = ₹ 20x$$

$$\text{According to given, } 7x = 420 \Rightarrow X = 60$$

$$\therefore \text{Total amount} = ₹ (20 \times 60) = ₹ 1,200$$

$$1^{\text{st}} \text{ part} = ₹ (4 \times 60) = ₹ 240 \text{ and}$$

$$3^{\text{rd}} \text{ part} = ₹ (9 \times 60) = ₹ 540$$

**Example 7 :**

Aditi weighs 84 kg. If she reduces her weight in the ratio 7 : 6, find her new weight.

**Solution :**

Weight of Aditi = 84 kg

Since she reduces her weight in the ratio 7 : 6

$$\therefore \text{Her new weight} = \frac{6}{7} \text{ of } 84 \text{ kg} = \frac{6}{7} \times 84 = 72 \text{ kg}$$

**Example 8 :**

What should be subtracted from each term of the ratio 4 : 9 so that it becomes 1 : 3?

**Solution :**

Let  $X$  be the number to be subtracted from each term of the given ratio.

$$\text{Then } \frac{4 - X}{9 - X} = \frac{1}{3}$$

$$\Rightarrow 3(4 - X) = 1(9 - X)$$

$$\Rightarrow 12 - 3X = 9 - X$$

$$\Rightarrow -3X + X = 9 - 12$$

$$\Rightarrow -2X = -3$$

$$\Rightarrow X = \frac{3}{2} = 1\frac{1}{2}$$

**Example 9 :**

The ages of Aditya and Bhaskar are in the ratio 7 : 8. Six years ago, their ages were in the ratio 5 : 6. Find their present ages.

**Solution :**

Let the present ages of Aditya and Bhaskar be  $(7x)$  years and  $(8x)$  years respectively.

$$\text{Aditya's age 6 years ago} = (7x - 6) \text{ years}$$

$$\text{Bhaskar's age 6 years ago} = (8x - 6) \text{ years}$$

$$\therefore (7x - 6) : (8x - 6) = 5 : 6$$

$$\Rightarrow \frac{7x - 6}{8x - 6} = \frac{5}{6}$$

$$\Rightarrow 6(7x - 6) = 5(8x - 6)$$

$$\Rightarrow 42x - 36 = 40x - 30$$

$$\Rightarrow 42x - 40x = -30 + 36$$

$$\Rightarrow 2x = 6$$

$$\Rightarrow x = 3$$

$$\therefore \text{Aditya's present age} = 7 \times 3 = 21 \text{ years}$$

$$\text{Bhaskar's present age} = 8 \times 3 = 24 \text{ years}$$

**Example 10 :**

The work done by  $(X - 3)$  men in  $(2X + 1)$  days and the work done by  $(2X + 1)$  men in  $(X + 4)$  days are in the ratio 3 : 10, find the value of  $X$ .

**Solution :**

Work done by  $(X - 3)$  men in  $(2X + 1)$  days

$$= (X - 3)(2X + 1) \text{ times the work done by 1 man in 1 day.}$$

Work done by  $(2X + 1)$  men in  $(X + 4)$  days

$$= (2X + 1)(X + 4) \text{ times the work done by 1 man in 1 day}$$

$$\therefore \frac{(X - 3)(2X + 1)}{(2X + 1)(X + 4)} = \frac{3}{10} \Leftrightarrow \frac{X - 3}{X + 4} = \frac{3}{10}$$

$$\Rightarrow 10(X - 3) = 3(X + 4)$$

$$\Rightarrow 10X - 30 = 3X + 12$$

$$20X - 3X = 12 + 30$$

$$7X = 42$$

$$\mathbf{X = 6}$$

**Example 11 :**

The ratio of the number of girls to the number of boys in a school of 720 students is 3 : 5. If 18 new boys are admitted in the school, find how many new girls may be admitted so the ratio of number of girls to the number of boys may change to 2 : 3.

**Solution :**

The ratio of the number of girls to the number of boys is 3 : 5

$$\text{Sum of the terms of the ratio} = 3 + 5 = 8$$

$$\therefore \text{The number of girls in the school} = \frac{3}{8} \times 720 = 270$$

$$\text{The number of boys in the school} = \frac{5}{8} \times 720 = 450$$

Let the number of new girls admitted be  $X$ , then the number of girls become  $(270 + x)$

After admitting 18 new boys, the number of boys become  $450 + 18 = 468$ .

According to given statement :

$$\frac{270 + x}{468} = \frac{2}{3}$$

$$\Rightarrow 3(270 + x) = 2 \times 468$$

$$\Rightarrow 810 + 3x = 936$$

$$\Rightarrow 3x = 936 - 810$$

$$\Rightarrow 3x = 126 \Rightarrow x = 42$$

Hence, the number of new girls admitted = 42.

### PROPORTION

A statement of equality between two ratios is called a proportion.

For example : 2 : 3 and 4 : 6 are in proportion since the ratio of 2 to 3 is equal to the ratio 4 : 6.

$$\text{i.e. } 2 : 3 = 4 : 6$$

$$\text{or } 2 : 3 :: 4 : 6$$

In general, if four quantities  $a, b, c$  and  $d$  are such that the ratio of the first to the second i.e.  $a : b$  is same as the ratio of the third to the fourth, i.e.  $c : d$ , then the four quantities  $a, b, c$  and  $d$  are said to be in proportion. We express it by writing

$$a : b :: c : d \text{ or } a : b = c : d. \text{ It is read as "a is to b as c is to d".}$$

In  $a : b = c : d$ ;  $a, b, c$  and  $d$  are called its first, second, third and fourth terms respectively. Also, first and fourth terms are called extremes (or extreme terms) and second and third terms are called means (or middle terms)

**Fourth proportional** : If  $a, b, c$  and  $d$  are in proportion then  $d$  is called the fourth proportional to  $a, b$  and  $c$ .

$$\Rightarrow \frac{a}{b} = \frac{c}{d} \quad \text{i.e. } ad = bc$$

i.e. product of extremes = product of means.

**Continued proportion** : Three quantities of the same kind are said to be in continued proportion if the ratio of the first to the second is equal to the ratio of second to the third.

$$\text{i.e., three quantities } a, b, c \text{ will be in continued proportion if } \frac{a}{b} = \frac{b}{c} \text{ or } b^2 = ac$$

In general, quantities are said to be in continued proportion when the first is to the second, as the second is to the third, as the third to the fourth and so on. Thus  $a, b, c, d, e, \dots$  are in continued proportion when

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = \dots$$

**Mean proportion** : If three quantities be in continued proportion, then the second quantity is called the mean proportional between the first and the third. If  $a, b, c$  are in the continued proportion, then  $b$  is called the mean proportion between  $a$  and  $c$ .

$$\text{i.e. } \frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = ac \Rightarrow b = \sqrt{ac}$$

**Third proportion** : If  $a, b, c$  are in continued proportion then the third quantity  $c$  is called the third proportional to first and second i.e.  $a$  and  $b$ .

### DIRECT AND INVERSE PROPORTION

**Direct variation** : Two quantities are said to vary directly, if the increase (or decrease) in one quantity causes the increase (or decrease) in the other quantity.

For example : The number of workers and wages to be paid to them.

**In direct variation, the ratio of one kind of like terms is equal to the ratio of second kind of like terms.**

**Inverse variation :** Two quantities are said to vary inversely if the increase (or decrease) in one quantity causes the decrease (or increase) in the other quantity.

For example : Speed of a car and time taken to cover a distance.

**In inverse variation, the ratio of one kind of like terms is equal to the inverse ratio of second kind of like terms.**

**Example 12 :**

Find the fourth proportional to 6, 10 and 12.

**Solution :**

Let X be the fourth proportional

Then,  $6 : 10 = 12 : X$

$$\Rightarrow 6X = 12 \times 10 \quad [\text{Product of means} = \text{product of extremes}]$$

$$\Rightarrow X = 20$$

Hence, the required fourth proportional is 20.

**Example 13 :**

Find the third proportional to 8 and 16.

**Solution :**

Let the third proportional be X.

Then 8, 16 and X are in continued proportion.

$$\therefore 8 : 16 = 16 : x$$

$$\Rightarrow 8x = 16 \times 16$$

$$\Rightarrow X = 32$$

Hence, the required third proportion is 32.

**Example 14 :**

Find the mean proportion of 1.25 and 1.8.

**Solution :**

Let X be the mean proportion of 1.25, 1.8

$$\therefore 1.25 : X = X : 1.8$$

$$\Rightarrow X^2 = 1.25 \times 1.8$$

$$X = \sqrt{1.25 \times 1.8} = \sqrt{2.25} = 1.5$$

Hence, the required mean proportion is 1.5.

**Example 15 :**

What least quantity must be added to each of the numbers 16, 7, 79 and 43, so that the resulting numbers are in proportion?

**Solution :**

Let the number to be added be  $x$ , then

$$(16 + x) : (7 + x) :: (79 + x) : (43 + x)$$

$$\Rightarrow \frac{16 + x}{7 + x} = \frac{79 + x}{43 + x}$$

$$\Rightarrow (16 + x)(43 + x) = (7 + x)(79 + x)$$

$$\Leftrightarrow x^2 + 58x + 688 = x^2 + 86x + 553$$

$$\Leftrightarrow 688 - 553 = 86x - 58x$$

$$\Leftrightarrow 27x = 135 \quad \Leftrightarrow x = \frac{135}{27} = 5$$

Hence, the required number is 5.

### Example 16

In a school, the ratio between the number of girls and the number of boys is 6 : 7. If there are 504 girls in the school, find

- i) the number of boys in the school.
- ii) the total no. of students in the school.

#### Solution :

- i) Let there be  $x$  boys in the school, then the ratio of number of girls in the school to that of boys is 504 :  $x$

According to the statement, 504 :  $x = 6 : 7$

$$\Rightarrow \frac{504}{x} = \frac{6}{7} \Rightarrow 6x = 504 \times 7$$

$$\therefore x = \frac{504 \times 7}{6}$$

$$\Rightarrow x = 84 \times 7 = 588$$

Hence there are 588 boys in the school.

- ii) The total no. of students in the school = 504 + 588 = 1092

### Example 17 :

Ages of Manav, Abhishek and Dilip are in continued proportion. If Manav is 4 years old and Dilip is 9 years old, what is the age of Abhishek?

#### Solution :

Let Abhishek be  $X$  years old.

$\therefore 4 : X :: X : 9$  (since they are in continued proportion)

$$\text{i.e. } \frac{4}{X} = \frac{X}{9} \Rightarrow X^2 = 36$$

$$\therefore X = 6$$

Hence, the age of Abhishek is 6 years.

### Example 18 :

The ratio of the length to its width of a park is 5 : 2. If the width of the park is 14 m, what is its length?

#### Solution :

Let the length be  $X$ m.

According to statement

$$5 : 2 :: x : 14$$

$$\Rightarrow \frac{5}{2} = \frac{x}{14} \Rightarrow 2x = 14 \times 5$$

$$x = \frac{14 \times 5}{2} = 35$$

$\therefore$  Length of the park is 35m.

### Example 19 :

A bag contains 50P, 25P and 10P coins in the ratio 5 : 9 : 4, amounting to ₹ 206. Find the number of coins of each type respectively.

**Solution :**

Let the ratio be  $X$ .

Hence the number of coins be  $5x$ ,  $9x$ ,  $4x$  respectively.

Now, given total amount = ₹ 206.

$$\Rightarrow (0.50)(5x) + (0.25)9x + (0.10)(4x) = 206$$

$$\Rightarrow 2.5x + 2.25x + 0.4x = 206$$

$$\Rightarrow 5.15x = 206 \quad \Rightarrow X = 40$$

$$\Rightarrow \text{No. of 50p coins} = 5 \times 40 = 200$$

$$\text{No. of 25p coins} = 9 \times 40 = 360$$

$$\text{No. of 10p coins} = 4 \times 40 = 160$$

**Example 20 :**

The sum of three numbers is 98. If the ratio of the first to second is  $2 : 3$  and of the second to the third is  $5 : 8$ , then find the second number.

**Solution :**

Let the three numbers be  $A$ ,  $B$ ,  $C$ . Then,

$$A : B = 2 : 3 \quad B : C = 5 : 8$$

LCM of 3 and 5 is 15

$$\Rightarrow \frac{2 \times 5}{3 \times 5} = \frac{5 \times 3}{8 \times 3}$$

$$A : B = 10 : 15, \quad B : C = 15 : 24$$

$$\therefore A : B : C = 10 : 15 : 24$$

$$A + B + C = 10 + 15 + 24 = 49$$

$$\therefore \text{Second number} = \frac{15}{49} \times 98 = 30$$

**Example 21 :**

A hostel had rations for 150 students for 60 days. After 12 days, 30 more students join the hostel. How long will the remaining food last?

**Solution :**

After 12 days, the ration is sufficient for 150 students for  $(60 - 12)$  i.e. 48 days.

$$\text{No. of students after 12 days} = 150 + 30 = 180$$

Now, let the no. of days for which food last be  $x$ .

This is a case of inverse variation

As the number of students increases in the ratio  $150 : 180$  i.e.  $5 : 6$ , the no. of days for which the ratio lasts decreases in the ratio  $6 : 5$ .

$\therefore$  the number of days for which the remaining ration lasts.

$$= \frac{5}{6} \text{ of } 48 = 40 \text{ days.}$$

**Example 22 :**

Three partners  $A$ ,  $B$  and  $C$  invested ₹ 1,00,000, ₹ 80,000 and ₹ 1,80,000 respectively in a startup business. At the end of the year,  $A$  received ₹ 16,000 as her share in the profit. Find the total profit. Also, find  $B$ 's and  $C$ 's share in the profit.

**Solution :**

The profit will be distributed among the partners in the proportion to the capitals invested.

i.e. in the ratio  $1,00,000 : 80,000 : 1,80,000$

i.e. in the ratio 5 : 4 : 9

Let the common ratio be  $x$

∴ Profits are  $5x : 4x : 9x$

Also, profit of A = ₹ 16,000

$$\Rightarrow 5x = 16,000$$

$$\Rightarrow x = 16,000/5 = 3,200$$

Total profit =  $5x + 4x + 9x = 18x$

$$= 18 \times 3,200 = 57,600$$

B's share =  $4x = 4 \times 3,200 = 12,800$

C's share =  $9x = 9 \times 3,200 = 28,880$

### Example 23 :

If 200 children consume 400 litres of milk in 30 days, in how many days will 300 children consume 540 litres of milk?

#### Solution :

No. of children	Quantity of Milk (litres)	Number of days
200	400	30
300	540	$x$

More children consume the same quantity in less days

$$\Rightarrow 300 : 200 :: 30 : x$$

More quantity of milk will be consumed in more days

$$\Rightarrow 400 : 540 :: 30 : x$$

Combining both the proportions, we get

$$\begin{array}{ccc} \overbrace{300 : 200}^X & : & \overbrace{30 : x}^X \\ \underbrace{400 : 540}_X & & \end{array} \quad \left[ \begin{array}{l} \text{Product of extremes} = \\ \text{Product of means} \end{array} \right]$$

Therefore,  $300 \times 400 \times x = 200 \times 540 \times 30$

$$x = \frac{200 \times 540 \times 30}{300 \times 400} = 27 \text{ days}$$

### VARIATION

Equations with direct and inverse variation sound a little intimidating, but really, they are just two different ways of talking about how one number changes relative to another number.

In direct variation, as one number increases, so does the other. This is also called direct proportion, they are the same thing. An example of this is relationship between age and height. As the age in years of a child increases, the height will also increase.

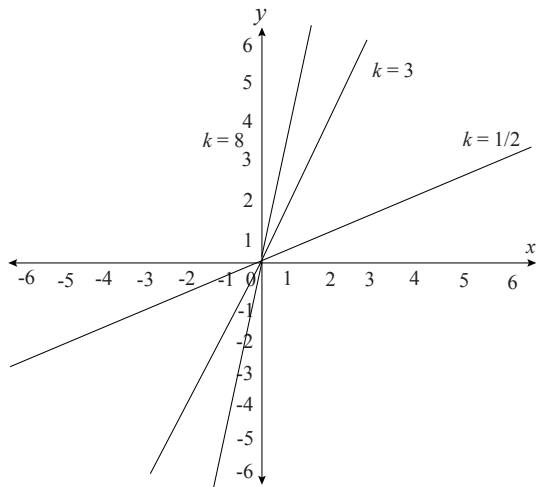
In the abstract, we can express direct variation by using the equation  $y = Kx$ ,

$X$  and  $y$  are two quantities – in our example, they'd be the age and the height of the child.  $K$  is called the constant of proportionality. It tells us specifically how much bigger  $y$  will get for every increase in  $X$ .

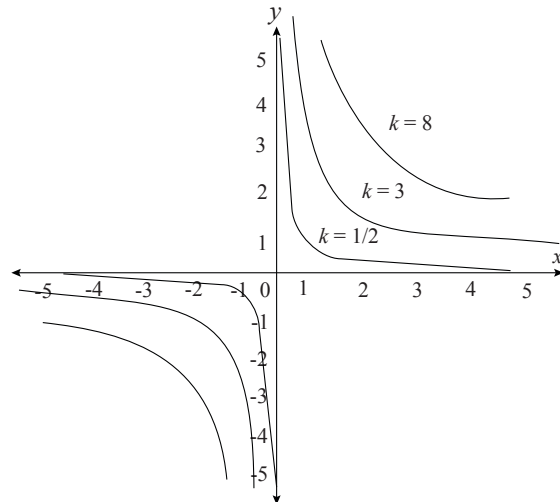
In inverse variation, it is exactly the opposite : as one number increases, the other decreases. This is also called inverse proportion. An example would be the relationship between time spent goofing off in class and the grade in the exam.

$$y = K/x$$

The bigger the value of  $x$ , the smaller the value of  $y$  will be.



Direct Variation Equation for  
3 different values of k



Inverse Variation Equation for  
3 different values of

**Example 24 :**

If  $A \propto B$  and  $A = 4$  when  $B = 6$ , find the value of  $A$  when  $B = 27$ .

**Solution :**

$$A \propto B \quad \dots(1)$$

$$\therefore A = KB$$

When  $A = 4$ ,  $B = 6$

$$\therefore 4 = 6K \quad \therefore K = \frac{4}{6} = \frac{2}{3} B$$

$$\therefore \text{From (1), } A = \frac{2}{3} B$$

$\therefore$  When  $B = 27$

$$A = \frac{2}{3} \times 27 \Rightarrow A = 18$$

**Example 25 :**

If  $X$  varies directly as  $y$  and inversely as  $Z$  and  $X = 12$  when  $y = 9$  and  $Z = 16$ . Find  $y$  when  $X = 9$  and  $Z = 24$ .

**Solution :**

$$X \propto y \quad \text{and} \quad X \propto 1/Z$$

$$\therefore X \propto \frac{y}{Z} \quad \therefore X = \frac{Ky}{Z} \quad \dots(1)$$

When  $X = 12$ ,  $y = 9$ ,  $Z = 16$ ,

$$12 = \frac{9K}{16} \quad \therefore K = \frac{64}{3}$$

$$\therefore \text{from (1), } X = 64 \frac{y}{3}$$

When  $X = 9$ ,  $Z = 24$ .

$$9 = \frac{64y}{3} \times 24$$

$$\therefore y = \frac{81}{8}$$

**Example 26 :**

A ball is dropped from a balloon floating above the surface of the earth. The distance  $S$  the ball falls is proportional to the square of the time  $t$  that has passed since the ball's release. If the ball falls 144 feet during the first 3 seconds, how far does the ball fall in 9 seconds?

**Solution :**

Given that  $S$  is proportional to the square of  $t$ ,

$$\Rightarrow S \propto t^2 \quad \Rightarrow S = Kt^2 \quad \dots(1)$$

Where 'K' is constant of proportionality.

When  $S = 144$ ,  $t = 3$

$$144 = K \times 3^2 \Rightarrow K = \frac{144}{9} = 16$$

From (1)

$$S = 16t^2$$

Now when  $t = 9$

$$S = 16 \times 9^2 = 1296$$

Thus the ball falls 1296 feet during first 9 seconds.

**Example 27 :**

The intensity  $I$  of light is inversely proportional to the square of the distance ' $d$ ' from the light source. If the light intensity 5 feet from the light source is 3-foot candles, what is the intensity of the light 15 feet from the light source?

**Solution :**

Given that the intensity  $I$  of the light is inversely proportional to the square of the distance from the light source.

$$I \propto \frac{1}{d^2} \quad I = \frac{K}{d^2} \quad \dots(I)$$

Where  $K$  is the proportionality constant.

When  $d = 5$ ,  $I = 3$

$$\therefore 3 = \frac{K}{5^2} \Rightarrow K = 3 \times 25 = 75$$

$$\text{From (I)} \quad I = \frac{75}{d^2}$$

Now, when,  $d = 15$ ,

$$I = \frac{75}{15 \times 15} = \frac{1}{3}$$

i.e. intensity of light source is  $\frac{1}{3}$  foot candle.

**EXERCISES**

1. Three numbers are in the ratio 2 : 3 : 5 and the sum of their squares is 950. Find the numbers.  
[Ans. 10, 15, 25]
2. A ratio is equal to 5 : 7. If its antecedent is 35, what is the consequent?  
[Ans. 49]
3. Two numbers are in the ratio 3 : 5. If 8 is added to each number, the ratio becomes 2 : 3. Find the numbers.  
[Ans. 24, 40]

4. What number must be added to each term of the ratio 3 : 5 to make it 11 : 12?  
[Ans. – 19]
5. Divide 3,600 among A, B, C in the ratio  $1/3 : 1/4 : 1/6$   
[Ans. 1600, 1200, 800]
6. Divide 3,740 into three parts in such a way that half of the first part, one third of the second part, and one-sixth of the third part are equal.  
[Ans. 680, 1020, 2040]
7. In a mixture of 63 litres, the ratio of milk and water is 5 : 2. How much water must be added to this mixture to make the ratio 3 : 2?  
[Ans. 12 litres]
8. An employer reduces the number of employees in the ratio 10 : 7 and increases their wages in the ratio of 14 : 15. In what ratio, the wage bill is increased or decreased?  
[Ans. Decreased in ratio 4 : 3]
9. A purse contains one rupee, 50 paise and 25 paise coins in the ratio 2 : 3 : 4. If the total amount in the purse is ₹ 180, find the number of coins of each kind. [Ans. 80, 120, 160]
10. Ritika reduces her weight in the ratio 3 : 2. What is her weight now, if originally it was 81 kg?  
[Ans. 54 kg]
11. The length and width of a rectangle are in the ratio 5 : 8. If the perimeter of the rectangle is 156 feet. What are the length and width of the rectangle? [Ans.  $l = 30$  feet  $w = 48$  feet]
12. A, B, C start a business by investing ₹ 20,000, ₹ 35,000 and ₹ 45,000 respectively and share the profit of ₹ 10,000 at the end of the year. Find the share in profit of each partner.  
[Ans. A – ₹ 2,000, B – ₹ 3,500, C – ₹ 4,500]
13. Mr. Manish, Mr. Gopal and Mr. Kumar started a transport business of investing ₹ 1 lakh each. Mr. Manish left after 5 month from the commencement of business and Mr. Gopal left 3 months later. At the end of the year the business realized a profit of ₹ 75,000. Find the share of profit of each partner.  
[Ans. Manish – ₹ 15,000, Gopal – ₹ 24,000, Kumar – ₹ 36,000]
14. The difference of squares of two numbers which are in the ratio 3 : 5 is 144. Find the numbers.  
[Ans. 9 and 15]
15. Find the fourth proportional to 6, 10 and 12. [Ans. 120]
16. Find the third proportional to 8 and 16. [Ans. 32]
17. Find the mean proportion of 3.6 and 4.9. [Ans. 4.2]
18. What least number must be added to each of the numbers 16, 7, 79 and 43, so that the resulting numbers are in proportion? [Ans. 5]
19. If  $(X - 2)$ ,  $(X + 2)$ ,  $(2X + 1)$  and  $(2X + 19)$  are in proportion, find the value of X. [Ans.  $X = 4$ ]
20. The ratio of income to the expenditure of a company is 5 : 3. If the company spends 2,700, what is its income? How much is its saving? [Ans. ₹ 4,500]
21. If 4 men can do a work in 2 days, in how many days 2 men can do this work? [Ans. 4 days]
22. If 48 men can dig a pit in 14 days, how long will it take 28 men to dig the similar pit.  
[Ans. 24 days]
23. A garrison is provided with food for 80 soldiers to last for 60 days. Find how long would the food last if 20 additional soldiers join them after 15 days. [Ans. 36 days]
24. The rent for 3 rooms for 9 months is ₹ 135. What will be the rent for 9 rooms for 3 months?  
[Ans. ₹ 135]
25. If 10 horses consume 18 bushels in 36 days, how long will 24 bushels last for 30 horses?  
[Ans. 16 days]

26. Given that  $y$  is proportional to  $x$  and that  $y = 21$  when  $X = 9$ , determine the value of  $y$  when  $X = 27$ . [Ans. 63]
27. A ball is dropped from the edge of a cliff on a certain planet. The distance  $S$ , the ball falls is proportional to the square of the time  $t$  that has passed since the ball's release. If the ball falls 50 feet during the first 5 seconds, how far does the ball fall in 8 seconds. [Ans. 128 feet]
28. Given that  $y$  is inversely proportional to ' $x$ ' and that  $y = 5$  when  $X = 8$ , determine the value of  $y$  when  $X = 10$ . [Ans. 4]
29. The intensity  $I$  of light is inversely proportional to the square of the distance  $d$  from the light source. If the light intensity 4 feet from a light source 2 foot-candles, what is the intensity of the light 8 feet from the light source? [Ans.  $\frac{1}{2}$  foot-candle]
30. Suppose that the price per person for a camping experience is inversely proportional to the number of people who sign up for the experience. If 10 people sign up, the price per person is 350. What will be the price per person if 50 people sign up? [Ans. 70]

### PERCENTAGE

We keep on seeing the advertisements in newspaper, television and hoardings etc. of the following type :

"Voters turnout in the poll was over 70%".

"Banks have lowered the rate of interest on fixed deposits from 8.5% to 7.5%".

"Ishaan got 98% aggregate in class XII examination".

In all the above statements, the important word is 'percent'. The word 'percent' has been derived from the Latin word 'percentum' meaning per hundred or out of hundred.

A fraction expressed with 100 as its denominator is called a percentage and the numerator which expresses the number of hundredths is called the rate percent. The symbol % or the letter P.C. are used as an abbreviation for the words per cent.

**i) To change percentage into fraction :** To change a percentage into fraction, divide the number by 100 and remove the symbol%. Reduce the fraction to its lowest terms.

$$\text{For e.g. : i) } 26\% = \frac{26}{100} = \frac{13}{50}$$

$$\text{ii) } 20\% = \frac{20}{100} = \frac{1}{5}$$

**ii) To change fraction into percentage :** Multiply the fraction by 100 and insert the symbol %.

$$\text{For e.g. : i) } \frac{7}{15} = \left(\frac{7}{15} \times 100\right)\% = 46\frac{2}{3}\%$$

$$\text{ii) } \frac{12}{25} = \left(\frac{12}{25} \times 100\right)\% = 48\%$$

**iii) To find percentage of a given quantity :**  $X\%$  of given quantity =  $\frac{X}{100}$  of given quantity.

$$\text{For e.g. : i) } 35\% \text{ of } 800 = \frac{7}{15} \text{ of } 800 = 280$$

$$\text{ii) } 29\% \text{ of } 67 = \frac{29}{100} \times 67 = 19.43$$

**iv) One quantity as a percentage of another :** To express ' $a$ ' as a percentage of ' $b$ ',  
percentage =  $\left\{\left(\frac{a}{b} \times 100\right)\right\}\%$

$$\text{For e.g. : i) What percent is 3 of 5?}$$

$$\frac{3}{5} \times 100 = 60\%$$

ii) What percent of 80 is 16?

$$\frac{16}{80} \times 100 = 20\%$$

<b>REMEMBER</b>
-----------------

$$\text{Percentage increase} = \left( \frac{\text{Increase in quantity}}{\text{Original quantity}} \times 100 \right) \%$$

$$\text{Percentage decrease} = \left( \frac{\text{Decrease in quantity}}{\text{Original quantity}} \times 100 \right) \%$$

If a quantity increases by  $a\%$ , then

new quantity = original quantity +  $a\%$  of original quantity.

If a quantity decrease by  $a\%$ , then

new quantity = original quantity -  $a\%$  of original quantity.

**Example 28 :**

Saurabh obtained 18 marks in a test of 25 marks. What was her percentage of marks?

**Solution :**

$$\text{Total marks} = 25$$

$$\text{Marks obtained} = 18$$

$$\therefore \text{Fraction of marks obtained} = 18/25$$

$$\therefore \text{Marks obtained in percent} = \frac{18}{25} \times 100 = 72\%$$

**Example 29 :**

One fourth of the total number of toys in a shop were on discount sale. What percentage of the shoes were there on normal price?

**Solution :**

$$\text{Fraction of the total no. of shoes on sale} = \frac{1}{4}$$

$$\therefore \text{Fraction of the total no. of shoes on the}$$

$$\text{Normal price} = 1 - 1/4 = 3/4$$

$$= \frac{3}{4} \times 100 = 75\%$$

**Example 30 :**

Out of 40 students in a class, 32 opted to go for a picnic. What percentage of students opted for picnic?

**Solution :**

$$\text{Total no. of students in a class} = 40$$

$$\text{No. of students who opted for picnic} = 32$$

$$\therefore \text{percentage of students who opted for picnic} = \frac{32}{40} \times 100 = 80\%$$

**Example 31 :**

An estate agent charged 2% commission on sales and thus earned ₹ 3,38,000. Find the value of the sales.

**Solution :**

Let the total sales be ₹  $x$ , then,

$$2\% \text{ of } X = \frac{2}{100} \text{ of } X = 3,38,000$$

$$\therefore X = 3,38,000 \times \frac{100}{2} = 1,69,00,000$$

$\therefore$  The sales are worth ₹ 1,69,00,000

**Example 32 :**

What percentage of 360 is 144?

**Solution :**

Let  $X\%$  of 360 = 144?

$$\therefore \frac{X}{100} \times 360 = 144$$

$$\therefore X = \frac{144}{360} \times 100 = 40\%$$

**Example 33 :**

A's income is 25% more than that of B. B's income is 8% more than that of C. If A's income is ₹ 20,250, then find the income of C.

**Solution :**

Let income of C be ₹  $x$ .

$$\begin{aligned} \text{Income of B} &= x + 8\% \text{ of } x \\ &= x + \frac{8x}{100} = \frac{108x}{100} \end{aligned}$$

$$\begin{aligned} \text{Income of A} &= \frac{108x}{100} + 25\% \text{ of } \frac{108x}{100} \\ &= \frac{108x}{100} \times \frac{125}{100} \end{aligned}$$

$$\frac{108x}{100} \times \frac{125}{100} = 20,250$$

$$\text{or } X = \frac{20,250 \times 100 \times 100}{108 \times 125} = 15,000$$

$\therefore$  Income of C is ₹ 15,000.

**Example 34 :**

A shirt is sold for ₹ 480 after allowing a discount of 20% on its marked price. Find the marked price.

**Solution :**

$$SP = ₹ 480, \quad \text{Discount} = 20\%$$

Let marked price be ₹  $x$  then

Selling price – Discount = Amount paid

$$\therefore x - 20\% \text{ of } x = ₹ 480$$

$$80\% \text{ of } x = 480 \Rightarrow \frac{80}{100} \text{ of } x = 480$$

$$x = \frac{480 \times 100}{80}$$

$$x = 600$$

$\therefore$  Marked price = ₹ 600.

<b>PROFIT AND LOSS</b>
------------------------

The concept of profit and loss is a vital concept for all businesses. The two important concepts related to profit and loss is cost price and selling price. The amount at which a shopkeeper purchases an article is called the Cost Price (C.P.). The amount at which the shopkeeper sells an article is called the selling price (S.P). The profit and loss is generally expressed as a percentage of the cost price.

$$\text{Profit} = \text{SP} - \text{CP} \quad \text{When } \text{S.P} > \text{C.P}$$

$$\text{Profit} = \text{CP} - \text{SP} \quad \text{When } \text{CP} > \text{SP}$$

However, to compare the profit earned on different items, we have to calculate the profit obtained on a C.P. of ₹ 100 i.e. profit percent.

$$\therefore \text{Profit \%} = \frac{\text{Profit}}{\text{CP}} \times 100 \quad \text{or} \quad \text{Profit} = \frac{\text{CP} \times \text{Profit \%}}{100}$$

Similarly,

$$\text{Loss \%} = \frac{\text{Loss}}{\text{CP}} \times 100$$

$$\text{Also, } \text{SP} = \left( \frac{100 + \text{Profit \%}}{100} \right) \times \text{CP}$$

$$\text{SP} = \left( \frac{100 - \text{Loss \%}}{100} \right) \times \text{CP}$$

$$\text{CP} = \left( \frac{100}{100 + \text{Profit \%}} \right) \times \text{SP}$$

$$\text{CP} = \left( \frac{100}{100 - \text{Loss \%}} \right) \times \text{SP}$$

**Example 35 :**

A shopkeeper buys an article for ₹ 360 and sells it for ₹ 270. Find his gain or loss percent.

**Solution :**

$$\text{Here C.P} = ₹ 360, \quad \text{S.P} = ₹ 270$$

Since  $\text{CP} > \text{SP}$   $\therefore$  there is a loss

$$\text{Loss} = \text{CP} - \text{SP} = ₹ (360 - 270) = ₹ 90$$

$$\therefore \text{Loss \%} = \left( \frac{\text{Loss}}{\text{CP}} \right) \times 100 = \frac{90}{360} \times 100 = 25\%$$

**Example 36 :**

A almirah was sold for ₹ 356.40 and 8% profit was earned on it. Find the cost price at which it was bought?

**Solution :**

$$\text{SP} = ₹ 356.40, \quad \text{Profit \%} = 8$$

$$\begin{aligned} \therefore \text{CP} &= \left( \frac{100}{100 + \text{Profit \%}} \right) \times \text{SP} \\ &= \frac{100}{100 + 8} \times 356.40 = \frac{100}{108} \times 356.40 \\ &= ₹ 330 \end{aligned}$$

$\therefore$  Cost price was ₹ 330.

**Example 37 :**

A man sells a watch for ₹ 350.50. He made a profit of 25% on the transaction for how much should he have sold it to gain 30% profit.

**Solution :**

$$SP = ₹ 350.50, \quad \text{Profit \%} = 25\%$$

$$\begin{aligned} \therefore CP &= \left( \frac{100}{100 + P \%} \right) \times SP \\ &= \left( \frac{100}{100 + 25} \right) \times 350.50 \\ &= \frac{100}{125} \times 350.50 = ₹ 280.40 \end{aligned}$$

$$\text{Required profit} = 30\%$$

$$\begin{aligned} \therefore \text{Required SP} &= \left( \frac{100 + P \%}{100} \right) \times CP \\ &= \left( \frac{100 + 30}{100} \times 280.40 \right) \\ &= \frac{130 \times 280.40}{100} = ₹ 364.52 \end{aligned}$$

$\therefore$  Thus, the man should have sold the watch for ₹ 364.52.

**Example 38 :**

A watch was sold at a profit of 12%. Had it been sold for ₹ 33 more, the profit would have been 14%. Find the cost price of the watch.

**Solution :**

Let the cost price of the watch be ₹  $x$ .

$$\therefore SP = x \left( \frac{100 + P \%}{100} \right) \times CP = \left( \frac{100 + 12}{100} \right) \times x = \frac{112x}{100}$$

$$\text{If the watch is sold for ₹ 33 more than SP} = \left( \frac{112x}{100} + 33 \right)$$

New profit = 14%

$$\therefore CP = x = \frac{\left( \frac{112x}{100} + 33 \right) \times 100}{114}$$

$$\Rightarrow 114x = 112x + 3,300 \quad \Rightarrow 2x = 3,300$$

$$\therefore X = ₹ 1,650$$

$$\therefore CP = ₹ 1,650$$

**Example 39 :**

A man sold two horses for ₹ 29,700 each. On one he lost 10% while he gained 10% on the other. Find his total gain or loss percent in the transaction.

**Solution :**

$$SP \text{ of first horse} = ₹ 29,700$$

$$\text{Loss} = 10\%$$

$$\begin{aligned} \therefore CP &= \left( \frac{100}{100 + L \%} \right) \times SP \\ &= \left( \frac{100}{100 - 10} \right) \times 29,700 \\ &= \frac{100 \times 29,700}{90} = ₹ 33,000 \end{aligned}$$

...(1)

$$SP \text{ of second Horse} = ₹ 29,700$$

Gain = 10%

$$\begin{aligned}\therefore \text{CP} &= \left(\frac{100}{100 + P\%}\right) \times \text{SP} \\ &= \left(\frac{100}{100 + 10}\right) \times 29,700 \\ &= \frac{100 \times 29,700}{110} = ₹ 27,000 \quad \dots(2)\end{aligned}$$

Total CP = ₹ (33,000 + 27,000) = ₹ 60,000

Total SP = 2 × 29,700 = ₹ 59,400

Net loss = 60,000 – 59,400 = ₹ 600

#### Example 40 :

A number is increased by 20% and then it is decreased by 20% find the next increase or decrease percent.

#### Solution :

Let the number be 100.

$$\text{Increased number} = \left(\frac{100 + 20}{100}\right) \times 100 = 120$$

Now, 120 is decreased by 20%

$$\therefore \text{Decreased No} = \left(\frac{100 - 20}{100}\right) \times 120 = \frac{80 + 120}{100} = 96$$

Net decrease = 100 – 96 = 4

$$\text{Hence, the net percentage decrease} = \left(\frac{4}{100} \times 100\right)\% = 4\%$$

#### EXERCISE

1. There are 20 eggs in a fridge and 6 of them are brown. What percent of eggs are not brown? [Ans. 70%]
2. 44% of the students of a class are girls. If the number of girls is 6 less than the number of boys, how many students are there in the class? [Ans. 50]
3. A man donated 5% of his monthly income to a charity and deposited 12% of the rest in Bank. If he has ₹ 11,704 left with him, what is his monthly income? [Ans. ₹ 14,000]
4. The passing marks in a examination is 35%. If a student gets 190 marks, falls short by 20 marks, then find the total marks of the examination conducted. [Ans. 600]
5. The salary of a technician has been increased by 30%. By what percent must the new salary be reduced to restore the original salary? [Ans.  $23\frac{1}{13}\%$ ]
6. Samir bought a bicycle for ₹ 1,724 and after three months he sold it at a profit of 25% for what price did he sell the bicycle? [Ans. ₹ 2,155]
7. By selling an article for ₹ 4,200, the shopkeeper lost 20%. At what price should he have sold it to gain 16%? [Ans. ₹ 6,090]
8. By selling a soft set for ₹ 3,825, a shopkeeper loses 15% on it. Find the price at which it was bought. [Ans. ₹ 4,500]
9. Rajnish sells a pair of shoes at a profit of 30%. Find its cost price if the selling price is ₹ 650. [Ans. ₹ 500]
10. A number is increased by 10% and then the increased number is decreased by 10% find the net increase or decrease percent. [Ans. 1%]

11. In an examination a student scored 176 marks and failed by 34 marks. Find the maximum marks of the examination if a student must score at least 40% to pass the examination. [Ans. 52s]
12. A shopkeeper purchases two calculators A and B at a total cost of ₹ 550. He sells calculator A at 15% profit and calculator B at a loss of 20% and gets the same selling price for both the calculators. Find the cost price of each one of the two calculators. [Ans. A – ₹ 225.64; B – ₹ 324.36]
13. Soham bought grapes worth ₹ 500 and sold half of them at a gain of 8%. At what gain percent must he sell the remaining grapes so as to get a gain of 20% on the whole? [Ans. 32%]
14. A dealer buys a table listed at ₹ 1,800 and gets a discount of 25%. He spends ₹ 150 on transportation and sells it at a profit of 10%. Find the selling price of table. [Ans. ₹ 1,650]
15. A shopkeeper marks his goods 50% more than the C.P. and allows a discount of 40%, find his gain or loss percent. [Ans. 10% loss]