What is Arithmetic Progression?

In mathematics, there are three different types of progressions. They are:

- Arithmetic Progression (AP)
- Geometric Progression (GP)

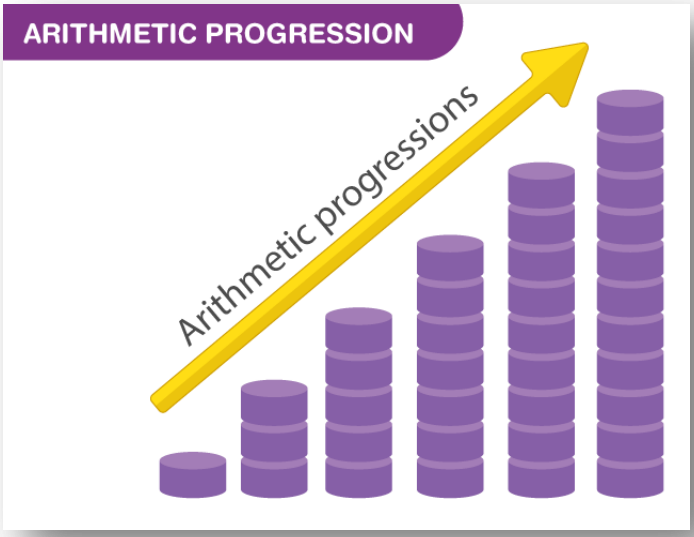
A progression is a special type of sequence for which it is possible to obtain a formula for the nth term. The Arithmetic Progression is the most commonly used sequence in maths with easy to understand formulas.

Definition 1: A mathematical sequence in which the difference between two consecutive terms is always a constant and it is abbreviated as AP.

Definition 2: An arithmetic sequence or progression is defined as a sequence of numbers in which for every pair of consecutive terms, the second number is obtained by adding a fixed number to the first one.

The fixed number that must be added to any term of an AP to get the next term is known as the common difference of the AP. Now, let us consider the sequence, 1, 4, 7, 10, 13, 16,…

It is considered as an arithmetic sequence (progression) with a common difference 3.



Notation in Arithmetic Progression

In AP, we will come across some main terms, which are denoted as:

- First term (a)
- Common difference (d)
- nth Term (a_n)
- Sum of the first n terms (S_n)

All three terms represent the property of Arithmetic Progression. We will learn more about these three properties in the next section.

First Term of AP

The AP can also be written in terms of common differences, as follows;

$a, a + d, a + 2d, a + 3d, a + 4d, \ldots\ldots\ldots, a + (n - 1) d$

where "a" is the **first term** of the progression.

Common Difference in Arithmetic Progression

In this progression, for a given series, the terms used are the first term, the common difference and nth term. Suppose, $a_1, a_2, a_3, \ldots\ldots\ldots\ldots\ldots, a_n$ is an AP, then; the **common difference " d "** can be obtained as;

$$d = a_2 - a_1 = a_3 - a_2 = \ldots\ldots\ldots = a_n - a_{n-1}$$

Where "d" is a common difference. It can be positive, negative or zero.

Also, check:

- Geometric Progression Sum Of Gp
- Arithmetic Progression For Class 10
- Important Questions Class 10 Maths Chapter 5 Arithmetic Progressions

General Form of an AP

Consider an AP to be: $a_1, a_2, a_3, \ldots\ldots\ldots\ldots\ldots, a_n$

Position of Terms	Representation of Terms	Values of Term
1	a_1	$a = a + (1-1) d$
2	a_2	$a + d = a + (2-1) d$
3	a_3	$a + 2d = a + (3-1) d$
4	a_4	$a + 3d = a + (4-1) d$

Position of Terms	Representation of Terms	Values of Term
.	.	.
n	a_n	a + (n-1)d

Arithmetic Progression Formulas

There are two major formulas we come across when we learn about Arithmetic Progression, which is related to:

- The nth term of AP
- Sum of the first n terms

Let us learn here both the formulas with examples.

nth Term of an AP

The formula for finding the n-th term of an AP is:

$\mathbf{a_n = a + (n - 1) \times d}$

Where

a = First term

d = Common difference

n = number of terms

a_n = nth term

Example: Find the nth term of AP: 1, 2, 3, 4, 5…., a_n, if the number of terms are 15.

Solution: Given, AP: 1, 2, 3, 4, 5…., a_n

n=15

By the formula we know, $a_n = a+(n-1)d$

First-term, a =1

Common difference, d=2-1 =1

Therefore, $a_n = a_{15} = 1+(15-1)1 = 1+14 = 15$

Note: The behaviour of the sequence depends on the value of a common difference.

- If the value of “d” is positive, then the member terms will grow towards positive infinity
- If the value of “d” is negative, then the member terms grow towards negative infinity

Types of AP

Finite AP: An AP containing a finite number of terms is called **finite AP**. A finite AP has a last term.

For example: 3,5,7,9,11,13,15,17,19,21

Infinite AP: An AP which does not have a finite number of terms is called **infinite AP.** Such APs do not have a last term.

For example: 5,10,15,20,25,30, 35,40,45……

Sum of N Terms of AP

For an AP, the sum of the first n terms can be calculated if the first term, common difference and the total terms are known. The formula for the arithmetic progression sum is explained below:

Consider an AP consisting “n” terms.

$$\mathbf{S_n = n/2[2a + (n - 1) \times d]}$$

This is the AP sum formula to find the sum of n terms in series.

Proof: Consider an AP consisting “n” terms having the sequence a, a + d, a + 2d, ……., a + (n – 1) × d

Sum of first n terms = a + (a + d) + (a + 2d) + ……. + [a + (n – 1) × d] ——————-(i)

Writing the terms in reverse order,we have:

S_n= [a + (n – 1) × d] + [a + (n – 2) × d] + [a + (n – 3) × d] + ……. (a) ————(ii)

Adding both the equations term wise, we have:

$2S_n$ = [2a + (n – 1) × d] + [2a + (n – 1) × d] + [2a + (n – 1) × d] + ……. + [2a + (n – 1) ×d] (n-terms)

$2S_n = n \times [2a + (n - 1) \times d]$

$S_n = n/2[2a + (n - 1) \times d]$

Example: Let us take the example of adding natural numbers up to 15 numbers.

AP = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

Given, a = 1, d = 2-1 = 1 and $a_n = 15$

Now, by the formula we know;

$S_n = n/2[2a + (n - 1) \times d]$

$S_{15} = 15/2[2.1+(15-1).1]$

$= 15/2[2+14]$

$= 15/2\ [16]$

$= 15 \times 8$

$= 120$

Hence, the sum of the first 15 natural numbers is 120.

Sum of AP when the Last Term is Given

Formula to find the sum of AP when first and last terms are given as follows:

S = n/2 (first term + last term)

List of Arithmetic Progression Formulas

The list of formulas is given in a tabular form used in AP. These formulas are useful to solve problems based on the series and sequence concept.

General Form of AP	a, a + d, a + 2d, a + 3d, . . .
The nth term of AP	$a_n = a + (n - 1) \times d$
Sum of n terms in AP	$S = n/2[2a + (n - 1) \times d]$
Sum of all terms in a finite AP with the last term as 'l'	n/2(a + l)

Arithmetic Progressions Solved Examples

Below are the problems to find the nth term and the sum of the sequence, which are solved using AP sum formulas in detail. Go through them once and solve the practice problems to excel in your skills.

Example 1: Find the value of n, if a = 10, d = 5, $a_n = 95$.

Solution: Given, a = 10, d = 5, $a_n = 95$

From the formula of general term, we have:

$a_n = a + (n - 1) \times d$

$95 = 10 + (n - 1) \times 5$

$(n - 1) \times 5 = 95 - 10 = 85$

$(n - 1) = 85/ 5$

$(n - 1) = 17$

$n = 17 + 1$

$n = 18$

Example 2: Find the 20th term for the given AP:3, 5, 7, 9, ……

Solution: Given,

3, 5, 7, 9, ……

$a = 3, d = 5 - 3 = 2, n = 20$

$a_n = a + (n - 1) \times d$

$a_{20} = 3 + (20 - 1) \times 2$

$a_{20} = 3 + 38$

$\Rightarrow a_{20} = 41$

Example 3: Find the sum of the first 30 multiples of 4.

Solution:

The first 30 multiples of 4 are: 4, 8, 12, ….., 120

Here, $a = 4, n = 30, d = 4$

We know,

$S_{30} = n/2 \, [2a + (n - 1) \times d]$

$S_{30} = 30/2[2 \, (4) + (30 - 1) \times 4]$

$S_{30} = 15[8 + 116]$

$S_{30} = 1860$

Practice Problems on AP

Question 1: Find the a_n and 10th term of the progression: 3, 1, 17, 24, ……

Question 2: If $a = 2, d = 3$ and $n = 90$. Find a_n and S_n.

Question 3: The 7th term and 10th terms of an AP are 12 and 25. Find the 12th term.