Unit 2 — Index Numbers (Price / quantity)

	Price	quantity
Base year → lower/previous	P_0	q_0
Current year → New	P_1	q_1

Index No

- Simple Index
  - Simple Aggregative Index: $\frac{\Sigma P_1}{\Sigma P_0} \times 100$
  - Simple Avg of price relative Index (I): $= \frac{\Sigma I}{N}$, $I = \frac{P_1}{P_0} \times 100$, N → No of commodities
- Weighted
  - Weighted Aggregative Index: $\frac{\Sigma P_1 W}{\Sigma P_0 W} \times 100$
  - Weighted Avg of price relative: $\frac{\Sigma (IW)}{\Sigma W}$

Q1 For the given data find Simple Agg. Index, Simple avg of price Relative Index, Weighted Agg. Index, Weighted avg of price Relative Index.

Commodities	Price P_0 (2010)	Price P_1 (2020)	Weights (W)	I	IW	P_1W	P_0W
Rice	60	90	6	150	900	540	360
Pulses	45	80	9	177.77	1600.02	720	405
Milk	30	70	3	233.33	700.02	210	90
Fuel	40	80	4	200	800	320	160
Electricity	10	18	6	180	1080	108	60
Misc.	40	15	2	37.5	75	30	80
	225	353					

Laspeyre's Index (I_L) $= \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times 100$

Paasche's Index (I_P) $= \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \times 100$

Fisher's Index (I_F) $= \sqrt{\left(\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}\right)} \times 100$

Dorbish - Bowley's Index (I_{DB}) $= \left(\frac{\frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} + \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1}}{2}\right) \times 100$

Marshall - Edgeworth (I_{ME}) $= \left(\frac{\Sigma P_1 q_0 + \Sigma P_1 q_1}{\Sigma P_0 q_0 + \Sigma P_0 q_1}\right) \times 100$

where I_F is geometric mean of I_L & I_P

I_{DB} is arithmetic mean of I_L & I_P.

Common.

	2005		2010					
	P_0	q_0	P_1	q_1	$P_1 q_0$	$P_0 q_0$	$P_1 q_1$	$P_0 q_1$
A	40	6	60	8	360	240	480	320
B	70	10	105	15	1050	700	1575	1050
C	30	4	48	10	192	120	480	300
D	50	7	60	6	420	350	360	300
E	70	6	42	10	252	420	420	700
					2274	1830	3315	2670

$I_L = \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times 100 = \frac{2274}{1830} \times 100 = 124.26$

$I_P = \frac{\Sigma P_1 q_1}{\Sigma P_0 q_1} \times 100 = \frac{3315}{2670} \times 100 = 124.15$

$I_F = \sqrt{I_L \times I_P} = \sqrt{124.26 \times 124.15} = 124.20$

$I_{DB} = \frac{I_L + I_P}{2} = \frac{124.26 + 124.15}{2} = 124.205$

$I_{ME} = \frac{\Sigma P_1 q_0 + \Sigma P_1 q_1}{\Sigma P_0 q_0 + \Sigma P_0 q_1} \times 100 = \frac{2274 + 3315}{1830 + 2670} \times 100$

$= \frac{5588}{4500} \times 100$

$= 124.22$

Comm.	Price 2005 p_0	Price 2008 p_1	q/y 2005 q_0	q/y 2008 q_1	p_1q_0	p_0q_0	p_1q_1	p_0q_1
A	40	70	6	8	420	240	560	320
B	70	105	15	10	1575	1050	1050	700
C	30	42	8	10	336	240	420	300
D	50	100	4	6	400	200	600	300
E	70	105	10	15	1050	700	1575	1050
					3781	2430	4205	2670

$$IL = \frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times 100 = \frac{3781}{2430} \times 100 = 155.59$$

$$IP = \frac{\Sigma p_1q_1}{\Sigma p_0q_1} \times 100 = \frac{4205}{2670} \times 100 = 157.49$$

$$IF = \sqrt{IL \times IP} = \sqrt{155.59 \times 157.49} = 156.53$$

$$IDB = \frac{IL + IP}{2} = \frac{155.59 + 157.49}{2} = 156.54$$

$$IME = \frac{\Sigma p_1q_0 + \Sigma p_1q_1}{\Sigma p_0q_0 + \Sigma p_0q_1} \times 100 = \frac{3781 + 4205}{2430 + 2670} \times 100 = 156.53$$

$$= \frac{7986}{5100} \times 100$$

Commodities	2010 p_0	2010 q_0	2018 p_1	2018 q_1	p_1q_0	p_0q_0	p_0q_1	p_1q_1
A	38	4	46	6	184	152	276	228
B	40	3	35	2	105	120	70	80
C	46	5	55	4	275	230	220	184
D	20	6	30	7	180	120	210	140
E	15	8	10	10	80	120	100	150
					824	742	876	782

$$IL = \frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times 100 = \frac{824}{742} \times 100 = 111.05$$

$$IP = \frac{\Sigma p_1q_1}{\Sigma p_0q_1} \times 100 = \frac{876}{782} \times 100 = 112.02$$

$$IF = \sqrt{IL \times IP} = \sqrt{111.05 \times 112.02} = 111.53$$

$$IDB = \frac{IL + IP}{2} = \frac{111.05 + 112.02}{2} = 111.535$$

$$IME = \frac{\Sigma p_1q_0 + \Sigma p_1q_1}{\Sigma p_0q_0 + \Sigma p_0q_1} = \frac{824 + 876}{742 + 782} \times 100 =$$

$$= \frac{1700}{1524} \times 100 = 111.548$$

11

Base shifting:-

Year	Index	New Index
2002	104	$\frac{104}{120} \times 100 = 86.67$
2003	110	$\frac{110}{120} \times 100 = 91.67$
2004	106	$\frac{106}{120} \times 100 = 88.33$
2005	118	$\frac{118}{120} \times 100 = 98.33$
2006	116	$\frac{116}{120} \times 100 = 96.67$
2007	120	$\frac{120}{120} \times 100 = 100$
2008	114	$\frac{114}{130} \times 100 = 95$
2009	130	$\frac{130}{120} \times 100 = 108.33$

Reconstruct the new index taking year 2007 as the new base year.

$$\text{New Index} = \frac{\text{Index of current year}}{\text{Index of new base year}} \times 100$$

Cost of living index:

a) Family budget method

$$= \frac{\Sigma IW}{\Sigma W}$$

where $I = \frac{P_1}{P_0} \times 100$

b) Aggregate Expenditure method:

$$= \frac{\Sigma P_1 q_0}{\Sigma P_0 q_0} \times 100 \quad (L_p)$$

Com	P_0	P_1	W	$I = \frac{P_1}{P_0} \times 100$	IW
A	50	60	10	120	1200
B	70	50	18	71.42	1285.56
C	40	75	16	187.5	3000
D	35	50	12	142.85	1714.2
E	45	30	14	66.66	933.24
F	70	105	20	150	3000
			90		11133

$$\frac{\Sigma IW}{\Sigma W} = \frac{11133}{90}$$

$$\approx 123.70$$

Cost of living

Agg. Expenditure Method

comm.	price P_0 2004	price P_1 2014	quantity Q_0 2004	quantity 2014	P_0Q_0	P_1Q_0
A	30	45	6		180	270
B	45	70	5		225	350
C	60	90	10		600	900
D	70	40	4		280	160
E	50	30	15		750	450
					2035	2130

$$CLI = \frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times 100$$

$$= \frac{2130}{2035} \times 100$$

$$= 104.668$$

Real income

working required

Year	Income	price Index	Real Income
2004	18000	103	17475.718
05	18800	112	16785.714
06	20200	114	17719.298
07	23000	110	20909.090
08	25500	118	21610.169
09	28700	122	23524.590
10	34000	140	24285.714

Real $= \frac{\text{Income}}{\text{Index}} \times 100$

Chain base Index

year	Price	Relative Index
08	40	$\frac{40}{40} \times 100 = 100$
09	70	$\frac{70}{40} \times 100 = 175$
10	60	$\frac{60}{70} \times 100 = 85.71$
11	80	$\frac{80}{60} \times 100 = 133.33$
12	90	$\frac{90}{80} \times 100 = 112.5$
13	110	$\frac{110}{90} \times 100 = 122.22$
14	130	$\frac{130}{110} \times 100 = 118.18$

Year	price	link relative	chain base index.
2012	90	$\frac{90}{90} \times 100 = 100$	100
13	88	$\frac{88}{90} \times 100 = 97.77$	$\frac{97.77 \times 100}{100} = 97.77$
14	95	$\frac{95}{88} \times 100 = 107.95$	$\frac{107.95 \times 97.77}{100} = 107.95$
15	102	$\frac{102}{95} \times 100 = 107.36$	$\frac{107.36 \times 105.55}{100} = 107.36$
16	100	$\frac{100}{102} \times 100 = 98.03$	$\frac{98.03 \times 113.32}{100} = 98.03$
17	107	$\frac{107}{100} \times 100 = 107$	$\frac{107 \times 111.06}{100} = 107$
18	110	$\frac{110}{107} \times 100 = 102.8$	$\frac{102.8 \times 118.83}{100} = 102.8$

~~Ans~~ where link relative is

$$\frac{\text{current year}}{\text{prev. year}} \times 100.$$

$$\text{Chain base index} = \frac{\text{link rel. CY} \times \text{Chain Base in Prev. Year}}{100}.$$

Year	Price	link relative	Chain base Index
2001	120	$\frac{120}{120} \times 100 = 100$	100
02	115	$\frac{115}{120} \times 100 = 95.83$	$\frac{95.85 \times 100}{100} = 95.83$
03	117	$\frac{117}{115} \times 100 = 101.73$	$\frac{101.73 \times 95.83}{100} = 97.48$
04	121	$\frac{121}{117} \times 100 = 103.41$	$\frac{103.41 \times 97.48}{100} = 100.8$
05	120	$\frac{120}{121} \times 100 = 99.17$	$\frac{99.17 \times 100.8}{100} = 99.96$
06	123	$\frac{123}{120} \times 100 = 102.5$	$\frac{102.5 \times 99.96}{100} = 102.45$
07	122	$\frac{122}{123} \times 100 = 99.18$	$\frac{99.18 \times 102.45}{100} = 101.6$
08	124	$\frac{124}{122} \times 100 = 101.63$	$\frac{101.63 \times 101.6}{100} = 103.25$

where link relative $= \frac{\text{current year}}{\text{prev. year}} \times 100$

$$\text{Chain base index} = \frac{\text{link rel. CY} \times \text{Chain base in Prev.}}{100}$$

Probability

In case of coin toss (1 coin)
$S = \{H, T\}$
Sample space
$n(S) = 2$

In case of coin toss (2 coins)
$S = \{HH, HT, TH, TT\}$
$n(S) = 4$

In case of 3 coins
$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$n(S) = 8$

When 2 dice are being thrown

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)$
$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)$
$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)$
$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)$
$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

$n(S) = 36$

& = *i multip^n
or = + rion.

Cards

52

- Red (26)
  - Hearts (13)
  - Diamonds (13)
- Black (26)
  - Spade (13)
  - (13) Clubs

Face cards (picture): King, Queen, Jack
Number cards: Ace, 2 to 10

AB, C
AB, BC, AC,
0 ⇒ 1
1 - A or B or C (3)
2 = AB or BC or AC (3)
3 = ABC (1)

A, B, C, D, E
0 ⇒ 1
1 → 5
2 → AB, BC, CD, DE
AC, AD, AE,
BD, BE,
CE. (10)
3 → ABC, ACD, ADE
BCD, BDE, BAD, BAE
CDE. (5)

1) A fair coin is tossed two times. Find the probability of getting :-
   1) No heads
   2) Atleast 1 head 0H 1H 2H
   3) Atmost 1 head 0H 1H 2H
   4) No tails

2) 3 coins are tossed simultaneously. Find the prob of getting :-
   1) Atleast 1 head
   2) Atleast 2 heads
   3) Atmost 1 head
   4) Atmost 2 heads
   5) Head only in the 2nd throw.

3) From a pack of cards numbered from 1 to 25, a card is drawn at random. Find the probability of getting :-
   1) Even numbered card
   2) Odd numbered card
   3) Squared numbered card
   4) Prime numbered card
   5) Cube numbered card
   6) Multiple of 3
   7) " of 2 or 3
   8) " " 2 & 3

1) $P(E) = \frac{n(E)}{n(S)}$

1) $P(\text{NoH}) = \frac{1}{4}$ (TT)

2) $P(\text{atleast 1H}) = \frac{3}{4}$ (1HT, TH, HH)

3) $P(\text{atmost 1H}) = \frac{3}{4}$ (1HT, TH, TT)

4) $P(\text{No tails}) = \frac{1}{4}$ (1H)

Rules of Derivatives.

1) $y = u + v$
$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

2) $y = u - v$
$\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$

3) $y = Ku$ K = constant.
$\frac{dy}{dx} = K\frac{du}{dx}$

4) $y = u \cdot v$
$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$ ☆

5) $y = \frac{u}{v}$ where $v \neq 0$
$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
(Quotient rule.

u – 1st fn
v – 2nd fn

Q1) $y = a^x + x^a + e^x$

$$\frac{dy}{dx} = \frac{d}{dx}(a^x + x^a + e^x)$$

$$= \frac{d\,a^x}{dx} + \frac{d}{dx}(x^a) + \frac{d}{dx}(e^x)$$

$$= a^x \log a \; a x^{a-1} + e^x$$

Q2) $y = 4x^7 - \log x + \sqrt{x}$

$$\frac{dy}{dx} = \frac{d}{dx}(4x^7 - \log x + \sqrt{x})$$

$$= \frac{d}{dx}(4x^7) - \frac{d}{dx}(\log x) + \frac{d}{dx}(\sqrt{x})$$

$$= 4 \times 7x^{7-1} - \frac{1}{x} + \frac{1}{2\sqrt{x}}$$

$$= 28x^6 - \frac{1}{x} + \frac{1}{2\sqrt{x}}$$

Q3) $y = (3x+4)\sqrt{x}$

$$\frac{dy}{dx} = \frac{d}{dx}[(3x+4)\sqrt{x}]$$

If $y = u.v$, then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$$\frac{dy}{dx} = (3x+4)\frac{d\sqrt{x}}{dx} + \sqrt{x}\,\frac{d(3x+4)}{dx}$$

$$= (3x+4)\left(\frac{1}{2\sqrt{x}}\right) + \sqrt{x}\left[\frac{d}{dx}(3x) + \frac{d}{dx}(4)\right]$$

$$= \frac{3x+4}{2\sqrt{x}} + \sqrt{x}\,[3+0]$$

$$= \frac{3x+4}{2\sqrt{x}} + 3\sqrt{x}$$

Q) $y = \frac{3x+5}{5x+3}$

If $y = \frac{u}{v}$, then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{3x+5}{5x+3}\right]$$

$$= \frac{(5x+3)\frac{d(3x+5)}{dx} - (3x+5)\frac{d(5x+3)}{dx}}{(5x+3)^2}$$

$$= \frac{(5x+3)(3) - (3x+5)(5)}{25x^2 + 30x + 9}$$

$$= \frac{15x + 9 - 15x - 25}{25x^2 + 30x + 9}$$

$$= \frac{-16}{25x^2 + 30x + 9}$$

Combination n_{C_r}

n_{P_r}

$$n_{C_r} = \frac{n!}{(n-r)! \times r!}$$

r will always be $\leq n$.

$5! = 5 \times 4 \times 3 \times 2 \times 1$

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$7! = 7!$ or $7 \times 6!$ or $7 \times 6 \times 5!$

If 52 cards se 2 nikalna hai,

52_{P_2}

or $^{52}C_2 = \frac{52!}{(52-2)! \times 2!}$

$= \frac{52!}{50! \times 2!}$

$= \frac{52 \times 51 \times 50!}{50! \times 2 \times 1}$

$= 1326$

$^{15}C_{11} = \frac{15!}{(15-11)! \times 11!}$

$= \frac{15!}{4! \times 11!}$

$= \frac{15 \times 14 \times 13 \times 12 \times 11!}{4 \times 3 \times 2 \times 1 \times 11!}$

$= 1365$ ways.

A bag contains 6 white, 5 black & 4 red balls. 3 balls are selected at random from the bag. Find the probability of drawing the balls in the following cases:-

1) All diff colour. 2) Same colour 3) No red ball
4) Only white 5) Red or black.

$n = 15, \; r = 3$