

(UNIVERSITY OF MUMBAI)

Syllabus for: T.Y.B.Sc./T.Y.B.A.

Program: B.Sc./B.A.

Course: Mathematics

Choice based Credit System (CBCS)

with effect from the
academic year 2018-19

SEMESTER V

Multivariable Calculus II				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 501, UAMT 501	I	Multiple Integrals	2.5	3
	II	Line Integrals		
	III	Surface Integrals		
Linear Algebra				
USMT 502 ,UAMT 502	I	Quotien spaces and Orthogonal Linear Transformations	2.5	3
	II	Eigen values and Eigen vectors		
	III	Diagonalisation		
Topology of Metric Spaces				
USMT 503/UAMT503	I	Metric spaces	2.5	3
	II	Sequences and Complete metric spaces		
	III	Compact Sets		
Numerical Analysis I(Elective A)				
USMT5A4 ,UAMT 5A4	I	Errors Analysis	2.5	3
	II	Transcendental and Polynomial & Equations		
	III	Linear System of Equations		
Number Theory and Its applications I (Elective B)				
USMT5B4 ,UAMT 5B4	I	Congruences and Factorization	2.5	3
	II	Diophantine equations and their & solutions		
	III	Primitive Roots and Cryptography		
Graph Theory (Elective C)				
USMT5C4 ,UAMT 5C4	I	Basics of Graphs	2.5	3
	II	Trees		
	III	Eulerian and Hamiltonian graphs		
Basic Concepts of Probability and Random Variables (Elective D)				
USMT5D4 ,UAMT 5D4	I	Basic Concepts of Probability and Random Variables	2.5	3
	II	Properties of Distribution function, Joint Density function		
	III	Weak Law of Large Numbers		
PRACTICALS				
USMT5P05/UAMTP05		Practicals based on USMT501/UAMT 501 and USMT 502/UAMT 502	3	6
USMT5P06/UAMTP06		Practicals based on USMT503/ UAMT 503 and USMT5A4/ UAMT 5A4 OR USMT5B4/ UAMT 5B4 OR USMT5C4/ UAMT 5C4 OR USMT5D4/ UAMT 5D4	3	6

SEMESTER VI

BASIC COMPLEX ANALYSIS				
Course Code	UNIT	TOPICS	Credits	L/Week
USMT 601, UAMT 601	I	Introduction to Complex Analysis	2.5	3
	II	Cauchy Integral Formula		
	III	Complex power series, Laurent series and isolated singularities		
ALGEBRA				
USMT 602 ,UAMT 602	I	Group Theory	2.5	3
	II	Ring Theory		
	III	Polynomial Rings and Field theory Homomorphism		
Topology of Metric Spaces and Real Analysis				
USMT 603 / UAMT 603	I	Continuous functions on Metric spaces	2.5	3
	II	Connected sets		
		Sequences and series of functions		
Numerical Analysis II(Elective A)				
USMT6A4 ,UAMT 6A4	I	Interpolation	2.5	3
	II	Polynomial Approximations and Numerical Differentiation		
	III	Numerical Integration		
Number Theory and Its applications II (Elective B)				
USMT6B4 ,UAMT 6B4	I	Quadratic Reciprocity	2.5	3
	II	Continued Fractions		
	III	Pell's equation, Arithmetic function & and Special numbers		
Graph Theory and Combinatorics (Elective C)				
USMT6C4 ,UAMT 6C4	I	Colorings of Graphs	2.5	3
	II	Planar graph		
	III	Combinatorics		
Operations Research (Elective D)				
USMT6D4 ,UAMT 6D4	I	Basic Concepts of Probability and Linear Programming I	2.5	3
	II	Linear Programming II		
	III	Queuing Systems		
PRACTICALS				
USMTTP07/ UAMTP07		Practicals based on USMT601/UAMT 601 and USMT 602/UAMT 602	3	6
USMTTP08/UAMTP08		Practicals based on USMT603/ UAMT 603 and USMT6A4/ UAMT 6A4 OR USMT6B4/ UAMT 6B4 OR USMT6C4/ UAMT 6C4 OR USMT6D4/ UAMT 6D4	3	6

- Note:**
- 1 . USMT501/UAMT501, USMT502/UAMT502, USMT503/UAMT503 are compulsory courses for Semester V.
 - 2 . Candidate has to opt one Elective Course from USMT5A4/UAMT5A4, USMT5B4/UAMT5B4, USMT5C4/UAMT5C4 and USMT5D4/UAMT5D4 for Semester V.
 - 3 . USMT601/UAMT601, USMT602/UAMT602, USMT603/UAMT603 are compulsory courses for Semester VI.
 - 4 . Candidate has to opt one Elective Course from USMT6A4/UAMT6A4, USMT6B4/UAMT6B4, USMT6C4/UAMT6C4 and USMT6D4/UAMT6D4 for Semester VI.
 - 5 . Passing in theory and practical shall be separate.

Teaching Pattern for T.Y.B.Sc/B.A.

1. Three lectures per week per course (1 lecture/period is of 48 minutes duration).
2. One practical of three periods per week per course (1 lecture/period is of 48 minutes duration).

Scheme of Examination

- I. Semester End Theory Examinations:** There will be a Semester-end external Theory examination of 100 marks for each of the courses USMT501/UAMT501, USMT502/UAMT502, USMT503 and USMT5A4 OR USMT5B4 OR USMT5C4 OR USMT 5D4 of Semester V and USMT601/UAMT601, USMT602/UAMT602, USMT603 and USMT6A4 OR USMT6B4 OR USMT 6C4 OR USMT 6D4 of semester VI to be conducted by the University.

1. Duration: The examinations shall be of 3 Hours duration.
2. Theory Question Paper Pattern:
 - a) There shall be FIVE questions. The first question Q1 shall be of objective type for 20 marks based on the entire syllabus. The next three questions Q2, Q2, Q3 shall be of 20 marks, each based on the units I, II, III respectively. The fifth question Q5 shall be of 20 marks based on the entire syllabus.
 - b) All the questions shall be compulsory. The questions Q2, Q3, Q4, Q5 shall have internal choices within the questions. Including the choices, the marks for each question shall be 30-32.
 - c) The questions Q2, Q3, Q4, Q5 may be subdivided into sub-questions as a, b, c, d & e, etc and the allocation of marks depends on the weightage of the topic.
 - d) The question Q1 may be subdivided into 10 sub-questions of 2 marks each.

II. Semester End Examinations Practicals:

There shall be a Semester-end practical examinations of three hours duration and 100 marks for each of the courses USMTP05/UAMTP05 of Semester V and USMTP06/UAMTP06 of semester VI.

In semester V, the Practical examinations for USMTP05/UAPTP05 and USMTP06/UAMTP06 are conducted by the college.

In semester VI, the Practical examinations for USMTP07/UAMTP07 and USMTP08/UAMTP08 are conducted by the University.

Question Paper pattern:

Paper pattern: The question paper shall have two parts A, B.
Each part shall have two Sections.

Section I Objective in nature: Attempt any Eight out of Twelve multiple choice questions. ($8 \times 3 = 24$ Marks)

Section II Problems: Attempt any Two out of Three. ($8 \times 2 = 16$ Marks)

Practical Course	Part A	Part B	Marks out of	duration
USMTP05/UAMTP05	Questions from USMT501/UAMT501	Questions from USMT502/UAMT502	80	3 hours
USMTP06/UAMTP06	Questions from USMT503/UAMT503	Questions from USMT504/UAMT504	80	2 hours
USMTP07/UAMTP07	Questions from USMT601/UAMT601	Questions from USMT602/UAMT602	80	3 hours
USMTP06/UAMTP08	Questions from USMT603/UAMT603	Questions from USMT604/UAMT604	80	2 hours

Marks for Journals and Viva:

For each course USMT501/UAMT501, USMT502/UAMT502, USMT503/UAMT503, USMT504/UAMT504, USMT601/UAMT601, USMT602/UAMT602 USMT603/UAMT603, and USMT604/UAMT604:

1. Journals: 5 marks.
2. Viva: 5 marks.

Each Practical of every course of Semester V and VI shall contain 10 (ten) problems out of which minimum 05 (five) have to be written in the journal. A student must have a certified journal before appearing for the practical examination.

SEMESTER V
MULTIVARIABLE CALCULUS II
Course Code: USMT501/UAMT501

ALL Results have to be done with proof unless otherwise stated.

Unit I-Multiple Integrals (15L)

Definition of double (resp: triple) integral of a function and bounded on a rectangle (resp:box). Geometric interpretation as area and volume. Fubini's Theorem over rectangles and any closed bounded sets, Iterated Integrals. Basic properties of double and triple integrals proved using the Fubini's theorem such as

- (i) Integrability of the sums, scalar multiples, products, and (under suitable conditions) quotients of integrable functions. Formulae for the integrals of sums and scalar multiples of integrable functions.

- (ii) Integrability of continuous functions. More generally, Integrability of functions with a “small set of (Here, the notion of “small sets should include finite unions of graphs of continuous functions.)
- (iii) Domain additivity of the integral. Integrability and the integral over arbitrary bounded domains. Change of variables formula (Statement only). Polar, cylindrical and spherical coordinates, and integration using these coordinates. Differentiation under the integral sign. Applications to finding the center of gravity and moments of inertia.

References for Unit I:

1. Apostol, Calculus, Vol. 2, Second Ed., John Wiley, New York, 1969 Section 1.1 to 11.8
2. James Stewart , Calculus with early transcendental Functions - Section 15
3. J.E.Marsden and A.J. Tromba, Vector Calculus, Fourth Ed., W.H. Freeman and Co., New York, 1996. Section 5.2 to 5.6.

Unit 2: Line Integrals (15L)

Review of Scalar and Vector fields on \mathbb{R}^n , Vector Differential Operators, Gradient, Curl, Divergence.

Paths (parametrized curves) in \mathbb{R}^n (emphasis on \mathbb{R}^2 and \mathbb{R}^3), Smooth and piecewise smooth paths. Closed paths. Equivalence and orientation preserving equivalence of paths. Definition of the line integral of a vector field over a piecewise smooth path. Basic properties of line integrals including linearity, path-additivity and behavior under a change of parameters. Examples.

Line integrals of the gradient vector field, Fundamental Theorem of Calculus for Line Integrals, Necessary and sufficient conditions for a vector field to be conservative. Greens Theorem (proof in the case of rectangular domains). Applications to evaluation of line integrals.

References for Unit II:

1. Lawrence Corwin and Robert Szczarba ,Multivariable Calculus, Chapter 12.
2. Apostol, Calculus, Vol. 2, Second Ed., John Wiley, New York, 1969 Section 10.1 to 10.5,10.10 to 10.18
3. James Stewart , Calculus with early transcendental Functions - Section 16.1 to 16.4.
4. J.E.Marsden and A.J. Tromba, Vector Calculus, Fourth Ed., W.H. Freeman and Co., New York, 1996. Section 6.1,7.1.7.4.

Unit III: Surface Integrals (15 L)

Parameterized surfaces. Smoothly equivalent parameterizations. Area of such surfaces.

Definition of surface integrals of scalar-valued functions as well as of vector fields defined on a surface.

Curl and divergence of a vector field. Elementary identities involving gradient, curl and divergence.

Stokes Theorem (proof assuming the general form of Greens Theorem). Examples.

Gauss Divergence Theorem (proof only in the case of cubical domains). Examples.

References for Unit III:

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1. Apostol, Calculus, Vol. 2, Second Ed., John Wiley, New York, 1969 Section 1.1 to 11.8
 2. James Stewart , Calculus with early transcendental Functions - Section 16.5 to 16.9
 3. J.E.Marsden and A.J. Tromba, Vector Calculus, Fourth Ed., W.H. Freeman and Co., New York, 1996 Section 6.2 to 6.4.

Other References :

1. T Apostol, Mathematical Analysis, Second Ed., Narosa, New Delhi. 1947.
2. R. Courant and F.John, Introduction to Calculus and Analysis, Vol.2, Springer Verlag, New York, 1989.
3. W. Fleming, Functions of Several Variables, Second Ed., Springer-Verlag, New York, 1977.
4. M.H. Protter and C.B.Morrey Jr., Intermediate Calculus, Second Ed., Springer-Verlag, New York, 1995.
5. G.B. Thomas and R.L Finney, Calculus and Analytic Geometry, Ninth Ed. (ISE Reprint), Addison- Wesley, Reading Mass, 1998.
6. D.V.Widder, Advanced Calculus, Second Ed., Dover Pub., New York. 1989.
7. A course in Multivariable Calculus and Analysis., Sudhir R.Ghorpade and Balmohan Limaye, Springer International Edition.

Linear Algebra

Course Code: USMT502/UAMT502

Unit I. Quotient Spaces and Orthogonal Linear Transformations (15L)

Review of vector spaces over \mathbb{R} , sub spaces and linear transformation.

Quotient Spaces: For a real vector space V and a subspace W , the cosets $v + W$ and the quotient space V/W , First Isomorphism theorem of real vector spaces (fundamental theorem of homomorphism of vector spaces), Dimension and basis of the quotient space V/W , when V is finite dimensional.

Orthogonal transformations: Isometries of a real finite dimensional inner product space, Translations and Reflections with respect to a hyperplane, Orthogonal matrices over \mathbb{R} , Equivalence of orthogonal transformations and isometries fixing origin on a finite dimensional inner product space, Orthogonal transformation of \mathbb{R}^2 , Any orthogonal transformation in \mathbb{R}^2 is a reflection or a rotation, Characterization of isometries as composites of orthogonal transformations and translation. Characteristic polynomial of an $n \times n$ real matrix. Cayley Hamilton Theorem and its Applications (Proof assuming the result $A(\text{adj}A) = I_n$ for an $n \times n$ matrix over the polynomial ring $\mathbb{R}[t]$).

Unit II. Eigenvalues and eigen vectors (15L)

Eigen values and eigen vectors of a linear transformation $T : V \rightarrow V$, where V is a finite dimensional real vector space and examples, Eigen values and Eigen vectors of $n \times n$ real matrices, The linear independence of eigenvectors corresponding to distinct eigenvalues of a linear transformation and a Matrix. The characteristic polynomial of an $n \times n$ real matrix and a linear transformation of a finite dimensional real vector space to itself, characteristic roots, Similar

matrices, Relation with change of basis, Invariance of the characteristic polynomial and (hence of the) eigen values of similar matrices, Every square matrix is similar to an upper triangular matrix. Minimal Polynomial of a matrix, Examples like minimal polynomial of scalar matrix, diagonal matrix, similar matrix, Invariant subspaces.

Unit III: Diagonalisation (15L)

Geometric multiplicity and Algebraic multiplicity of eigen values of an $n \times n$ real matrix, An $n \times n$ matrix A is diagonalizable if and only if has a basis of eigenvectors of A if and only if the sum of dimension of eigen spaces of A is n if and only if the algebraic and geometric multiplicities of eigen values of A coincide, Examples of non diagonalizable matrices, Diagonalisation of a linear transformation $T : V \rightarrow V$, where V is a finite dimensional real vector space and examples. Orthogonal diagonalisation and Quadratic Forms. Diagonalisation of real Symmetric matrices, Examples, Applications to real Quadratic forms, Rank and Signature of a Real Quadratic form, Classification of conics in \mathbb{R}^2 and quadric surfaces in \mathbb{R}^3 . Positive definite and semi definite matrices, Characterization of positive definite matrices in terms of principal minors.

Recommended Books.

1. S. Kumaresan, Linear Algebra: A Geometric Approach.
2. Ramachandra Rao and P. Bhimasankaram, Tata McGrawHillll Publishing Company.

Additional Reference Books

1. T. Banchoff and J. Wermer, Linear Algebra through Geometry, Springer.
2. L. Smith, Linear Algebra, Springer.
3. M. R. Adhikari and Avishek Adhikari, Introduction to linear Algebra, Asian Books Private Ltd.
4. K Hoffman and Kunze, Linear Algebra, Prentice Hall of India, New Delhi.
5. Inder K Rana, Introduction to Linear Algebra, Ane Books Pvt. Ltd.

Course: Topology of Metric Spaces
Course Code: USMT503/UAMT503

Unit I: Metric spaces (15 L)

Definition, examples of metric spaces \mathbb{R}, \mathbb{R}^2 , Euclidean space \mathbb{R}^n with its Euclidean, sup and sum metric, \mathbb{C} (complex numbers), the spaces l^1 and l^2 of sequences and the space $C[a, b]$, of real valued continuous functions on $[a, b]$. Discrete metric space.

Distance metric induced by the norm, translation invariance of the metric induced by the norm. Metric subspaces, Product of two metric spaces. Open balls and open set in a metric space, examples of open sets in various metric spaces. Hausdorff property. Interior of a set. Properties of open sets. Structure of an open set in \mathbb{R} . Equivalent metrics.

Distance of a point from a set, between sets, diameter of a set in a metric space and bounded sets. Closed ball in a metric space, Closed sets- definition, examples. Limit point of a set, isolated point, a closed set contains all its limit points, Closure of a set and boundary of a set.

Unit II: Sequences and Complete metric spaces (15L)

Sequences in a metric space, Convergent sequence in metric space, Cauchy sequence in a metric space, subsequences, examples of convergent and Cauchy sequence in finite metric spaces, \mathbb{R}^n with different metrics and other metric spaces.

Characterization of limit points and closure points in terms of sequences, Definition and examples of relative openness/closeness in subspaces. Dense subsets in a metric space and Separability Definition of complete metric spaces, Examples of complete metric spaces, Completeness property in subspaces, Nested Interval theorem in \mathbb{R} , Cantor's Intersection Theorem, Applications of Cantors Intersection Theorem:

- (i) The set of real Numbers is uncountable.
- (ii) Density of rational Numbers (Between any two real numbers there exists a rational number)
- (iii) Intermediate Value theorem: Let $f : [a, b] \rightarrow \mathbb{R}$ be continuous, and assume that $f(a)$ and $f(b)$ are of different signs say, $f(a) < 0$ and $f(b) > 0$. Then there exists $c \in (a, b)$ such that $f(c) = 0$.

Unit III: Compact sets 15 lectures

Definition of compact metric space using open cover, examples of compact sets in different metric spaces $\mathbb{R}, \mathbb{R}^2, \mathbb{R}^n$, Properties of compact sets: A compact set is closed and bounded, (Converse is not true). Every infinite bounded subset of compact metric space has a limit point. A closed subset of a compact set is compact. Union and Intersection of Compact sets. Equivalent statements for compact sets in \mathbb{R} :

- (i) Sequentially compactness property.
- (ii) Heine-Borel property: Let I be a closed and bounded interval. Let \mathcal{F} be a family of open intervals such that $I \subseteq \bigcup \mathcal{F}$. Then there exists a finite subset \mathcal{F}_0 such that $I \subseteq \bigcup \mathcal{F}_0$, that is, I is contained in the union of a finite number of open intervals of the given family.
- (iii) Closed and boundedness property.
- (iv) Bolzano-Weierstrass property: Every bounded sequence of real numbers has a convergent subsequence.

Reference books:

1. S. Kumaresan, Topology of Metric spaces.
2. E. T. Copson. Metric Spaces. Universal Book Stall, New Delhi, 1996.
3. Expository articles of MTTTS programme

Other references :

1. W. Rudin, Principles of Mathematical Analysis.
2. T. Apostol. Mathematical Analysis, Second edition, Narosa, New Delhi, 1974
3. E. T. Copson. Metric Spaces. Universal Book Stall, New Delhi, 1996.
4. R. R. Goldberg Methods of Real Analysis, Oxford and IBH Pub. Co., New Delhi 1970.
5. P.K.Jain. K. Ahmed. Metric Spaces. Narosa, New Delhi, 1996.
6. W. Rudin. Principles of Mathematical Analysis, Third Ed, McGraw-Hill, Auckland, 1976.
7. D. Somasundaram, B. Choudhary. A first Course in Mathematical Analysis. Narosa, New Delhi
8. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hi, New York, 1963.
9. Sutherland. Topology.

Course: Numerical Analysis I (Elective A)
Course Code: USMT5A4/UAMT5A4

N.B. Derivations and geometrical interpretation of all numerical methods have to be covered.

Unit I. Errors Analysis and Transcendental & Polynomial Equations (15L)

Measures of Errors: Relative, absolute and percentage errors. Types of errors: Inherent error, Round-off error and Truncation error. Taylors series example. Significant digits and numerical stability. Concept of simple and multiple roots. Iterative methods, error tolerance, use of intermediate value theorem. Iteration methods based on first degree equation: Newton-Raphson method, Secant method, Regula-Falsi method, Iteration Method. Condition of convergence and Rate of convergence of all above methods.

Unit II. Transcendental and Polynomial Equations (15L)

Iteration methods based on second degree equation: Muller method, Chebyshev method, Multipoint iteration method. Iterative methods for polynomial equations; Descarts rule of signs, Birge-Vieta method, Bairstrow method. Methods for multiple roots. Newton-Raphson method. System of non-linear equations by Newton- Raphson method. Methods for complex roots. Condition of convergence and Rate of convergence of all above methods.

Unit III. Linear System of Equations (15L)

Matrix representation of linear system of equations. Direct methods: Gauss elimination method.

Pivot element, Partial and complete pivoting, Forward and backward substitution method, Triangularization methods-Doolittle and Crouts method, Choleskys method. Error analysis of direct methods. Iteration methods: Jacobi iteration method, Gauss-Siedal method. Convergence analysis of iterative method. Eigen value problem, Jacobis method for symmetric matrices Power method to determine largest eigenvalue and eigenvector.

Recommended Books

1. Kendall E. and Atkinson, An Introduction to Numerical Analysis, Wiley.
2. M. K. Jain, S. R. K. Iyengar and R. K. Jain, Numerical Methods for Scientific and Engineering Computation, New Age International Publications.
3. S.D. Comte and Carl de Boor, Elementary Numerical Analysis, An algorithmic approach, McGrawHill International Book Company.
4. S. Sastry, Introductory methods of Numerical Analysis, PHI Learning.
5. Hildebrand F.B., Introduction to Numerical Analysis, Dover Publication, NY.
6. Scarborough James B., Numerical Mathematical Analysis, Oxford University Press, New Delhi.

Course: Number Theory and its applications I (Elective B)

Course Code: USMT5B4 / UAMT5B4

Unit I. Congruences and Factorization (15L)

Review of Divisibility, Primes and The fundamental theorem of Arithmetic.

Congruences : Definition and elementary properties, Complete residue system modulo m , Reduced residue system modulo m , Euler's function and its properties, Fermat's little Theorem, Euler's generalization of Fermat's little Theorem, Wilson's theorem, Linear congruence, The Chinese remainder Theorem, Congruences of Hillgher degree, The Fermat-Kraitichik Factorization Method.

Unit II. Diophantine equations and their solutions (15L)

The linear equations $ax + by = c$. The equations $x^2 + y^2 = p$, where p is a prime. The equation $x^2 + y^2 = z^2$, Pythagorean triples, primitive solutions, The equations $x^4 + y^4 = z^2$ and $x^4 + y^4 = z^4$ have no solutions $(x; y; z)$ with $xyz \neq 0$. Every positive integer n can be expressed as sum of squares of four integers, Universal quadratic forms $x^2 + y^2 + z^2 + t^2$. Assorted examples :section 5.4 of Number theory by Niven- Zuckermann-Montgomery.

Unit III. Primitive Roots and Cryptography (15L)

Order of an integer and Primitive Roots. Basic notions such as encryption (enciphering) and decryption (deciphering), Cryptosystems, symmetric key cryptography, Simple examples such as shift cipher, Affine cipher, Hill's cipher, Vigenere cipher. Concept of Public Key Cryptosystem; RSA Algorithm. An application of Primitive Roots to Cryptography.

Reference for Unit III:

Elementary number theory, David M. Burton, Chapter 8 sections 8.1, 8.2 and 8.3, Chapter 10, sections 10.1, 10.2 and 10.3

Recommended Books

1. Niven, H. Zuckerman and H. Montgomery, An Introduction to the Theory of Numbers, John Wiley & Sons. Inc.
2. David M. Burton, An Introduction to the Theory of Numbers. Tata McGrawHillll Edition.
3. G. H. Hardy and E.M. Wright. An Introduction to the Theory of Numbers. Low priced edition. The English Language Book Society and Oxford University Press, 1981.
4. Neville Robins. Beginning Number Theory. Narosa Publications.
5. S.D. Adhikari. An introduction to Commutative Algebra and Number Theory. Narosa Publishing House.
6. N. Koblitz. A course in Number theory and Cryptography, Springer.
7. M. Artin, Algebra. Prentice Hall.
8. K. Ireland, M. Rosen. A classical introduction to Modern Number Theory. Second edition, Springer Verlag.
9. William Stalling. Cryptology and network security.

Course: Graph Theory (Elective C)
Course Code: USMT5C4 / UAMT5C4

Unit I. Basics of Graphs (15L)

Definition of general graph, Directed and undirected graph, Simple and multiple graph, Types of graphs- Complete graph, Null graph, Complementary graphs, Regular graphs Sub graph of a graph, Vertex and Edge induced sub graphs, Spanning sub graphs. Basic terminology- degree of a vertex, Minimum and maximum degree, Walk, Trail, Circuit, Path, Cycle. Handshaking theorem and its applications, Isomorphism between the graphs and consequences of isomorphism between the graphs, Self complementary graphs, Connected graphs, Connected components. Matrices associated with the graphs Adjacency and Incidence matrix of a graph- properties, Bipartite graphs and characterization in terms of cycle lengths. Degree sequence and Havel-Hakimi theorem, Distance in a graph- shortest path problems, Dijkstra's algorithm.

Unit II. Trees (15L)

Cut edges and cut vertices and relevant results, Characterization of cut edge, Definition of a tree and its characterizations, Spanning tree, Recurrence relation of spanning trees and Cayley formula for spanning trees of K_n , Algorithms for spanning tree-BFS and DFS, Binary and m-ary tree, Prefix codes and Huffman coding, Weighted graphs and minimal spanning trees - Kruskal's algorithm for minimal spanning trees.

Unit III. Eulerian and Hamiltonian graphs (15L)

Eulerian graph and its characterization- Fleury's Algorithm-(Chinese postman problem), Hamiltonian graph, Necessary condition for Hamiltonian graphs using $G - S$ where S is a proper subset of $V(G)$, Sufficient condition for Hamiltonian graphs- Ore's theorem and Dirac's theorem, Hamiltonian closure of a graph, Cube graphs and properties like regular, bipartite, Connected and Hamiltonian nature of cube graph, Line graph of graph and simple results.

Recommended Books.

1. Bondy and Murty Graph, Theory with Applications.
2. Balkrishnan and Ranganathan, Graph theory and applications.
3. West D G. , Graph theory.

Additional Reference Book.

1. Behzad and Chartrand Graph theory.
2. Choudam S. A., Introductory Graph theory.

Course: Basic Concepts of Probability and Random Variables (Elective D)
Course Code: USMT5D4 / UAMT5D4

Unit I. Basic Concepts of Probability and Random Variables.(15 L)

Basic Concepts: Algebra of events including countable unions and intersections, Sigma field \mathcal{F} , Probability measure P on \mathcal{F} , Probability Space as a triple (Ω, \mathcal{F}, P) , Properties of P including Subadditivity. Discrete Probability Space, Independence and Conditional Probability, Theorem of Total Probability. Random Variable on (Ω, \mathcal{F}, P) Definition as a measurable function, Classification of random variables - Discrete Random variable, Probability function, Distribution function, Density function and Probability measure on Borel subsets of \mathbb{R} , Absolutely continuous random variable. Function of a random variable; Result on a random variable R with distribution function F to be absolutely continuous, Assume F is continuous everywhere and has a continuous derivative at all points except possibly at finite number of points, Result on density function f_2 of R_2 where $R_2 = g(R_1)$, h_j is inverse of g over a suitable subinterval $f_2(y) + \sum_{i=1}^n f_1(h_j(y))|h'_j(y)|$ under suitable conditions.

Reference for Unit 1, Sections 1.1-1.6, 2.1-2.5 of Basic Probability theory by Robert Ash, Dover Publication, 2008.

Unit II. Properties of Distribution function, Joint Density function (15L) Properties of distribution function F , F is non-decreasing, $\lim_{x \rightarrow \infty} F(x) = 1$, $\lim_{x \rightarrow -\infty} F(x) = 0$, Right continuity of F , $\lim_{x \rightarrow x_0} F(x) = P(\{R < x_0\})$, $P(\{R = x_0\}) = F(x_0) - F(x_0^-)$. Joint distribution, Joint Density, Results on Relationship between Joint and Individual densities, Related result for Independent random variables. Examples of distributions like Binomial, Poisson and Normal distribution. Expectation and k -th moments of a random variable with properties.

Reference for Unit II:

Sections 2.5-2.7, 2.9, 3.2-3.3, 3.6 of Basic Probability theory by Robert Ash, Dover Publication, 2008.

Unit III. Weak Law of Large Numbers

Joint Moments, Joint Central Moments, Schwarz Inequality, Bounds on Correlation Coefficient ρ , Result on ρ as a measure of linear dependence, $\text{Var}\left(\sum_{i=1}^n R_i\right) = \sum_{i=1}^n \text{Var}(R_i) + 2 \sum_{i=1}^n \sum_{i=1 \leq i < j \leq n} \text{Cov}(R_i, R_j)$, Method of Indicators to find expectation of a random variable, Chebyshev's Inequality, Weak

law of Large numbers.

Reference for Unit III

Sections 3.4, 3.5, 3.7, 4.1-4.4 of Basic Probability theory by Robert Ash, Dover Publication, 2008.

Additional Reference Books. Marek Capinski, Probability through Problems, Springer.

Course: Practicals (Based on USMT501 / UAMT501 and USMT502 / UAMT502)
Course Code: USMTP05 / UAMTP05

Suggested Practicals (Based on USMT501 / UAMT501)

1. Evaluation of double and triple integrals.
2. Change of variables in double and triple integrals and applications
3. Line integrals of scalar and vector fields
4. Greens theorem, conservative field and applications
5. Evaluation of surface integrals
6. Stokes and Gauss divergence theorem
7. Miscellaneous theory questions on units 1, 2 and 3.

Suggested Practicals (Based on USMT502 / UAMT502)

1. Quotient Spaces, Orthogonal Transformations.
2. Cayley Hamilton Theorem and Applications
3. Eigen Values & Eigen Vectors of a linear Transformation/ Square Matrices
4. Similar Matrices, Minimal Polynomial, Invariant Subspaces
5. Diagonalisation of a matrix
6. Orthogonal Diagonalisation and Quadratic Forms.
7. Miscellaneous Theory Questions

Course: Practicals (Based on USMT503 / UAMT503 and USMT5A4 / UAMT5A4 OR USMT5B4 / UAMT5B4 OR USMT5C4 / UAMT5C4 OR USMT5D4 / UAMT5D4)
Course Code: USMTP06 / UAMTP06

Suggested Practicals USMT503 / UAMT503:

1. Examples of Metric Spaces, Normed Linear Spaces,
2. Sketching of Open Balls in \mathbb{R}^2 , Open and Closed sets, Equivalent Metrics
3. Subspaces, Interior points, Limit Points, Dense Sets and Separability, Diameter of a set, Closure.

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4. Limit Points ,Sequences , Bounded , Convergent and Cauchy Sequences in a Metric Space
 5. Complete Metric Spaces and Applications
 6. Examples of Compact Sets
 7. Miscellaneous Theory Questions

Suggested Practicals on USMT5A4 / UAMT5A4

The Practicals should be performed using non-programmable scientific calculator. (The use of programming language like C or Mathematical Software like Mathematica, Matlab, MuPad, and Maple may be encouraged).

1. Newton-Raphson method, Secant method, Regula-Falsi method, Iteration Method
2. Muller method, Chebyshev method, Multipoint iteration method
3. Descarts rule of signs, Birge-Vieta method, Bairstrow method
4. Gauss elimination method, Forward and backward substitution method,
5. Triangularization methods-Doolittles and Crouts method, Choleskys method
6. Jacobi iteration method, Gauss-Siedal method
7. Eigen value problem: Jacobis method for symmetric matrices and Power method to determine largest eigenvalue and eigenvector

Suggested Practicals based on USMT5B4 / UAMT5B4

1. Congruences.
2. Linear congruences and congruences of Hilgher degree.
3. Linear diophantine equation.
4. Pythagorean triples and sum of squares.
5. Cryptosystems (Private Key).
6. Cryptosystems (Public Key) and primitive roots.
7. Miscellaneous theoretical questions based on full USMT5B4 / UAMT5B4.

Suggested Practicals based on USMT5C4 / UAMT5C4

1. Handshaking Lemma and Isomorphism.
2. Degree sequence and Dijkstra's algorithm
3. Trees, Cayley Formula
4. Applications of Trees
5. Eulerian Graphs.
6. Hamiltonian Graphs.

7. Miscellaneous Problems.

Suggested Practicals based on USMT5D4 / UAMT5D4

1. Basic concepts of Probability (Algebra of events, Probability space, Probability measure, combinatorial problems)
2. Conditional Probability, Random variable (Independence of events. Definition, Classification and function of a random variable)
3. Distribution function, Joint Density function
4. Expectation of a random variable, Normal distribution
5. Method of Indicators, Weak law of large numbers
6. Conditional density, Conditional expectation
7. Miscellaneous Theoretical questions based on full paper

**SEMESTER VI
BASIC COMPLEX ANALYSIS
Course Code: USMT501/UAMT501**

Unit I: Introduction to Complex Analysis (15 Lectures)

Review of complex numbers: Complex plane, polar coordinates, exponential map, powers and roots of complex numbers, De Moivre's formula, \mathbb{C} as a metric space, bounded and unbounded sets, point at infinity-extended complex plane, sketching of set in complex plane (No questions to be asked).

Limit at a point, theorems on limits, convergence of sequences of complex numbers and results using properties of real sequences. Functions $f : \mathbb{C} \rightarrow \mathbb{C}$, real and imaginary part of functions, continuity at a point and algebra of continuous functions. Derivative of $f : \mathbb{C} \rightarrow \mathbb{C}$, comparison between differentiability in real and complex sense, Cauchy-Riemann equations, sufficient conditions for differentiability, analytic function, f, g analytic then $f + g, f - g, fg$ and f/g are analytic, chain rule.

Theorem: If $f(z) = 0$ everywhere in a domain D , then $f(z)$ must be constant throughout D
Harmonic functions and harmonic conjugate.

Unit II: Cauchy Integral Formula (15 Lectures)

Explain how to evaluate the line integral $\int f(z) dz$ over $|z - z_0| = r$ and prove the Cauchy integral

formula : If f is analytic in $B(z_0, r)$ then for any w in $B(z_0, r)$ we have $f(w) = \frac{1}{2\pi i} \int \frac{f(z)}{z - w} dz$,
over $|z - z_0| = r$.

Taylor's theorem for analytic function, Möbius transformations: definition and examples
Exponential function, its properties, trigonometric function, hyperbolic functions.

Unit III: Complex power series, Laurent series and isolated singularities. (15 Lectures)

Power series of complex numbers and related results following from Unit I, radius of convergence, disc of convergence, uniqueness of series representation, examples.

Definition of Laurent series , Definition of isolated singularity, statement (without proof) of existence of Laurent series expansion in neighbourhood of an isolated singularity, type of isolated singularities viz. removable, pole and essential defined using Laurent series expansion, examples Statement of Residue theorem and calculation of residue.

Reference:

1. J.W. Brown and R.V. Churchill, Complex analysis and Applications : Sections 18, 19, 20, 21, 23, 24, 25, 28, 33, 34, 47, 48, 53, 54, 55 , Chapter 5, page 231 section 65, define residue of a function at a pole using Theorem in section 66 page 234, Statement of Cauchys residue theorem on page 225, section 71 and 72 from chapter 7.

Other References:

1. Robert E. Greene and Steven G. Krantz, Function theory of one complex variable
2. T.W. Gamelin, Complex analysis

Course: Algebra

Course Code: USMT602 / UAMT602

Unit I. Group Theory (15L)

Review of Groups, Subgroups, Abelian groups, Order of a group, Finite and infinite groups, Cyclic groups, The Center $Z(G)$ of a group G , Cosets, Lagranges theorem, Group homomorphisms, isomorphisms, automorphisms, inner automorphisms (No questions to be asked)

Normal subgroups: Normal subgroups of a group, definition and examples including center of a group, Quotient group, Alternating group A_n , Cycles. Listing normal subgroups of A_4, S_3 . First Isomorphism theorem (or Fundamental Theorem of homomorphisms of groups), Second Isomorphism theorem, third Isomorphism theorem, Cayleys theorem, External direct product of a group, Properties of external direct products, Order of an element in a direct product, criterion for direct product to be cyclic, Classification of groups of order ≤ 7 .

Unit II. Ring Theory (15L)

Motivation: Integers & Polynomials.

Definitions of a ring (The definition should include the existence of a unity element), zero divisor, unit, the multiplicative group of units of a ring. Basic Properties & examples of rings, including $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}, Mn(\mathbb{R}), \mathbb{Q}[X], \mathbb{R}[X], \mathbb{C}[X], \mathbb{Z}[i], \mathbb{Z}[\sqrt{2}], \mathbb{Z}[\sqrt{-5}], \mathbb{Z}_n$.

Definitions of Commutative ring, integral domain (ID), Division ring, examples. Theorem such as: A commutative ring R is an integral domain if and only if for $a, b, c \in R$ with $a \neq 0$ the relation $ab = ac$ implies that $b = c$. Definitions of Subring, examples. Ring homomorphisms, Properties of ring homomorphisms, Kernel of ring homomorphism, Ideals, Operations on ideals and Quotient rings, examples. Factor theorem and First and second Isomorphism theorems for rings, Correspondence Theorem for rings: (If $f : R \rightarrow R'$ is a surjective ring homomorphism, then there is a 1 – 1 correspondence between the ideals of R containing the $\ker f$ and the ideals of R . Definitions of characteristic of a ring, Characteristic of an ID.

Unit III. Polynomial Rings and Field theory (15L)

Principal ideal, maximal ideal, prime ideal, the characterization of the prime and maximal ideals

in terms of quotient rings. Polynomial rings, $R[X]$ when R is an integral domain/ Field. Divisibility in Integral Domain, Definitions of associates, irreducible and primes. Prime (irreducible) elements in $\mathbb{R}[X], \mathbb{Q}[X], \mathbb{Z}_p[X]$. Eisensteins criterion for irreducibility of a polynomial over \mathbb{Z} . Prime and maximal ideals in polynomial rings. Definition of field, subfield and examples, characteristic of fields. Any field is an ID and a finite ID is a field. Characterization of fields in terms of maximal ideals, irreducible polynomials. Construction of quotient field of an integral domain (Emphasis on \mathbb{Z}, \mathbb{Q}). A field contains a subfield isomorphic to \mathbb{Z}_p or \mathbb{Q} .

Recommended Books

1. P. B. Bhattacharya, S. K. Jain, and S. R. Nagpaul, Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995.
2. N. S. Gopalakrishnan, University Algebra, Wiley Eastern Limited.
3. N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.
4. M. Artin, Algebra, Prentice Hall of India, New Delhi.
5. J. B. Fraleigh, A First course in Abstract Algebra, Third edition, Narosa, New Delhi.
6. J. Gallian, Contemporary Abstract Algebra, Narosa, New Delhi.

Additional Reference Books:

1. S. Adhikari, An Introduction to Commutative Algebra and Number theory, Narosa Publishing House.
2. T.W. Hungerford. Algebra, Springer.
3. D. Dummit, R. Foote. Abstract Algebra, John Wiley & Sons, Inc.
4. I.S. Luthar, I.B.S. Passi. Algebra, Vol. I and II.
5. U. M. Swamy, A. V. S. N. Murthy Algebra Abstract and Modern, Pearson.
6. Charles Lanski, Concepts Abstract Algebra, American Mathematical Society
7. Sen, Ghosh and Mukhopadhyay, Topics in Abstract Algebra, Universities press

Course: Topology of Metric Spaces and Real Analysis Course Code: USMT603/ UAMT603

Unit I: Continuous functions on metric spaces (15 L) Epsilon-delta definition of continuity at a point of a function from one metric space to another. Characterization of continuity at a point in terms of sequences, open sets and closed sets and examples, Algebra of continuous real valued functions on a metric space. Continuity of composite continuous function. Continuous image of compact set is compact, Uniform continuity in a metric space, definition and examples (emphasis on \mathbb{R}). Let (X, d) and (Y, d) be metric spaces and $f : X \rightarrow Y$ be continuous. If (X, d) is compact metric, then $f : X \rightarrow Y$ is uniformly continuous. Contraction mapping and fixed point theorem, Applications.

Unit II: Connected sets: (15L)

Separated sets- Definition and examples, disconnected sets, disconnected and connected metric spaces, Connected subsets of a metric space, Connected subsets of \mathbb{R} . A subset of \mathbb{R} is connected if and only if it is an interval. A continuous image of a connected set is connected. Characterization of a connected space, viz. a metric space is connected if and only if every continuous function from X to $\{1, -1\}$ is a constant function. Path connectedness in \mathbb{R}^n , definition and examples. A path connected subset of \mathbb{R}^n is connected, convex sets are path connected. Connected components. An example of a connected subset of \mathbb{R}^n which is not path connected.

Unit III : Sequence and series of functions:(15 lectures)

Sequence of functions - pointwise and uniform convergence of sequences of real-valued functions, examples. Uniform convergence implies pointwise convergence, example to show converse not true, series of functions, convergence of series of functions, Weierstrass M-test. Examples. Properties of uniform convergence: Continuity of the uniform limit of a sequence of continuous function, conditions under which integral and the derivative of sequence of functions converge to the integral and derivative of uniform limit on a closed and bounded interval. Examples. Consequences of these properties for series of functions, term by term differentiation and integration. Power series in \mathbb{R} centered at origin and at some point in \mathbb{R} , radius of convergence, region (interval) of convergence, uniform convergence, term by-term differentiation and integration of power series, Examples. Uniqueness of series representation, functions represented by power series, classical functions defined by power series such as exponential, cosine and sine functions, the basic properties of these functions.

References for Units I, II, III:

1. S. Kumaresan, Topology of Metric spaces.
2. E. T. Copson. Metric Spaces. Universal Book Stall, New Delhi, 1996.
3. Robert Bartle and Donald R. Sherbert, Introduction to Real Analysis, Second Edition, John Wiley and Sons.
4. Ajit Kumar, S. Kumaresan, Introduction to Real Analysis
5. R.R. Goldberg, Methods of Real Analysis, Oxford and International Book House (IBH) Publishers, New Delhi.

Other references :

1. W. Rudin, Principles of Mathematical Analysis.
2. T. Apostol. Mathematical Analysis, Second edition, Narosa, New Delhi, 1974
3. E. T. Copson. Metric Spaces. Universal Book Stall, New Delhi, 1996.
4. R. R. Goldberg Methods of Real Analysis, Oxford and IBH Pub. Co., New Delhi 1970.
5. P.K.Jain. K. Ahmed. Metric Spaces. Narosa, New Delhi, 1996.
6. W. Rudin. Principles of Mathematical Analysis, Third Ed, McGraw-Hill, Auckland, 1976.
7. D. Somasundaram, B. Choudhary. A first Course in Mathematical Analysis. Narosa, New Delhi

8. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hi, New York, 1963.
9. Sutherland. Topology.

Course: Numerical Analysis II (Elective A)
Course Code: USMT6A4 / UAMT6A4

N.B. Derivations and geometrical interpretation of all numerical methods with theorem mentioned have to be covered.

Unit I. Interpolation (15L)

Interpolating polynomials, Uniqueness of interpolating polynomials. Linear, Quadratic and Higher order interpolation. Lagranges Interpolation. Finite difference operators: Shift operator, forward, backward and central difference operator, Average operator and relation between them. Difference table, Relation between difference and derivatives. Interpolating polynomials using finite differences Gregory-Newton forward difference interpolation, Gregory-Newton backward difference interpolation, Stirlings Interpolation. Results on interpolation error.

Unit II. Polynomial Approximations and Numerical Differentiation (15L)

Piecewise Interpolation: Linear, Quadratic and Cubic. Bivariate Interpolation: Lagranges Bivariate Interpolation, Newtons Bivariate Interpolation. Numerical differentiation: Numerical differentiation based on Interpolation, Numerical differentiation based on finite differences (forward, backward and central), Numerical Partial differentiation.

Unit III. Numerical Integration (15L)

Numerical Integration based on Interpolation. Newton-Cotes Methods, Trapezoidal rule, Simpson's 1/3rd rule, Simpson's 3/8th rule. Determination of error term for all above methods. Convergence of numerical integration: Necessary and sufficient condition (with proof). Composite integration methods; Trapezoidal rule, Simpson's rule.

Reference Books

1. Kendall E, Atkinson, An Introduction to Numerical Analysis, Wiley.
2. M. K. Jain, S. R. K. Iyengar and R. K. Jain,, Numerical Methods for Scientific and Engineering Computation, New Age International Publications.
3. S.D. Comte and Carl de Boor, Elementary Numerical Analysis, An algorithmic approach, McGrawHill International Book Company.
4. S. Sastry, Introductory methods of Numerical Analysis, PHI Learning.
5. Hildebrand F.B, .Introduction to Numerical Analysis, Dover Publication, NY.
6. Scarborough James B., Numerical Mathematical Analysis, Oxford University Press, New Delhi.

Course: Number Theory and its applications II (Elective B)
Course Code: USMT6B4 / UAMT6B4

Unit I. Quadratic Reciprocity (15 L)

Quadratic residues and Legendre Symbol, Gauss Lemma, Theorem on Legendre Symbol $\left(\frac{2}{p}\right)$, the result: If p is an odd prime and a is an odd integer with $(a, p) = 1$ then

$\left(\frac{a}{p}\right) = (-1)^t$ where $t = \sum_{k=1}^{\frac{p-1}{2}} \left[\frac{ka}{p}\right]$, Quadratic Reciprocity law. Theorem on Legendre Symbol $\left(\frac{3}{p}\right)$. The Jacobi Symbol and law of reciprocity for Jacobi Symbol. Quadratic Congruences with Composite moduli.

Unit II. Continued Fractions (15 L)

Finite continued fractions. Infinite continued fractions and representation of an irrational number by an infinite simple continued fraction, Rational approximations to irrational numbers and order of convergence, Best possible approximations. Periodic continued fractions.

Unit III. Pell's equation, Arithmetic function and Special numbers (15 L)

Pell's equation $x^2 - dy^2 = n$, where d is not a square of an integer. Solutions of Pell's equation (The proofs of convergence theorems to be omitted). Arithmetic functions of number theory: $d(n)$ (or $\tau(n)$), $\sigma(n)$, $\sigma_k(n)$, $\omega(n)$ and their properties, $\mu(n)$ and the Mbius inversion formula. Special numbers: Fermat numbers, Mersenne numbers, Perfect numbers, Amicable numbers, Pseudo primes, Carmichael numbers.

Recommended Books

1. Niven, H. Zuckerman and H. Montgomery. An Introduction to the Theory of Numbers. John Wiley & Sons. Inc.
2. David M. Burton. An Introduction to the Theory of Numbers. Tata McGraw-Hill Edition.
3. G. H. Hardy and E.M. Wright. An Introduction to the Theory of Numbers. Low priced edition. The English Language Book Society and Oxford University Press, 1981.
4. Neville Robins. Beginning Number Theory. Narosa Publications.
5. S. D. Adhikari. An introduction to Commutative Algebra and Number Theory. Narosa Publishing House
6. .N. Koblitz. A course in Number theory and Cryptography. Springer.
7. M. Artin. Algebra. Prentice Hall.
8. K. Ireland, M. Rosen. A classical introduction to Modern Number Theory. Second edition, Springer Verlag.
9. William Stalling. Cryptology and network security.

Course: Graph Theory and Combinatorics (Elective C)
Course Code: USMT6C4 /UAMT6C4

Unit I. Colorings of graph (15L)

Vertex coloring- evaluation of vertex chromatic number of some standard graphs, critical graph. Upper and lower bounds of Vertex chromatic Number- Statement of Brooks theorem. Edge coloring- Evaluation of edge chromatic number of standard graphs such as complete graph, complete bipartite graph, cycle. Statement of Vizing Theorem. Chromatic polynomial of graphs- Recurrence Relation and properties of Chromatic polynomials. Vertex and Edge cuts vertex and edge connectivity and the relation between vertex and edge connectivity. Equality of vertex and edge connectivity of cubic graphs. Whitney's theorem on 2-vertex connected graphs.

Unit II. Planar graph (15L)

Definition of planar graph. Euler formula and its consequences. Non planarity of K_5 ; $K(3; 3)$. Dual of a graph. Polyhedron in \mathbb{R}^3 and existence of exactly five regular polyhedra- (Platonic solids) Colorability of planar graphs- 5 color theorem for planar graphs, statement of 4 color theorem. Networks and flow and cut in a network- value of a flow and the capacity of cut in a network, relation between flow and cut. Maximal flow and minimal cut in a network and Ford-Fulkerson theorem.

Unit III. Combinatorics (15L)

Applications of Inclusion Exclusion Principle- Rook polynomial, Forbidden position problems Introduction to partial fractions and using Newton's binomial theorem for real power series, expansion of some standard functions. Forming recurrence relation and getting a generating function. Solving a recurrence relation using ordinary generating functions. System of Distinct Representatives and Hall's theorem of SDR. Introduction to matching, M alternating and M augmenting path, Berge theorem. Bipartite graphs.

Recommended Books.

1. Bondy and Murty Graph, Theory with Applications.
2. Balkrishnan and Ranganathan, Graph theory and applications. 3 West D G. , Graph theory.
3. Richard Brualdi, Introduction to Combinatorics.

Additional Reference Book.

1. Behzad and Chartrand Graph theory.
2. Choudam S. A., Introductory Graph theory. 3 Cohen, Combinatorics.

Course: Operations Research Elective D)
Course Code: USMT6D4 / UAMT6D4

Unit I. Linear Programming-I (15L)

Prerequisites: Vector Space, Linear independence and dependence, Basis, Convex sets, Dimension of polyhedron, Faces.

Formation of LPP, Graphical Method. Theory of the Simplex Method- Standard form of LPP, Feasible solution to basic feasible solution, Improving BFS, Optimality Condition, Unbounded solution, Alternative optima, Correspondence between BFS and extreme points. Simplex Method Simplex Algorithm, Simplex Tableau.

Reference for unit I

1. G. Hadley, Linear Programming, Narosa Publishing, (Chapter 3).

Unit II. Linear programming-II (15L)

Simplex Method Case of Degeneracy, Big-M Method, Infeasible solution, Alternate solution, Solution of LPP for unrestricted variable. Transportation Problem: Formation of TP, Concepts of solution, feasible solution, Finding Initial Basic Feasible Solution by North West Corner Method, Matrix Minima Method, Vogels Approximation Method. Optimal Solution by MODI method, Unbalanced and maximization type of TP.

Reference for Unit II

1. G. Hadley, Linear Programming, Narosa Publishing, (Chapter 4 and 9).
2. J. K. Sharma, Operations Research, Theory and Applications, (Chapter 4, 9).

Unit III. Queuing Systems (15L)

Elements of Queuing Model, Role of Exponential Distribution. Pure Birth and Death Models; Generalized Poisson Queuing Mode. Specialized Poisson Queues: Steady- state Measures of Performance, Single Server Models, Multiple Server Models, Self- service Model, Machine-servicing Model.

Reference for Unit III:

1. J. K. Sharma, Operations Research, Theory and Applications.
2. H. A. Taha, Operations Research, Prentice Hall of India.

Additional Reference Books:

1. Hillier and Lieberman, Introduction to Operations Research.
2. Richard Broson, Schaum Series Book in Operations Research, Tata McGrawHill Publishing Company Ltd.

Course: Practicals (Based on USMT601 / UAMT601 and USMT602 / UAMT602)

Course Code: USMTP07 / UAMTP07

Suggested Practicals (Based on USMT601 / UAMT601):

1. Limit continuity and derivatives of functions of complex variables,
2. Steriographic Projection , Analytic function, finding harmonic conjugate,
3. Contour Integral, Cauchy Integral Formula ,Mobius transformations
4. Taylors Theorem , Exponential , Trigonometric, Hyperbolic functions
5. Power Series , Radius of Convergence, Laurents Series

6. Finding isolated singularities- removable, pole and essential, Cauchy Residue theorem.
7. Miscellaneous theory questions.

Suggested Practicals (Based on USMT602 / UAMT602)

1. Normal Subgroups and quotient groups.
2. Cayleys Theorem and external direct product of groups.
3. Rings, Subrings, Ideals, Ring Homomorphism and Isomorphism
4. Prime Ideals and Maximal Ideals
5. Polynomial Rings
6. Fields.
7. Miscellaneous Theoretical questions on Unit 1, 2 and 3.

Course: Practicals (Based on USMT603 / UAMT603 and USMT6A4 / UAMT6A4 OR USMT6B4 / UAMT6B4 OR USMT6C4 / UAMT6C4 OR USMT6D4 / UAMT6D4)
Course Code: USMTP08 / UAMTP08

Suggested practicals Based on USMT603 / UAMT603:

- 1 Continuity in a Metric Spaces
- 2 Uniform Continuity, Contraction maps, Fixed point theorem
- 3 Connected Sets , Connected Metric Spaces
- 4 Path Connectedness, Convex sets, Continuity and Connectedness
- 5 Pointwise and uniform convergence of sequence functions, properties
- 6 Point wise and uniform convergence of series of functions and properties
- 7 Miscellaneous Theory Questions

Suggested Practicals based on USMT6A4 / UAMT6A4

The Practicals should be performed using non-programmable scientific calculator. (The use of programming language like C or Mathematical Software like Mathematica, Matlab, MuPad, and Maple may be encouraged).

- 1 Linear, Quadratic and Higher order interpolation, Interpolating polynomial by Lagranges Interpolation
- 2 Interpolating polynomial by Gregory-Newton forward and backward difference Interpolation and Stirling Interpolation.
- 3 Bivariate Interpolation: Lagranges Interpolation and Newtons Interpolation
- 4 Numerical differentiation: Finite differences (forward, backward and central), Numerical Partial differentiation

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- 5 Numerical differentiation and Integration based on Interpolation
 - 6 Numerical Integration: Trapezoidal rule, Simpsons 1/3rd rule, Simpsons 3/8th rule
 - 7 Composite integration methods: Trapezoidal rule, Simpsons rule.

Suggested Practicals based on USMT6B4 / UAMT6B4

1. Legendre Symbol.
2. Jacobi Symbol and Quadratic congruences with composite moduli.
3. Finite continued fractions.
4. Infinite continued fractions.
5. Pells equations and Arithmetic functions of number theory.
6. Special Numbers.
7. Miscellaneous Theoretical questions based on full USMT6B4 / UAMT6B4.

Suggested Practicals based on USMT6C4 / UAMT6C4

1. Coloring of Graphs
2. Chromatic polynomials and connectivity.
3. Planar graphs
4. Flow theory.
5. Inclusion Exclusion Principle and Recurrence relation.
6. SDR and Mathching.
7. Miscellaneous theoretical questions.

Suggested Practicals based on USMT6D4 / UAMT6D4

All practicals to be done manually as well as using software TORA / EXCEL solver.

1. LPP formation, graphical method and simple problems on theory of simplex method
2. LPP Simplex Method
3. Big-M method, special cases of solutions.
4. Transportation Problem
5. Queuing Theory; single server models
6. Queuing Theory; multiple server models
7. Miscellaneous Theory Questions.

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