UNIT 1

Chapter 1

RATIO, PROPORTION AND PERCENTAGE

RATIO:

Introduction: The relation between two quantities (a and b) of the same kind or the same type, which expresses one quantity (a) as a multiple or part of the other (b), is called a **ratio**. It is written as a:b or as a fraction a/b and is read as " a is to b".

The two quantities a and b are called the terms of the ratio a:b. The first term 'a' is called **antecedent** and the second term 'b' is called the **consequent**.

Rules of ratio:

1. The two quantities must be of the same kind. For example: we cannot the ratio of apples to milk. But we can write ratio of number of apples to the number of bottles of milk.
2. The units of both antecedent and consequent should be same. For example: we can write ratio of Rs.10 to Rs.25 but we cannot write Rs. 10 to 250 paise. Here we have to convert either both into Rs. or both into paise.
3. When the ratio of two quantities with the same units is written down, the units are not mentioned.
4. When a ratio is written as a fraction a/b, it is generally written in the lowest terms. For example: the ratio 10/25 should be written as 2/5.
5. When two quantities are in ratio a:b, for example: 2:5, it means that the two quantities are 2 times and 5 times of some

measure , say x. Hence, it can be assumed that the two quantities are 2x and 5x respectively.

6. If $a = b$, then the ratio a:b is known as the **ratio of equality**.
   If $a > b$, then the ratio a:b is known as the **ratio of greater inequality.**
   If $a < b$, then the ratio a:b is known as the **ratio of lesser inequality**.
7. If a, b, c are positive real numbers. Then the following are true:
   (a)If $a/b > 1$ then, $a/b > (a+c)/(b+c) > 1$
   (b)If $a/b < 1$ then, $a/b < (a+c)/(b+c) < 1$
   (a)For example: let a = 5, b = 2, c = 3

   $5/2 > 1$ and $(5+3)/(2+3) = 8/5$ Now $5/2 > 8/5 > 1$

   (b) Now let a = 2, b = 3, c = 4.

   $2/3 < 1$ and $(2+4)/(3+4) = 6/7$ Now $2/3 < 6/7 < 1$

8. Let a, b, c be positive real numbers such that c is less than both a and b i.e., $0 < c < a$, $0 < c < b$. Then the following are true:
   (a) If $a/b > 1$, then $(a-c)/(b-c) > a/b > 1$
   (b) If $a/b < 1$, then $(a-c)/(b-c) < a/b < 1$
   (a) For example: let a=5, b=2 and c=1, then
   $5/2 > 1$ and $(5-1)/(2-1) = 4/1$ and $4/1 > 5/2 > 1$
   (b) For example: let a=3, b=4 and c=1, then
   $3/4 < 1$ and $(3-1)/(4-1) = 2/3$ and $2/3 < 3/4 < 1$

COMPOUND RATIO:

When ratios are compounded by multiplying together their antecedents to form a new antecedent and their consequents to form a new consequent, then such a ratio is known as **compound ratio**.

For example: the compound ratio of three ratios 3/2, 1/5 and 3/7 is (3*1*3)/(2*5*7), i.e.,9/70

When a ratio a:b is compounded with itself, the resulting ratio $a^2:b^2$ is called the **duplicate ratio of a:b**.

Similarly, $a^3:b^3$ is called the **triplicate ratio of a:b.**

$\sqrt{a}:\sqrt{b}$ is called the **sub-duplicate ratio of a:b.**

INVERSE RATIO:

If a:b is a ratio, then (1/a):(1/b), i.e., b:a is called the **inverse ratio** or the **reciprocal ratio of a:b**.

CONTINUED RATIO:

If more than two quantities of the same kind are compared, then it is known as **continued ratio** or an **extended ratio**. For example: if the measures of angle A, B and C of a triangle ABC are 45^0, 60^0 and 75^0 respectively, then they are in the continued ratio 45:60:75 or 3:4:5.

Exercise 1:

1. A college collected Rs. 6,02,136 for charity which is to be divided between an orphanage and a school for autistic children in the ratio 7 : 11. What amount did each institution receive?

Solution:

Let the common ratio be x.

Let the share of orphanage be 7x and

Let the share of school for autistic children be 11x.

So , $7x + 11x = 6{,}02{,}136$

$\Rightarrow 18x = 6{,}02{,}136$

$\Rightarrow$ x = 6,02,136/18

$\Rightarrow$ x = 33,452

Hence, orphanage receives = 7x = Rs. 7 * 33,452 = Rs. 2,34,164

School for autistic children receives = 11x = Rs. 11*33452

= Rs. 3,67,972

2. There are 45 members on a students' council in an educational institution and the ratio of the number of boys to the number of girls is 2 : 1. How many more girls should be added to the council to make the ratio 3 : 2 ?

Solution:

Let the common ratio be x.

Then the number of boys = 2x

The number of girls = x

So, 2x + x = 45

$\Rightarrow$ 3x = 45

$\Rightarrow$ x = 45/3 = 15

Hence, number of boys = 2 * 15 = 30

And the number of girls = 15

Now, new number of boys = 3x

And , new number of girls = 2x

So, 3x + 2x = 45

$\Rightarrow$ 5x = 45

$\Rightarrow$ x = 45/5 = 9

Hence, number of girls = 2 * 9 = 18

So, more number of girls to be added in the council = 18 – 15 = 3

3. If the ratio of A to B is 3 : 2 and the ratio of B to C is 3 : 2, find the ratio A : B : C. Are A, B, C are in continued proportion?

Solution:

A : B = 3 : 2 B : C = 3 : 2

$\Rightarrow$ A/B = 3/2 $\Rightarrow$ B/C = 3/2

So, A/B = (3 * 3)/(2 * 3) $\Rightarrow$ B/C = (3 * 2)/(2 * 2)

= 9/6 = 6/4 [LCM of B=2*3=6]

Hence A :B : C = 9 : 6 : 4

Yes, A, B, C are in continued proportion.

PROPORTION:

Let a, b, c and d be four quantities such that a and b are of the same kind and c and d are of the same kind. If the two ratios a/b and c/d are equal then we say that a, b, c and d are in **proportion.** This is denoted by a : b :: c : d or a : b = c : d or a/b = c/d and is read as “a is to b as c is to d”.

In other words, proportion is a statement stating the equality of two ratios (or more than two ratios).

When a, b, c and d are in proportion , they are called the **first, second, third** and **fourth proportional respectively. a** and **d** are called **extremes** and **b** and **c** are called **means.**

When a, b, c and d are in proportion,

$a/b = c/d$

$\Rightarrow ad = bc$

$\Rightarrow$ product of two extremes = product of two means.

This helps us to determine an unknown quantity if the three quantities in the proportion are known. This is commonly known as " **The Rule of Three**".

The Rule of Three:

Suppose we have to find the fourth proportional to the given first three proportional a, b and c.

Let the fourth proportional be x. Then a, b, c and x are in proportion.

So, $a/b = c/x$

$\Rightarrow x = bc/a$

This is the " **Rule of Three**".

CONTINUED PROPORTION:

If a, b and c are three quantities of the same kind ,then they are said to be in **Continued Proportion** if **a : b :: b : c** or **a/b = b/c. c** is called the **third proportional, a** is called the **first proportional** and **b** is called the **mean proportional.**

When a, b and c are in continued proportion,

$a/b = b/c$

gives $ac = b^2$ i.e.

$b = \sqrt{ac}$

If $a_1, a_2, \ldots\ldots, a_{n-1}, a_n$ are in continued proportion ,then

$A_1/a_2 = a_2/a_3 = a_3/a_4 = \ldots\ldots\ldots\ldots\ldots = a_{n-1}/a_n = k$

=> $a_1 = ka_2$, $a_2 = ka_3$, $a_3 = ka_4, \ldots\ldots\ldots$, $a_{n-1} = ka_n$

=> $a_1 = ka_2 = k^2a_3 = k^3a_4 = \ldots\ldots\ldots\ldots = k^{n-1}a_n$.

DIRECT AND INVERSE PROPORTION:

A quantity A is said to be in **direct proportion** to another quantity B if an **increase (or decrease)** in one of them leads to the **increase (or decrease) respectively** in the other.

A quantity A is said to be in **inverse proportion** to another quantity B if an **increase (or decrease)** in one of them leads to the **decrease (or increase) respectively** in the other.

A, B are in direct proportion means $a_1/a_2 = b_1/b_2$

=> $\mathbf{a_1 : a_2 :: b_1 : b_2}$

A, B are in indirect proportion means $a_1/a_2 = b_2/b_1$

VARIATION:

If two variables A and B are such that the value of A can always be represented as a constant times the value of B, then we say that **A varies directly as B** or simply A varies as B and write as **A α B.** When

A α B , we have **A = kB** for some constant k, which is called the **constant of variation.**

If A varies as the **reciprocal** of B, then we say that **A varies inversely as B** and write as **A α 1/B** . In this case **A α k/B** where **k** is the **constant of variation.**

If a variable A varies as the product of variables B and C, then we say that **A varies jointly as B and C and write A α BC.** In this case,

A = kBC , where k is the constant of variation.

Some properties of variation:

Let A, B, C and D etc be variables.

(1) If A α B, then B α A.
(2) If A α B and B α C, then A α C.
(3) If A α BC , then B α (A/C) and C α (A/B)
(4) If A α C and B α C then A+B α C, A-B α C and AB α C^2
(5) If A α B and CA α CB for a constant C or a variable C
(6) If A α B and C α D then, AC α BD and A/C α B/D

PERCENTAGE:

Percentage is the shortened form of phrase per centum which means for every hundred. The ratio r/100 when written as percentage, is written as r% and read as ''r per cent''. Here r is the rate per cent. Most rates in business and Economics such as rates of interest, discount, commission, brokerage, profit, loss, dividend, tax, etc. are expressed as percentage.

PROFIT, LOSS, DISCOUNT, COMMISSION AND BROKERAGE

1. **PROFIT & LOSS:**
   The price at which an article is purchased is called its **Cost Price(C.P).** The price at which the article is actually sold is called its **Net Selling Price (N.S.P).**

If the selling price is **greater** than the cost price, then a **profit** is earned. If the selling price is **less** than the cost price, then a **loss** is incurred. When the selling price is **equal** to the cost price, neither profit nor loss is made. This is called **break-even point.**

Profit = N.S.P. – C.P.   when N.S.P. > C.P.

Loss = C.P. – N.S.P.   when N.S.P. < C.P.

Profit percentage or loss percentage is calculated when prifit or loss is compared with **Cost Price.**

Profit% = (Profit/C.P)* 100

Loss% = (Loss/C.P)* 100

N.S.P. = {(100 + Profit%)/100} * C.P   when a profit is earned.

N.S.P. = {(100 – Loss%)/100}* C.P.   when a loss is incurred.

2. DISCOUNT:

The price at which each good/item is sold in the market is known as **Printed Price** or **Marked Price.** This price is also called **List Price** or **Listed Price** or **Catalogue Price.**

When the seller (manufacturer or trader) offers a **reduction** in the **List Price,** it is known as **Discount.** After deducting the discount, an item is sold at the **Net Selling Price (N.S.P.)**

Hence,

N.S.P. = L.P. – Discount

Usually , discount is expressed as a percentage on the List Price.

N.S.P. = L.P. – Discount

= L.P. – Discount% on L.P.

= L.P. – (Discount%/100) * L.P.

= (1 - Discount%/100) * L.P.

={ (100 – Discount%)/100} * L.P.

AND

N.S.P. = {(100 + Profit%)/100} * C.P.

3.TRADE DISCOUNT AND CASH DISCOUNT:

When a trader is selling goods to another trader (e.g. wholesaler to retailer), usually a two-discounts structure is followed.

Firstly, a **Trade Discount (T.D)** is given to all traders. **T.D.** is the percentage on the **List Price . List Price minus the Trade Discount** is called the **Invoice Price (I.P)** or the **Reduced List Price.**

If the trader who is buying the goods offers immediate **Cash Discount (C.D), which is a percentage (not on the List Price)** but on the **Invoice Price.** The **Net Selling Price (N.S.P.) is I.P – C.D.**

I.P = {(100 – T.D%)/100} * L.P.

N.S.P. = {(100 – C.D.%)/100} * I.P.

4.COMMISSION AND BROKERAGE:

An **agent** or a **commission agent** is a person who buys and/or sells goods for another person. The person who employs the services of the agent is called the **Principal** and the remuneration given by the Principal to the agent is called **commission.**

A commission agent may sell goods for cash or on credit. If the sale takes place on credit, then there could be a risk of payment default.

A Del Credere agent sells goods and guarantees the collection of dues from the customers to the Principal. For this he or she charges extra commission which is also known as **del Credere.**

A Broker is an agent who brings together prospective buyers and sellers and negotiates a deal. For this, he or she charges a commission, called **Brokerage**, from both the buyer and the seller.

INTEREST

Introduction to Basic Notations:

The money paid for the use of a sum of money taken as a loan, is known as **interest.** The sum of money borrowed is called the **principal.** The total **amount** that is due from the borrower at any given time is given by the addition of principal and the interest. The **rate of interest** is usually expressed as a percentage of the principal.

If the interest is charged only on the principal, then it is called **simple interest.** If periodically the interest due is added to the principal and the interest for the next period is calculated on this addition (of principal and the earlier interest), then it is called **compound interest.**

The notations are as follows:

P = Principal , also known as the Present Value

n = no. of years

r = rate of interest per annum

A = Amount at the end of n years, also called the **Accumulated Value** or the **Future Value**

I = Interest for the period of n years

Often it is more convenient to use the following measure of i= r/100 = rate of interest per Re.1 per annum

For eg: if r = 9%, then i= 9/100 = 0.09

Simple Interest:

Basic formulae:

The simple interest on principal P for 1 year at r% per year is r% of P, i.e., P * r/100

Hence, the simple interest on principal P for n years at r% p.a. is n times the above sum, i.e.,

n (P * r/100) = P * n * r/100 =Pni

Hence, we have,

I = P * n * r/100

=>I = Pni

The amount due is the addition of the principal and the interest. So.

A = P + I

- ⇨ **A = P + P * n * r/100**
- ⇨ **A = P (1 + n * r/100)**
- ⇨ **A = P (1 + ni)**

[Note: When the principal and the rate of interest is same, then since

P * n * r/100 = n * (P * 1 * r/100), we have,

Simple Interest for n years = n * (Simple Interest for 1 year)

Also, SI for m yrs + SI for n yrs = SI for (m+n) yrs]

SIMPLE INTEREST FOR FRACTIONAL YEARS:

Even if the number of years , n, is not an integer, the formula

$I = P * n * r/100 = Pni$, still holds true.

COMPOUND INTEREST:

In compound interest, at the end of each year, the interest is added to the principal and the interest for the next year is calculated on this added amount.

Suppose the principal is P, the rate of interest is r% p.a. and the number of years is n. Let the interest in the 1st, 2nd, 3rd,….nth year be I_1, I_2, I_3,……, I_n and the amount at the end of the 1st, 2nd, 3rd,……, nth be A_1, A_2, A_3,……,A_n = A. Then for each year, the amount equals to the previous year's amount plus the interest for that year, so

At the end of the 1st year, $A_1 = P + I_1$

At the end of the 2nd year, $A_2 = A_1 + I_2$

At the end of the 3rd year, $A_3 = A_2 + I_3$

And so on……

At the end of the 1st year, the interest is r% of P, i.e.,

$I_1 = P * r/100$

$A_1 = P + I_1$

⇨ $A_1 = P + P * r/100$

⇨ $A_1 = P(1 + r/100)$

At the end of the 2nd year, r% interest is given on this amount, i.e., the interest in the 2nd year is r% of P(1 +r/100)

So, $I_2 = [P(1 +r/100)] * r/100$

Hence, the amount for the 2nd year is

$A_2 = A_1 + I_2$

$= P(1 + r/100) + [P(1 + r/100)] * r/100$

$= P (1 + r/100) (1 + r/100)$

$= P(1 + r/100)^2$

Similarly, after n years,

$I_n = P(1 + r/100)^{n-1} * (r/100)$

$A_n = P(1 + r/100)^n$

Or, $A = P(1 + i)^n$

Where i= r/100 is the rate per Re.1 per annum.

Or, $P = A/(1 +i)^n$

[Note: The Principal **P** is also called the **present value** and the amount **A** is called the **future value.]**

The compound interest for the entire period of n years is given by

$I = A – P$

⇨ $I = P(1 + r/100)^n – P$

⇨ $I = P[(1 + r/100)^n – 1]$

⇨ $I = P[(1 + i)^n – 1]$

Interest Compounded more than once a year; Nominal (Stated) and the Effective Rate of Interest:

In practice, the interest is compounded not annually, but more frequently: half-yearly(semi-annually), quarterly or monthly.

Let P, n, r, A, I and i have their usual meanings.

r = rate of interest percent p.a.

i = r/100 = rate of interest per Re.per annum

n = number of years

Let, m = number of items the interest is compounded per year.

m= 4 if the interest is compounded quarterly.

m= 12 if the interest is compounded monthly.

i.e., the interest is compounded when (1/m) year is over(e.g., $1/4^{th}$ year).

(1) In 1 year, the interest is compounded m times.

Hence, in n years, the interest is compounded (m*n) times.

(2) The rate of interest per Re. 1 per 1 year is i.

Hence, the rate of interest per Re.1 per 1/m year is i/m.

So the accumulated amount A at the end of n years is given by:

$A = (1 + i/m)^{mn}$

i.e., $A = (1 + r/100m)^{mn}$

RELATION BETWEEN NOMINAL AND EFFECTIVE RATE OF INTEREST:

Suppose the interest is compounded m times a year. Let the nominal or the stated rate of interest per Re.1 per year be i as usual and let the effective rate of interest per Rs. 1 per year be denoted by i_e. Let us consider the principal be Re.1 and the number of years to be 1, i.e n=1. We can calculate the accumulated amount A after 1 year in two ways:

(1) Using the nominal rate of interest, we get

$$A = (1 + i/m)^m$$

(2) Using the effective rate of interest, we get

$$A = (1 + i_e)^1 = 1 + i_e$$

Equating the two, we get

$$1 + i_e = (1 + i/m)^m$$

⇨ $$I_e = (1 + i/m)^m – 1$$

CONTINUOUS COMPOUNDING:

When the frequency of compounding within one year is increased, the effective rate of interest also increases. If the frequency of compounding is increased to infinity, that is, if the interest is compounded at each instant, then this process is called **continuous compounding** and the effective rate is called **continuous rate of interest.**

Let us suppose

P = Principal Sum

r = nominal rate of interest percent p.a.

n = number of years

A = Amount accumulated at the end of n years of continuous compounding

i = r/100 = rate of interest per Re.1 per annum.

Then the formula for amount after n years of continuous compounding is given by:

$A = Pe^{rn/100}$

⇨ $A = Pe^{in}$

If i_e= effective rate of interest with continuous compounding,

Then $i_e = e^i – 1$

Here , e is an irrational number lying between 2 and 3. Its approximate value is 2.7182

Depreciation Of Assets:

Suppose a machinery or an asset gets depreciated at the rate of i per 1 year per annum. Suppose its present value is P.

Then its value after depreciation at the end of 1 year is P(1 - i), at the end of 2 years it is $P(1 – i)^2$,……, at the end of n years it is $P(1 – i)^n$.

If we denote the depreciated value at the end of n years by D, then

$$D = P(1 – I)^n$$

EMI using Reducing Balance Method and Flat Interest Rate Method:

Nowadays, many consumer goods are sold with a loan to be repaid in Equated Monthly Instalments (EMI). There are two methods of calculating the EMI- the flat interest rate method that uses simple interest and the reducing balance method that uses compound interest.

Suppose the loan, i.e., the principal is **P**, which has to be repaid in n months. It is more convenient to take the interest rate per month. Let i be the interest rate per Re.1 per month.

In the flat interest rate method, the simple interest on P for n months is considered, which is Pni. Hence, the borrower must return

Principal + Interest

$= P + Pni$

$= P(1 + ni)$

This repayment is flatly divided over n months. Hence, EMI using the Flat Interest Rate is:

$EMI = [P(1 + ni)]/n$

In the reducing balance method, compound interest is calculated. Each time an instalment is paid, it is considered that a part of the principal P has been paid and hence the interest for the next month is charged only on the reduced balance principal to be paid. The resulting EMI is less than the EMI calculated using the flat interest rate method.

Hence ,

$EMI = Pi/[1 - (1 + i)^{-n}]$

Where i is the interest rate per Re.1 per month.

ANNUITY

Time Value Of Money:

The value of money changes over time. The same amount of money's worth at different times is different. The purchasing power that Rs.1,000 had a hundred years ago was much more than the purchasing power today of the same amount. Hence for individuals and institutions, it is necessary and beneficial to understand the present value of amounts due in future and the future value of amounts held today.

Future Value and Present Value:

Suppose a principal sum of money P gives an amount A after n years. Then **A** is the **Future Value** that P would give after n years. Also **P** is the **Present Value** of the amount A due at the end of n years.

The future value is also referred to as the **Accumulated Value** or the **Amount** that **P** would give after n years.

Suppose an amount A is due after n years and to settle it today (instead of after n years) we calculate the amount P that one would have to invest today (at a given rate of interest)to get an amount A after n years. Then **A**, the **Amount**, is called the **Sum Due**, **P,** its **Present Value** is called the **Present Worth** or the **Discounted Value** and the process of finding the present value of a sum due is called **Discounting.**

Compounding Factor, Growth Factor and Discount Factor:

When one has to find out the future value of a sum, one has to multiply the sum by the factor $(1 + i)^n$ where i = rate of interest p.a. and n = number of years. Here , $(1 + i)^n$ is called the **Compounding Factor** or the **Growth Factor**.

present value or the discounted value of a sum due in future, one multiplies it by the factor $1/(1+i)^n$. Hence, $1/(1+i)^n$ is called the **Discounting Factor.**

Net Present Value:

In capital budgeting , in order to make long-term investment decisions, companies and institutions employ certain evaluation techniques to evaluate projects. **NPV** and **IRR** are two such simple methods.

NPV stands for **Net Present Value** of a project. Suppose the cash flows associated with a project are C_0, C_1, C_2….., C_p. Usually C_0 is the initial investment which is a cash outflow and C_1, C_2, C_3,….., C_p are cash inflows at the end of 1st, 2nd, 3rd,….pth year respectively. All the cash outflows are taken to be positive. Suppose the cost of capital expressed as a compound rate of interest p.a. is i. Then the discount factor is $1/(1+i)^n$.

Multiplying C_n by $1/(1+i)^n$ gives the present value of C_n. This can be done for each cash flow C_n, with n=1, 2, 3,….., p. By adding all the positive and negative present values, we get the Net Present Value(NPV) for the project.

NPV = $C_0 + C_1/(1+i) + C_2/(1+i)^2 + C_3/(1+i)^3 + \ldots\ldots + C_p/(1+i)^p$

Accept-Reject Criterion:

1) A project is acceptable if its NPV > 0
2) For comparing two projects with equal investment, the project with higher NPV is better.

Annuities:

INTRODUCTION TO IMMEDIATE ANNUITY:

An **annuity** is a sequence of payments made over successive periods or intervals of time. The unit period is usually a year, but need not always be so.

If all payments are equal, then the annuity is called an **uniform annuity** or **a level annuity.** If all the payments are not equal, then it is known as **variable annuity.**

Annuities can also be classified according to how long the sequence of payments continue. If the total number of time periods for which the annuity payments are made, is fixed in advance, the annuity is called an **annuity certain.** If the payments are to be made as long as a person is alive, it is called **life annuity.** An annuity whose number of payments depend on the happening of an event is called a

contingent annuity. A life annuity is an example of a contingent annuity. An annuity that is supposed to go on perpetually or endlessly is called a **perpetual annuity** or a **perpetuity.**

Annuity Certain:

Such annuities are annuities which are uniform and certain. This means that payments are all equal and for a fixed number of times.

They can be classified into two categories. If the payments are made at the **end** of the successive periods, then the annuity is called the **immediate annuity** or an **ordinary annuity.** If the payments are made at the beginning of each successive period, then the annuity is called an **annuity due.**

The **present value of an annuity** is the sum total of all the present values of each payment at a given rate of compound interest.

The **accumulated value** or the **amount of an annuity certain** is the sum total of all the accumulated value or future values of each payment calculated at the end of the last period of the annuity, at a given rate of compound interest.

BASIC FORMULAE FOR AN IMMEDIATE ANNUITY:

Suppose an immediate annuity has n payments, each of amount Rs.C, made at the end of each time period(usually a year), for n time periods(i.e., usually for n years). We take the letter C as it represent the uniform **cash flows** or payments of the annuity.

Let the rate of compound interest be r% per each time period. As before, we have i= r/100.

Let the present value of an immediate annuity be represented by the letter **P** and its accumulated value by the letter **A.**

Let us suppose that the rate of compounding is 10% p.a. Then the amount of the 3rd payment of Rs. 10,000 paid at the end of 3 years is Rs. 10,000 itself.

The 2nd payment of Rs. 10,000 is paid at the end of the 2nd year. Its amount at the end of 3 years [using the formula $A=P(1+i)^n$] is

Rs. $10{,}000(1 + 0.10)^1$ = Rs. 11,000

The 1st payment of Rs. 10,000 is paid at the end of the 1st year, hence at the end of 3 years, i.e., after 2 years, its amount will be

Rs. $10{,}000(1 + 0.10)^2$ = Rs. 12,100

Hence the accumulated value of the annuity, which is the sum total of all the future values of the 3 payments is

Rs.(10,000+11,000+12,100) = Rs. 33,100

In general, accumulated value of annuity is

$A = C + C(1+i) + C(1+i)^2 + C(1+i)^3 + \ldots\ldots\ldots C(1+i)^{n-1} \ldots\ldots\ldots (1)$

Multiplying (1) by (1+i) we get,

$(1+i)A = C(1+i) + C(1+i)^2 + C(1+i)^3 + \ldots\ldots\ldots + C(1+i)^n \ldots\ldots\ldots (2)$

Subtracting (1) from (2) we get,

$(1+i)A - A = -C \qquad\qquad\qquad\qquad + C(1+i)^n$

$\Rightarrow A + iA - A = C(1+i)^n - C$

$\Rightarrow iA = C\,[(1+i)^n - 1]$

$\Rightarrow A = C/i[(1+i)^n - 1]$

And the present value P is given by

$P = C/i[\, 1 - (1 + i)^{-n}\,]$

At the rate of interest r%, the present value and the accumulated value are obviously related by the formula

$A = P\,(\,1 + r/100\,)^n$

$\Rightarrow A = P\,(1 + i)^n$

Also, $(1/P) - (1/A) = i/C$

STATED (NOMINAL) AND EFFECTIVE ANNUAL RATE OF INTEREST IN ANNUITY PROBLEMS:

(Annuities with frequency other than with which the interest is convertible)

If the payments of an annuity are made once a year but the rate of compounding is more frequently than once a year, then instead of the stated or nominal rate of compounding, we must take the effective rate of compounding per annum and use r = effective rate % p.a. with i=r/100.

ACCUMULATED VALUE OF IMMEDIATE ANNUITY WITH PAYMENTS MORE THAN ONCE A YEAR:

When the annuity time period is half-yearly, quarterly or monthly, the same formula is used **provided the interest is compounded exactly at the same time periods, i.e., half-yearly, quarterly or monthly** respectively.

r=rate of compound interest per cent per time period.

n= number of time periods.

i= r/100 (i is the interest per 1 Rupee per time period)

C= payment at the end of each time period.

The formulae remain the same as before.

PRESENT VALUE OF IMMEDIATE ANNUITY WITH PAYMENT MORE THAN ONCE A YEAR:

As in the case of accumulated value, we can use the formulae for present value same as before with C, n and i referring to the appropriate time periods instead of a year.

Redemption of a Loan:

The process of gradual elimination of a loan through regular instalments that are sufficient to cover both the principal and the interest, is called as **amortization** of a loan.

Nowadays, many consumer goods are sold with a loan to be repaid in **Equated Monthly Instalments (EMI).** That is, when a loan is repaid as an immediate annuity with unit time period being a month, then the sum of one monthly payment is known as **EMI**. The method employed of using the present value of the annuity using compound interest to calculate the EMI is called the method of **reducing balance.**

Each instalment per month contains two components, one is the repayment of the principal and the other is the payment of the interest. Such calculations are necessary for the tax payment of the lender since the interest is taxed, not the principal repayment.

The interest for the month, when subtracted from the EMI, gives the principal repayment that is taking place that month.

Flat Interest Rate Method:

Method of calculating equal instalments of a loan repayment in which the principal as well as the simple interest is equally divided among the total number of (unit) time periods.

Sinking Fund:

A fund created to accumulate a specific sum of money at some definite date in future by paying regular and equal payments at compound interest.

Variable Annuity:

An annuity in which not all the payments are equal.

SHARES AND MUTUAL FUND

SHARES

When a single person or a few persons set up a business the capital required is usually beyond the means of a few individuals and has to be raised from a large number of people. In such a case, a few persons take the initiative to form a company. These persons are called the **promoters** of the company. The company is established under the Companies Act, 1956. The promoters decide to raise a certain amount of capital to start the company. They divide this amount into small parts called shares.

A **share** is the smallest unit of the capital of a company. **Stock** is the American term for share. For example, the promoters could decide to raise a capital of Rs. 10 crores. This amount can be divided into 10 lakh shares of Rs. 100 each or 1 crore shares of Rs. 10 each or 10 crore shares of Re.1 each, etc. Usually a share is of value Rs. 100 or Rs. 50 or Rs. 10 or Rs. 5 or Rs. 2 or Re.1. This value is called the **Face Value** of the share. These shares are sold to the public. This sale is called the **Initial Public Offer (IPO)** of the company. People apply through application forms and the promoters allot (sell) or partially allot or not allot them the shares, depending on the availability.

The company issues **share certificates** to the persons from whom it accepts the money to raise the capital. This certificate is the evidence of the ownership of the company. Persons who have paid money to form the capital are called **shareholders.** The shareholders have purchased shares and are part-owners of the company in the proportion of their holdings, i.e. their shares. Nowadays the shares are not in the form of paper , but in electronic **dematerialised (demat)** form, hence the allotment of shares is done directly in the demat account, without a certificate.

Face Value or Nominal Value or Par Value is the value of a share printed on the share certificate or in its initial public offer. Now-a-days, since shares exist not on paper but in electronic de-materialised (demat) form, one can say that the face value is the value stated in the I.P.O. subscription form.

The shareholders share the profits (if any) of the company, after providing for the taxes and the other reserve funds. This is called the **dividend**. It is usually expressed as a percentage of the face value of a share.

Types of Shares:

Shares are mainly of two types: (i) Preference Shares and (ii) Equity Shares or Ordinary Shares or Common Shares.

(i) **Preference Shares :** These are the shares that have a priority over the equity shares. From the profits, a dividend at a **fixed rate** is paid to them first, before distributing any part of the profits to the equity shares. Also, if and when the company is closed down, then while returning of the capital, the preference shareholders get a preference. Preference shares are mainly of two types:

(a) Cumulative Preference Shares: If due to loss or inadequate profit, the preference shareholders are not paid their fixed rate of dividend, then the dividend is accumulated in the subsequent year/years to the cumulative preference shareholders and is paid preferentially whenever possible.

(b)Non-Cumulative Preference Shares: Those are Preference Shares for which the unpaid dividends do not accumulate.

(ii) Equity Shares or Ordinary Shares or Common Shares: These are shares for whom the dividend and the return of capital is paid after paying the preference shareholders. The rate of dividend to equity shares is not fixed and is decided by the Board of Directors.

Just like a commodity, a shareholder is allowed to buy and sell shares. If a particular company is doing well and its shares are more in demand then selling a share for a higher price than its face value is legal. The share prices are allowed to be subject to the market forces of demand and supply and thus the prices at which they are traded can be above or below the face value. The place at which the shares are bought and sold is called a **Share Market** or a **Stock Exchange** and the price at which a share or stock is traded is called its **Market Price** or the **Market Value.**

If the market price of a share is same as its face value, then the share is said to be traded **at par.**

If the market price of a share is greater than its face value, then the share is said to be available at a **premium or above par** and is called a **premium share** or **above par share**.

If the market price of a share is lower than its face value, then the share is said to be available at a **discount or below par** and is called a **discount share** or **below par share**.

The shares can be purchased and sold through authorized brokers. The brokers are usually companies like Kotak Securities Ltd., ICICI direct.com etc. They charge a commission on the purchase and the sale of shares, which is called **brokerage.**

The brokerage is charged as a percentage of the market price of the share. Usually it is below 1%, i.e., some paise per hundred rupees.

The rate of dividend is calculated on the face value . But a shareholder, i.e., an investor, usually buys the shares **not** at the face value, but at the market value.

Some Important Assumptions and Points:

(1) If the face value of a share is not mentioned in the problem, we will assume it to be Rs. 100.

(2) If the brokerage is not mentioned in the problem, then we will assume that there was no brokerage involved. This situation arises, for example, when a broker buys shares for himself/herself.

(3) ‘ A 5% share at 120’ or ‘ A 5% stock at 120’ means a share (with face value Rs. 100) with 5% dividend (on the face value) and whose market price is Rs. 120.

(4) ‘A share at 10% premium’ or ‘ A share at Rs. 10 premium’ or ‘A share at 10% above par’ or ‘ A share at Rs. 10 above par’ means a share with face value Rs. 100 and market price Rs. 110.

(5) ‘ A share at 10% discount’ or ‘ A share at Rs. 10 discount’ or ‘ A share at 10% below par’ or ‘ A share at Rs.10 below par’ means a share with face value Rs. 100 and market value Rs. 90.

(6) The rate of dividend is always calculated as a percentage of the face value and the rate of return on investment is always calculated as a percentage of investment (investment is market value plus brokerage).

(7) If shares are bought and sold on the same day and the problem mentions single brokerage without stating if the brokerage is charged on the purchase or on the sale, then we usually assume that the brokerage is on the sale.

(8) While purchasing , the brokerage is added to the market price to give the purchase cost of a share.

(9) While selling , the brokerage is subtracted from the market price to give the amount received on the sale.

Bonus Shares: Sometimes, when a company's free reserves are high, it may choose to capitalize a part of it by converting it into shares. This is done by issuing bonus shares to existing shareholders. The ratio of bonus shares to the existing shares is fixed. These bonus shares are issued free of cost.

Getting bonus shares increases the number of shares of shareholders. But since this applies to all the shareholders that too in a fixed ratio, hence the percentage of a shareholder's ownership of the company remains same as before.

Splitting of Shares: Sometimes companies split the face value of a share and break it up into smaller units. For example, a Rs. 100 share can be split into 10 shares each of face value Rs. 10 or a Rs.10 share can be split into two shares of face value Rs. 5 each. Usually this does not affect a shareholder's wealth. However , it can make selling of a part of the holdings easier.

Earlier, having Rs.100 share was more of a norm. Now-a -days, the tendency is to keep the face value of newly floated shares small as also split earlier shares of large face value into smaller shares of smaller face value.

MUTUAL FUNDS

A **mutual fund** is a pool of money collected from large number of investors with a promise to invest the money in a particular manner by professional managers. The investors are allotted 'Units' of the mutual fund and share the gains and losses of the fund. The organisation that manages the investments for the benefit of the investors is called the **Asset Management Company.** The employees of the company who actually manage the funds are called the **fund managers.**

In other words, a mutual fund is an indirect way of investing where investors pool their money and the fund managers invest the money on their behalf. This suits the investors who may not have the time or the expertise to study the market and invest directly on one's own.

The pooled money of a mutual fund may be invested into debt instruments like government securities or in equities, i.e., the stock market. Accordingly, funds can be broadly classified as **debt funds** and **equity funds.** There are mixed or hybrid funds which invest in both debt and equity. The offer document gives the guidelines/constraints under which the fund managers would operate. For example: investment in equity 80% to 100%, investment in money markets 0% to 20% etc.

In India, the mutual funds are governed by the Securities and Exchange Board of India (SEBI). There are different companies, called the 'Fund Houses' like SBI or Reliance or HDFC which float different mutual funds. Each such fund is called a 'Scheme'. For example, HDFC has schemes like 'HDFC Tax Saver', 'HDFC Long-Term Advantage' etc.

Similar to an Initial Public Offer (I.P.O.) of a company's share, a mutual fund scheme starts by having a **'New Fund Offer' (N.F.O.).** Investors can invest by purchasing **Units** of the mutual funds. Usually a unit is of Rs. 10 . A share is the smallest unit of a company's capital. But in mutual funds, even a fraction of a unit can be purchased after the N.F.O.

After collecting this money, most of it is invested in different avenues as per the promises in the fund's offer document. Some part of the money gets used for the AMC's expenses, brokerage etc. The total value of the investments may increase or decrease-accordingly there will be a gain or loss. This gain or loss has to be shared by all the investors in proportion of their investments, i.e., in proportion of their units. Hence, at a given time, the total value is divided by the total number of units to get the value of a single unit at a given time. This is called the **Net Asset Value (NAV).**

The NAV of a mutual fund scheme is calculated and disclosed to the public for every working day. The calculations are based on the closing prices of the stocks and other securities in which the scheme has invested.

The NAV changes daily. Investors try to invest when the NAV is low and sell the units and get gains when the NAV is high. Shares are purchased and sold at the stock market. Most mutual fund schemes are not traded in the stock market. This means that an investor must purchase the units from the AMC and sell back to the AMC itself. This sale is called **'redemption'** of units.

Some mutual fund schemes collect a charge when the investors purchase or redeem units. These are percentages of NAV. The charge levied while purchasing a unit is called the **entry load** and the charge collected on redemption is called the **exit load.** There are regulatory limits on the loads that can be charged.

Usually , the debt funds have no loads. The equity funds, typically have a 2.25% entry load currently, for investments below Rs. 5 crores. Exit loads usually are 0.5 to 1% of NAV if redemption is within 6 months of investment, for investments below 5 crores. These are not necessarily followed by all fund schemes. The brokers in the mutual fund business are called **mutual fund distributor.** Even if an investor invests in a mutual fund through a distributor, there is no brokerage charged directly to the investor , since the money from the entry load is used to pay the broker. Some funds give an option to the investor : Invest through a distributor and pay an entry load or invest online through the internet and don't pay any entry load.

In calculations, the load plays a role similar to that of the brokerage. Thus , the **entry load is added to the NAV** while purchasing to get the **Purchase Price. The exit load is subtracted from the NAV** while redeeming to get the **Redemption Price.**

Mutual Funds can be broadly categorised into two types: '**Dividend**' Funds which offer a dividend and '**Growth**' Funds which do not offer a dividend.

Unlike shares, in mutual funds, the dividend given has no direct relation to the profit. The mutual fund invests the money in different shares that may or may not give a dividend at different times and different rates. The mutual fund's dividend has no relation to the dates and the rates of the share dividend either.

The fund manager may at any arbitrary point, decide to give a part of the units' value back to the investors. This is called **dividend**.

If a fund's NAV is Rs. 20 per unit and a dividend of Rs. 5 per unit is distributed, then the NAV comes down to Rs. 15 per unit. Thus the AMC had Rs. 20 per unit belonging to the investor, it is divided into two parts: Rs. 5 per unit with the investor and Rs. 15 per unit still with the AMC.

For a growth fund, the NAV does not come down this way due to dividends. It goes up or down purely on the basis of the gains or losses of the securities that the fund has invested in.

For a growth fund, the gains per unit are purely from the difference between the redemption price and the purchase price, i.e., the total gain is purely the capital gain. For a dividend fund, the total gain is purely the capital gain. For a dividend fund, the total gain is the addition of the capital gain and the dividend.

Some mutual fund schemes offer both options- growth and dividend to the investor. Hence we have names like 'Birla Sunlife Equity-Growth Plan' and 'Birla Sunlife Equity- Dividend Plan'.

Some Important Points:

(1) The entry load and exit load calculated as a percentage of the NAV.

(2)The NAV, the Purchase Price and Redemption Price could be calculated upto two or three or four decimal points. Different funds have different conventions. But within a single mutual fund scheme, all three would have the same number of decimal places. For e.g., If the NAV given in a problem is calculated upto 4 decimal places, then calculate the purchase price and the redemption price also upto 4 decimal places.

(3)The number of units are usually calculated upto three decimal places. Some liquid (debt) funds calculate them upto 4 decimal places. Within a single mutual fund scheme, the same convention will be followed.

(4)The amount invested , the dividend and the amount received after redemption are rounded off to the nearest paisa, i.e., to 2 decimal places. This is because any money transaction that has to take place must be done in the multiple of the least unit , 1 paisa; it cannot have a fraction of a paisa.

(5)The tax charged in each scheme is different as per the type of fund (debt/equity fund), the type of capital gain (short term- within a year/ long term – greater duration), the tax regulations for the year, etc.

The Dividend Reinvestment Option:

Many mutual fund schemes offer a dividend plan under a reinvestment option. This means that the investor is actually not paid the dividend but the dividend amount is immediately reinvested in the same scheme at the ex-dividend NAV. When the dividend amount is reinvested, there is no entry load.

Suppose the NAV of a mutual fund just before declaring the dividend is Rs.40.(This can be called the pre-dividend NAV) . Suppose a dividend of Rs.3 per unit is declared. Then the NAV comes down to Rs. 37.(This is called the ex-dividend NAV.)

In the dividend re-investment option , the dividend of Rs. 3 per unit is again invested back into the same scheme, i.e., the mutual fund still has Rs. 40 of the investor as it had before the dividend. The difference is that since the NAV has come down from Rs. 40 to Rs. 37, the number of units owned by the investor would increase. There is no entry load for purchase of reinvested dividend units.