Practical-3.1

Order, Degree of Differential equations and Exact equations

Objective Questions

1. The order of differential equation is always
   (a) Positive integer (b) Negative integer
   (c) Rational number (d) Whole number.

2. The order and degree of the differential equation $\frac{d^2y}{dx^2} + 5xy\frac{dy}{dx} = 6x^2$ is
   (a) 2 and 1 (b) 2 and 2
   (c) 1 and 2 (d) 1 and 1.

3. The order and degree of the differential equation $8\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 5\int y\,dx = \frac{7x^2}{2}$ is
   (a) 2 and 1 (b) 2 and 2
   (c) 1 and 2 (d) 3 and 1.

4. The order and degree of the differential equation $y = x\frac{dy}{dx} + \left(5/\frac{dy}{dx}\right)$ is
   (a) 2 and 1 (b) 2 and 2
   (c) 1 and 2 (d) 1 and 1.

5. The order and degree of the differential equation $y = x\frac{dy}{dx} + 3\sqrt{1 + \left(\frac{dy}{dx}\right)^4}$ is
   (a) 4 and 4 (b) 2 and 4
   (c) 1 and 4 (d) 4 and 1.

6. The order and degree of the differential equation $y = x\frac{dy}{dx} + 5\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is
   (a) 2 and 1 (b) 2 and 2
   (c) 1 and 2 (d) 1 and 1.

7. The order and degree of the differential equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + \int y\,dx = e^{2x}$ is
   (a) 2 and 1 (b) 2 and 2
   (c) 1 and 2 (d) 3 and 1.

8. The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + 3\frac{dy}{dx} + 7y = 0$ is
   (a) 2 and 3 (b) 2 and 2
   (c) 3 and 2 (d) 3 and 3.

9. The function $f(x, y) = 4x^2 - 7xy + \frac{x^3}{y}\tan\frac{y}{x}$ is homogenous function of degree ____ in *x*, *y*.
   (a) one (b) two
   (c) three (d) four.

10. The solution of the equation $\frac{d^2z}{dt^2}+5\frac{dz}{dt}+6z=0$, where a and b are arbitrary constants is
(a) $z=ae^{-3t}-be^{-2t}$ (b) $z=ae^{-3t}+be^{-2t}$
(c) $z=ae^{3t}-be^{2t}$ (d) $z=ae^{3t}+be^{2t}$

11. The solution of the equation $\frac{d^2y}{dx^2}=4y$, where a and b are arbitrary constants is
(a) $y=ae^{-2x}+be^{2x}$ (b) $y=ae^{-3x}+be^{-2x}$
(c) $y=ae^{-2x}-be^{2x}$ (d) $y=ae^{2x}+be^{2x}$

12. The necessary and sufficient condition for the equation $M\,dx+N\,dy=0$, where M and N are functions of x and y, to be exact is
(a) $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ (b) $\frac{\partial M}{\partial x}=\frac{\partial N}{\partial y}$
(c) $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial y}$ (d) $\frac{\partial M}{\partial x}=\frac{\partial N}{\partial x}$

13. If $M\,dx+N\,dy=0$ is a homogenous differential equation and if $Mx+Ny\neq 0$, then the integrating factor of the equation is
(a) $\frac{1}{Mx+Ny}$ (b) $\frac{1}{Mx-Ny}$
(c) $e^{\int f(x)dx}$ (d) $e^{\int f(y)dy}$

14. If $M\,dx+N\,dy=0$ isof the type $f_1(xy)ydx+f_2(xy)xdy=0$ and if $Mx-Ny\neq 0$, then the integrating factor of the equation is
(a) $\frac{1}{Mx+Ny}$ (b) $\frac{1}{Mx-Ny}$
(c) $e^{\int f(x)dx}$ (d) $e^{\int f(y)dy}$

15. If $\frac{1}{N}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)$ is a function of x alone, say, $f(x)$, then the integrating factor of the equation $M\,dx+N\,dy=0$ is
(a) $\frac{1}{Mx+Ny}$ (b) $\frac{1}{Mx-Ny}$
(c) $e^{\int f(x)dx}$ (d) $e^{\int f(y)dy}$

16. If $\frac{1}{M}\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)$ is a function of y alone, say, $f(y)$, then the integrating factor of the equation $M\,dx+N\,dy=0$ if
(a) $\frac{1}{Mx+Ny}$ (b) $\frac{1}{Mx-Ny}$
(c) $e^{\int f(x)dx}$ (d) $e^{\int f(y)dy}$

17. Which of the following equations is an exact Differential equation?
(a) $(x^2+1)dx-xydy=0$ (b) $xdy+(3x-2y)dx=0$
(c) $2xydx+(2+x^2)dy=0$ (d) $x^2ydy-ydx=0$

18. Which of the following equations is a variable separable Differential equation?
(a) $(x + x^2y)dy = (2x + xy^2)dx$ (b) $(x + y)dx - 2ydy = 0$
(c) $2ydx = (x^2 + 1)dy$ (d) $y^2dx + (2x - 3y)dy = 0$

19. The order of the differential equation of all circles of given radius *a* is
(a) 1 (b) 2
(c) 3 (d) 4

20. The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represents a family of
(a) Straight lines (b) Circles
(c) Parabolas (d) Ellipses.

21. Which of the following is not a homogeneous function of *x* and *y*?
(a) $x^2 + 2xy$ (b) $2x - y$
(c) $cos^2\left(\frac{y}{x}\right) + \frac{y}{x}$ (d) $\sin x - \cos y$

22. Solution of the differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ is
(a) $\frac{1}{x} + \frac{1}{y} = 0$ (b) $\log x . \log y = c$
(c) $xy = c$ (d) $x + y = c$

23. A differential equation is considered to be ordinary if it has
(a) one dependent variable (b) more than one dependent variable
(c) one independent variable (d) more than one independent variable

Descriptive Questions

1. Solve the following equations by variable separable form.

i. $\frac{dy}{dx} + \sqrt{\frac{1 - y^2}{1 - x^2}} = 0$

ii. $sec^2x \tan y \, dx + sec^2y \tan x \, dy = 0$

iii. $x \, cos^2 \, y \, dx = ycos^2x \, dy = 0$

iv. $(e^y + 1) \cos x \, dx + e^y \sin x \, dy = 0$

v. $(\sin x + \cos x)dy + (\cos x - \sin x)dx = 0$

vi. $y \, sec^2x \, dx + (y + 7) \tan x \, dy = 0$

vii. $(x^2 - yx^2)dy + (y^2 + xy^2)dx = 0$

viii. $(1 - x^2)(1 - y)dx = xy(1 + y)dy$

ix. $(2x + 3y - 5)dy + (3x + 2y - 5)dx = 0$

x. $(x + y)dy + (x - y)dx = 0$

xi. $(y + yx^2)dy - (2x - 2xy^2)dx = 0$

xii. $(x-y)^2 \frac{dy}{dx} = a^2$

xiii. $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$

xiv. $(x+y)^2\left(x\frac{dy}{dx} + y\right) = xy\left(1 + \frac{dy}{dx}\right)$

xv. $(x^3 + y^3)dx - 3xy^2dy = 0.$

xvi. $(3y - 7x + 7)dx + (7y - 3x + 3)dy = 0$

2. Determine whether the following differential equations are exact or not. If exact then solve it, otherwise find the integrating factor and then solve it.

i. $\frac{dy}{dx} = \frac{y+1}{(y+2)e^y - x}$

ii. $\left[\frac{y}{(x-y)^2} - \frac{1}{2\sqrt{1-x^2}}\right]dx - \frac{x}{(x-y)^2}dy = 0$

iii. $\cos y - x\sin y\ \frac{dy}{dx} = sec^2x.$

iv. $(2x - y + 1)dx + (2y - x - 1)dy = 0$

v. Find *r* if x^r is an Integrating factor of the equation $(x + y^3)dx + 6xy^2dy = 0$ and hence solve it.

vi. $x^2y\ dx - (x^3 + y^3)dy = 0$

vii. $(xy + 2x^2y^2)y\ dx + (xy - x^2y^2)x\ dy = 0$

viii. $(x - y^2)dx + 2xy\ dy = 0$

ix. $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$

x. $(2y\ dx + 3x\ dy) + 2xy(3y\ dx + 4x\ dy) = 0$

xi. $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$

xii. $(3xy^2 - y^3)dx - (2x^2y - xy^2)dy = 0$

xiii. $(xy\sin xy)y\ dx + (xy\sin \mathrm{xy} - \cos xy)x\ dy = 0$

xiv. $(x^2 + y^2 + 2x)dx + 2y\ dy = 0$

xv. $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$

xvi. $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$

Practical-3.2

First order linear differential equations & applications to orthogonal trajectories

Objective Questions

1. Which is the linear differential equation?
   (a) $\frac{dy}{dx} + ycosx = sinx$ (b) $\frac{d^4y}{dx^4} = (k + (y')^2)^{3/2}$
   (c) $\frac{d^4y}{dx^4} = cos\frac{dy}{dx}$ (d) None of these.

2. The differential equation $2\frac{dy}{dx} + x^2y = 2x + 3, y(0) = 5$ is
   (a) linear (b) nonlinear
   (c) linear with fixed constants (d) undeterminable to be linear or nonlinear

3. Radium decomposes at a rate proportional to the amount at any instant. In 100 years, 100 mg of radium decomposes to 96 mg. How many mg will be left after 100 years?
   (a) 88.60 (b) 95.32
   (c) 92.16 (d) 90.72.

4. Radium decomposes at a rate proportional to the amount present. If the half of the original amount disappears after 1000 years, what is the percentage lost in 100 years?
   (a) 6.70% (b) 4.50%
   (c) 5.35% (d) 4.30%

5. The population of a country doubles in 50 years. How many years will it be five times as much? Assume that the rate of increase is proportional to the number of inhabitants.
   (a) 100 years (b) 116 years
   (c) 120 years (d) 98 years

6. An object falls from rest in a medium offering a resistance. The velocity of the object before the object reaches the ground is given by the differential equation $dV / dt + V / 10 =$ 32, ft/sec. What is the velocity of the object one second after if falls?
   (a) 40.54 ft/sec (b) 38.65 ft/sec
   (c) 30.45 ft/sec (d) 34.12 ft/sec

7. According to Newton's law of cooling, the rate at which a substance cools in air is directly proportional to the difference between the temperatures of the substance and that of air. If the temperature of the air is 30° and the substance cools from 100° to 70° in 15 minutes, how long will it take to cool from 100° to 50°?
   (a) 33. 59 min (b) 43.60 min
   (c) 35.39 min (d) 45.30 min

8. Find the equation of the family of orthogonal trajectories of the system of parabolas$y^2 = 2x + C$.
   (a) $y = ce^{-x}$ (b) $y = ce^{2x}$
   (c) $y = ce^{x}$ (d) $y = ce^{-2x}$

1. Solve the following first order differential equations.

i. $1+x^2\frac{dy}{dx}+y=e^{tan^{-1}x}$

ii. $(y-2\log x)\ dx+x\log x\ dy=0$

iii. $\frac{dy}{dx}+xsin2y=x^3cos^2y$

iv. $(x+2y^3)\frac{dy}{dx}=y$

v. $dx+xdy=e^{-y}logy\ dy$

vi. $(3y^2-x)\frac{dy}{dx}=y$

vii. $\frac{dy}{dx}+\frac{y}{x}+x^2=0$

viii. $\frac{1}{1+x^2}\frac{dy}{dx}+\frac{y}{x(1+x^2)}=4$

ix. $\frac{dy}{dx}(1+x^2)+2xy-4x^2=0$

x. $sinx\frac{dy}{dx}+2y\ cosx=cosx$

xi. $\frac{dy}{dx}+2ytanx=sinx$

xii. $\frac{dy}{dx}-\frac{2y}{x}=-\frac{1}{x^2e^{1/x^2}}$

xiii. $x\log x\frac{dy}{dx}+y=2\log x$

xiv. $(x^2-x)\frac{dy}{dx}+(1-2x)y+x^2=0$

xv. $\frac{dy}{dx}-\frac{2}{x+1}y=(x+1)^3$

2. Solve the following:

i. Find the orthogonal trajectories of the series of hyperbolas $xy=k^2$.

ii. Find the orthogonal trajectories of the series of parabolas whose equation is $y^2=4ax$.

iii. Find the orthogonal trajectories of astroids$x^{2/3}+y^{2/3}=a^{2/3}$.

iv. Find the orthogonal trajectories of $\frac{x^2}{a^2}+\frac{y^2}{a^2+\lambda}=1$, where λ is a parameter.

v. Find the orthogonal trajectories of the family of coaxial circles $x^2+y^2+2gx+c=0$, where g is a parameter and c constant.

vi. Find the orthogonal trajectories of the family of semi cubical parabolas $ay^2=x^3$.

vii. Prove that the system of confocal and coaxial parabolas $y^2=4a(x+a)$ is self orthogonal.

viii. Prove that the system of confocal conics $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$ is self-orthogonal.

ix. Find the orthogonal trajectories of the cardioid $r = a(1 - cos\theta)$, where a is a parameter.

x. Find the orthogonal trajectories of the cardioid $r = a(1 + cos\theta)$, where a is a parameter.

xi. Find the orthogonal trajectories of the system of curves $r^n = a^n \cos n\theta,$ where a is a parameter.

xii. Find the orthogonal trajectories of the system of curves $r^n = a^n \sin n\theta,$ where a is a parameter.

xiii Find the orthogonal trajectories of the system of curves $r^n \sin n\theta = a^n,$ where a is a parameter.

xiv. Find the orthogonal trajectories of the system of curves $r^n \cos n\theta = a^n,$ where a is a parameter.

xv. Find the orthogonal trajectories of the parabolas $\frac{l}{r} = 1 + cos\theta$, where l is a parameter.

xvi. Find the orthogonal trajectories of the series of curves $r = a + \sin 5\theta.$

xvii Find the equation of the system of orthogonal trajectories of a series of confocal and coaxial parabolas $r = \frac{2a}{1+cos\theta}$.

Practical 3.3

General solution of Second order homogeneous and non-homogeneous equations and Wronskian

Objective questions

1. Which of the following functions are linearly independent to each other?
   (a) $y_1 = 2\sin^2 x\,, y_2 = 1 - \cos^2 x$ (c) $y_1 = \log x\,, y_2 = \log x^3$
   (b) $y_1 = x, y_2 = x^2$ (d) $y_1 = x^2, y_2 = 5x^2$
2. General solution of $y'' - 6y' + 4y = 0$ is
   (a) $y = e^{3x}(c_1 \cos\sqrt{5}\,x + c_2 \sin\sqrt{5}\,x)$ (c) $y = c_1 e^{(3+\sqrt{5})x} + c_2 e^{(3-\sqrt{5})x}$
   (b) $y = e^{3x}(c_1 \cos 5x + c_2 \sin 5x)$ (d) None of these
3. Which of the following functions are linearly dependent to each other?
   (a) $y_1 = e^x, y_2 = xe^x$ (c) $y_1 = e^{3x}, y_2 = e^{5x}$
   (b) $y_1 = 2\sin^2 x\,, y_2 = 1 - \cos^2 x$ (d) $y_1 = \cos x\,, y_2 = \sin x$
4. General solution of $y'' + y' - 6y = 0$ is
   (a) $y = c_1 e^{3x} + c_2 e^{2x}$ (c) $y = c_1 e^{-3x} + c_2 e^{-2x}$
   (b) $y = c_1 e^{-3x} + c_2 e^{2x}$ (d) $y = c_1 e^{3x} + c_2 e^{-2x}$
5. General solution of $3y'' = 5y'$ is
   (a) $y = c_1 x + c_2 e^{\frac{5x}{3}}$ (c) $y = c_1 + c_2 e^{5x/3}$
   (b) $y = (c_1 + c_2 x)e^{5x/3}$ (d) None of these
6. For what value of n, the equation $(x + ye^{2xy})dx + nxe^{2xy}dy = 0$ is exact?
   (a) 1 (b) 2 (c) 3 (d) 4
7. If $y_1 = x^2$ is a solution of $x^2y'' + xy' - y = 0$, then the other linearly independent solution is
   (a) $y_2 = 1/x^2$ (c) $y_2 = x$
   (b) $y_2 = 1/\,x$ (d) None of these
8. If $y_1 = e^{x^2}$ is a solution $y'' - 4xy' + (4x^2 - 2)y = 0$, then the other linearly independent solution is
   (a) $y_2 = e^{x^{-2}}$ (c) $y_2 = xe^{x^2}$
   (b) $y_2 = xe^{x^{-2}}$ (d) None of these
9. Wronskian determinant $W(y_1, y_2)$ with usual symbols is equal to
   (a) $y_1y_2' - y_2y_1'$ (c) $y_1y_2' + y_2{y_1}'$
   (b) $y_1y_1' - y_2{y_2}'$ (d) $y_1y_1' + y_2{y_2}'$
10. Wronskian of $y_1 = \cos 2x$ and $y_2 = \sin 2x$ is
    (a) 0 (b) 1 (c) 2 (d) None of these
11. Which of the following are two linearly independent solutions of $y'' + 3y' + 2y = 0$?
    (a) $y_1 = e^x, y_2 = e^{2x}$ (c) $y_1 = e^{-x}, y_2 = e^{-2x}$
    (b) $y_1 = e^{3x}, y_2 = e^{2x}$ (d) None of these
12. Which of the following are two linearly independent solutions of $y'' - 2y' - 2y = 0$?
    (a) $y_1 = e^x \cos x\,,\ y_2 = e^x \sin x$ (c) $y_1 = e^x \cos 2x\,, y_2 = e^x \sin 2x$
    (b) $y_1 = e^{2x} cosx\,,\ y_2 = e^{2x} \sin x$ (d) None of these.

13. If $y_1(x)$ and $y_2(x)$ are two independent solutions for the equation $y'' + P(x)y' + Q(x)y = 0$ then
    (a) $y_1(x) = k\, y_2(x)$ where k is a constant.
    (b) $y_1(x)y_2(x) = k$ where k is a constant.
    (c) $y_2(x) = u(x)y_1(x)$, $u(x)$ is not constant.
    (d) None of these.
14. If $y_1 = e^{bx}$ and $y_2 = e^{-bx}$ are linearly independent then
    (a) b is any real number
    (b) $b \neq 2$
    (c) $b \neq 1$
    (d) $b \neq 0$

Descriptive Questions

1. By eliminating the constants c_1 and c_2, find the differential equation of the following family of curves:
   (a) $y = c_1x + c_2x^2$
   (b) $y = c_1x + c_2x^{-1}$
   (c) $y = c_1 + c_2e^{-2x}$
   (d) $y = c_1x + c_2 \sin x$
   (e) $y = c_1 \sin kx + c_2 \cos kx$
   (f) $y = c_1e^{kx} + c_2e^{-kx}$
2. Show that $y(x) = ax^2 + bx + 3$ is solution for the equation $x^2y'' - 2xy' + 2y = 6$, hence find particular solution,if $y(1) = 0$, $y'(1) = 1$
3. Show that $y = c_1x + c_2x^2$ is the general solution of $x^2y'' - 2xy' + 2y = 0$ on any interval not containing 0, and find the particular solution for which $y(1) = 3$ and $y'(1) = 5$.
4. Show that $y = c_1e^x + c_2e^{2x}$ is the general solution of $y'' - 3y' + 2y = 0$ on any interval, find the particular solution for which $y(0) = -1$ and $y'(0) = 1$.
5. Using Wronskian determinant method, determine whether the following functions are linearly dependent or independent:
   (a) $y_1 = e^{2x} \sin 3x$ and $y_2 = e^{2x} \cos 3x$
   (b) $y_1 = x^4$ and $y_2 = x^4 \log x$
   (c) $y_1 = x^2$ and $y_2 = \sqrt{x}\,, x \neq 0$
6. Verify that y_1 is one solution of the following equation and find another linearly independent solution y_2 and the general solution:
   (a) $x^2y'' + xy' - 4y = 0,$ $\quad y_1 = x^2$
   (b) $\sin^2 x\, y'' - 2y = 0,$ $\quad y_1 = \cot x$
   (c) $x^2y'' - xy' = 0,$ $\quad y_1 = x^2$
   (d) $(1 + x^2)y'' - 2xy' + 2y = 0,$ $\quad y_1 = x$
   (e) $x^2y'' - 7xy' + 15y = 0,$ $\quad y_1 = x^3$ on $(0, \infty)$
   (f) $x^2y'' + xy' - 9y = 0,$ $\quad y_1 = x^3$
   (g) $xy'' - (2x + 1)y' + (x + 1)y = 0,$ $\quad y_1 = e^x$

7. Find two linearly independent solutions of the differential equation $x^2y'' - 2y = 0$ using $y = x^n$as a solution. Hence determine the particular solution satisfying the initial conditions $y(1) = 1,\ y'(1) = 8$.
8. Find the general solution of each of the following equations:
   (a) $y'' + y' - 6y = 0$
   (b) $y" + 2\,y' +\ y = 0$
   (c) $y" + 8y = 0$
   (d) $y" - 4y' - 4y = 0$
   (e) $4y" - 12y' + 9y = 0$
   (f) $y" = 4y$
   (g) $4y" - 8y' + 7y = 0$
   (h) $2y" + y' - y = 0$
9. Find the solution of the following initial value problems:
   (a) $y'' - 5y' + 6y = 0,$ $y(1) = e^2$and $y'(1) = 3e^2$.
   (b) $y" - 6y' + 9y = 0,$ $y(0) = 0$ and $y'(0) = 5$
   (c) $y" + 4y' + 5y = 0,$ $y(0) = 1$ and $y'(0) = 0$
   (d) $y'' + y = 0,$ $y(0) = 2$ and $y'(0) = 3$
   (e) $y'' + 4y' + 2y = 0,$ $y(0) = -1$ and $y'(0) = 2 + 3\sqrt{2}$
10. Solve the following equations using substitution $x = e^z$:
   (a) $x^2y'' + 3xy' + 10y = 0$
   (b) $x^2y'' - 3xy' + 4y = 0$
   (c) $x^2y'' + 2xy' - 6y = 0$
   (d) $x^2y'' + xy' - 16y = 0$

Practical 3.4

Method of Undetermined Coefficients and Method of Variation of Parameters

Objective questions

1. For the differential equation $y'' - 9\,y' + 14\,y = \cos 4x$, trial function for calculation of particular integral is
   (a) $A\,e^{4x}$ (b) $A\cos 4x$ (c)$A\sin 4x$ (d) $A\sin 4x + B\cos 4x$
2. Trial function for calculating particular integral to differential equation $y'' + 4\,y = x$ is
   (a) $Ax + B$ (c)$A\,e^{x} + B$
   (b) $A\sin x + B\cos x$ (d) $A\,x^2 + Bx + c$
3. If $R = 0$, for differential equation $y'' + P(x)y' + Q(x)y = R$, differential equation is called
   (a) Partial (c) Non-homogenous
   (b) Homogeneous (d) None of these
4. Trial function for calculating particular integral of the differential equation $y'' - 5\,y' + 6y = e^{4x}$ is $y =$
   (a) $A\,e^{2x} + Be^{3x}$ (c)$A\,e^{-4x}$
   (b) Ae^{4x} (d) $Ae^{-5x} + Be^{6x}$
5. If $y = A\sin 2x + B\cos 2x$ is the trail solution of $y'' - 5y' + 6y = \sin 2x$, then
   (a) $A = \frac{1}{52}, B = -\frac{5}{52}$
   (b) $A = \frac{1}{52}, B = \frac{5}{52}$
   (c) $A = -\frac{1}{52}, B = \frac{5}{52}$
   (d) $A = 1, B = 5$
6. Trial function for calculating particular integral of the differential equation$y'' - 2y' + y = e^{x} + 1$ is
   (a) $Ae^{x} + B$ (c)$Ae^{x} + Bxe^{x}$
   (b) Ae^{x} (d) $Ax^2e^{x} + B$
7. The undetermined coefficients in the trial solution of $y'' - 4y' - 12y = xe^{5x}$ are
   (a) $A = -\frac{1}{7}, B = -\frac{6}{7}$ (c)$A = \frac{1}{7}, B = -\frac{6}{49}$
   (b) $A = -\frac{1}{7}, B = -\frac{6}{49}$ (d) $A = \frac{1}{7}, B = \frac{6}{49}$
8. Particular solution of $y'' + y = \tan x$ is
   (a) $y_p = -\cos x\ \log(\sec x + \tan x)$
   (b) $y_p = -\sin x\ \log(\sec x + \tan x)$
   (c) $y_p = -\cos x\ \log(\sec x)$
   (d) $y_p = -\cos x\ \log(\sec x - \tan x)$
9. Using variation of parameters method of finding particular integral $y_p = v_1y_1 + v_2y_2$ (with usual notations) for the differential equation $y'' - y = \frac{2}{1+e^{x}}$, we get$v_1 =$
   (a) $-e^{-x} + \log(e^{-x} + 1)$ (c)$e^{-x} + \log(e^{-x} + 1)$
   (b) $\log(1 + e^{x})$ (d) None of these

10. Using variation of parameters method of finding particular integral $y_p = v_1 y_1 + v_2 y_2$ (with usual notations) for the differential equation $y'' + y = \sec x$, we get $v_2 =$
    (a) x (b) $\log \sec x$ (c) $\log \cos x$ (d) x^2
11. Using variation of parameters method of finding particular integral $y_p = v_1 y_1 + v_2 y_2$ (with usual notations) for the differential equation $y'' - 4y = e^{2x}$, we get $v_1 =$
    (a) $\frac{x}{4}$ (b) $-\frac{x}{4}$ (c) x (d) $\frac{e^{4x}}{16}$
12. Using variation of parameters method of finding particular integral $y_p = v_1 y_1 + v_2 y_2$ (with usual notations) for the differential equation $y'' - 4y' + 4y = \frac{e^{2x}}{x}$, we get $v_1 =$
    (a) x (b) $-x$ (c) $\log x$ (d) x^2
13. Using variation of parameters method of finding particular integral $y_p = v_1 y_1 + v_2 y_2$ (with usual notations) for the differential equation $y'' + y = \sec x$, we get $v_1 =$
    (a) x (b) $\log \sec x$ (c) $\log \cos x$ (d) x^2
14. Using variation of parameters method of finding particular integral $y_p = v_1 y_1 + v_2 y_2$ (with usual notations) for the differential equation $y'' - y = e^x \sin x$, we get $v_1 =$
    (a) $-\frac{1}{2}\sin x$ (b) $-\frac{1}{2}\cos x$ (c) $\frac{1}{2}\cos x$ (d) None of these

Descriptive questions

1. Solve the following non-homogenous differential equations by method of undetermined coefficients:
   (a) $y'' - 5y' + 6y = 10 \sin x$
   (b) $y'' + 5y' + 6y = e^{4x}$
   (c) $y'' - y' - 2y = 4x^2$
   (d) $y'' - 6y' + 9\,y = \cos 2x$
   (e) $y'' + 4y' + 5\,y = x + 12$
   (f) $y'' - y = \sin x - \cos x$
   (g) $y'' + 10y' + 25y = 14e^{-5x}$
   (h) $y'' - 2y' + 5y = 25x^2 + 12$
   (i) $y'' - 2y' + 2y = e^x \sin x$
   (j) $y'' + y' = 10x^4 + 2$
2. Using method of variation of parameter, solve the following differential equations:
   (a) $y'' + y = \csc x$
   (b) $y'' - 5y' + 6y = xe^{4x}$
   (c) $y'' + 4y = \sin x$
   (d) $y'' - 6y' + 9y = e^{3x}x^{-2}$
   (e) $y'' + y = \sec x \csc x$
   (f) $y'' + y = \sec x$
   (g) $y'' + 2y' + y = e^{-x} \log x$
   (h) $2y'' + 3y' + y = e^{-3x}$
   (i) $y'' + y = \cot^2 x$
   (j) $y'' + 3y' + 2y = (1 + e^{-x})^{-1}$

Practical 3.5
Second Order Linear Differential Equations

Given one solution find another linearly independent solution for the following differential equations:

1. $y_1 = \frac{x}{(x-1)^2}$‘ $x(x - 1)\, y'' + 3xy' + y = 0$
2. $y_1 = \mathrm{x} + \frac{1}{\mathrm{x}}$ ‘ $x^2y'' + xy' - y = 0.$
3. $y_1 = \frac{\mathrm{sinx}}{\sqrt{\mathrm{x}}},$ $x^2y'' + 4xy' + (4x^2 - 1)y = 0.$

Solve the following second order linear Differential equations. Mentions the methods used **(Here D=d/dx)**

1. $(D^2 + 4)y = e^{2\,x} \sin 3x.$
2. $(D^2 + 9)y = e^{3\,x} + \cos 3x$
3. $(D^2 - 2D + 1)y = x^2e^{3x}$
4. $(D^2 + 4)y = \sin 3x + e^x + x^2$
5. $(D^2 + 9)y = e^{3x} + \cos 4x$
6. $(D^2 - 1)\, y = 3x^4$
7. $(D^2 + D - 2)y = e^x$
8. $(D^2 - 2D + 1)y = x^2e^{3x}$
9. $(D^2 - 1)y = 8xe^x$
10. $(D^2 - 2D + 4)y = e^x \cos x$
11. $(D^2 + 4D - 12)y = e^{2x}(x - 1)$
12. $(D^2 - 2D + 4)y = e^x \sin x$
13. $(D^2 - 1)y = e^x \sin 3x$
14. $(D^2 + 2D + 4)y = e^x\ \sin 2x$
15. $(D^2 - 4D + 3)y = e^x \cos 2x + \cos 3x$
16. $(D^2 + 2D + 1)y = x \sin x$
17. $(D^2 + 4)y = x \sin x$
18. $(D^2 + 3D + 2)y = x \cos 2x$
19. $(D^2 - 2D + 1)y = xe^x \sin x$
20. $(D^2 +\ D)y = x + \cos x.$
21. $(D^2 - 3D + 2)\, y = e^{5\,x}$
22. $(D^2 + 4)y = e^x \sin x$
23. $(\mathrm{D}^2 - 7\mathrm{D} + 12\mathrm{y}) = \sin 2\mathrm{x}$
24. $(\mathrm{D}^2 + 1)\mathrm{y} = \mathrm{x}^2 + 1$
25. $(D^2 + 2D - 1)\, y = 2cos\, x + 3sin\, x$

Solve the following second order linear Differential equations. Mention the method used

1. $y'' - 2y' + 3y = \frac{e^x}{e^x+1}$
2. $\mathrm{y}'' - 2\mathrm{y}' + \mathrm{y} = \frac{e^x}{x}$
3. $\mathrm{y}'' - 2\mathrm{y}' + \mathrm{y} = \frac{e^x}{x^2}$

4. $y'' + 4y' + 5y = x + 2$
5. $y'' - 2y' + y = \frac{e^x}{x^3}$
6. $y'' - y' - 2y = 4x^2$
7. $y'' + 25y = 2\sin 2x$
8. $y'' + y' = x$
9. $y'' + y = \tan x$
10. $y'' + y = \sin x$
11. $y'' - 2y' + y = x\sin x$
12. $y'' + 4y = xsin2x$
13. $y'' + 2y = xcosx$
14. $y'' - 6y' + 9y = \frac{e^{3x}}{x}.$
15. $y'' + 9y' = secx$
16. $y'' + 2y' + y = e^x$
17. $y'' + 2y + y = e^{-x}$
18. $y'' - y = x$
19. $y'' - y = e^x$
20. $y'' - y = \sin\ x$

Find the particular solution by using the method of undetermined coefficients

1. $x'' + x' = 2t + 3$
2. $x' - 7x' = (3 - 36t)e^{4t}$
3. $x'' + 25x = 2\sin\ 2t$
4. $x'' + x = t\cos\ t.$

Practical 3.6

Solving a system of first order linear Ordinary Differential Equations

Objective Questions

1. The Wronskian of two solutions $(x_1(t), y_1(t))$ &$(x_2(t), y_2(t))$ for the linear system of first order homogeneous differential equations is

(a) $\begin{vmatrix} x_1(t) & y_1(t) \\ x_1'(t) & y_1'(t) \end{vmatrix} \times \begin{vmatrix} x_2(t) & y_2(t) \\ x_2'(t) & y_2'(t) \end{vmatrix}$ (b) $\begin{vmatrix} x_1(t) & y_1(t) \\ x_1'(t) & y_1'(t) \end{vmatrix} + \begin{vmatrix} x_2(t) & y_2(t) \\ x_2'(t) & y_2'(t) \end{vmatrix}$

(c) $\begin{vmatrix} x_1(t) & y_1(t) \\ x_2(t) & y_2(t) \end{vmatrix}$ (d) None of these

2. The auxiliary equation of the following linear system of homogeneous differential equations $\frac{dx}{dt} = a_1x + b_1y$ and $\frac{dy}{dt} = a_2x + b_2y$ is
(a) $m^2 - (a_1 + b_2)\, m \; + a_1b_2 - a_2b_1 = 0$
(b) $m^2 - (a_2 + b_1)m + a_1b_2 - a_2b_1 = 0$
(c) $m^2 + (a_1 + b_2)m \; + a_1b_2 - a_2b_1 = 0$
(d) $m^2 - (a_2 + b_1)m + a_2b_1 - a_1b_2 = 0$

3. The auxiliary equation of the linear system of homogeneous differential equations $\frac{dx}{dt} = 3x + 2y$ and $\frac{dy}{dt} = -5x + y$ has
(a) real and distinct roots (b) roots which are complex conjugates
(c) real and repeated roots (d) no roots

4. The auxiliary equation of the linear system of homogeneous differential equations $\frac{dx}{dt} = 5x + 4y$ and $\frac{dy}{dt} = -x + y$ has
(a) real and distinct roots (b) roots which are complex conjugates
(c) real and repeated roots (d) no roots

5. $(e^{4t}, e^{4t})$and$(e^{-2t}, -e^{-2t})$ are linearly independent solutions of $\frac{dx}{dt} = x + 3y$ &$\frac{dy}{dt} = 3x + y$ then particular solution for which $x(0) = 5, y(0) = 1$ is
(a)$(3e^{4t} + 2e^{4t}, 3e^{-2t} - 2e^{-2t})$ (b) $(4e^{4t} + e^{4t}, 4e^{-2t} - 3e^{-2t})$
(c) $(3e^{4t} + 2e^{-2t}, -2e^{4t} + e^{-2t})$ (d) $(3e^{4t} + 2e^{-2t}, 3e^{4t} - 2e^{-2t})$

6. The Wronskian of two solutions $\begin{cases} x = e^{2t} \\ y = 2e^{2t} \end{cases}$ and $\begin{cases} x = e^{3t} \\ y = 2e^{3t} \end{cases}$, of a homogeneous linear system of differential equations, is equal to
(a) $4e^{5t}$ (b) $2e^{2t}$ (c) 0 (d) None of these

7. Amongst the following, the pair of linearly independent solutions is

(a) $\begin{cases} x = e^t \\ y = e^t \end{cases}$ and $\begin{cases} x = 2e^{3t} \\ y = e^{3t} \end{cases}$ (b) $\begin{cases} x = -e^{3t} \\ y = -e^{2t} \end{cases}$ and $\begin{cases} x = e^{3t} \\ y = e^{2t} \end{cases}$

(c) $\begin{cases} x = e^t \\ y = -e^{3t} \end{cases}$ and $\begin{cases} x = -e^{3t} \\ y = e^{3t} \end{cases}$ (d) None of these

8. One of the solutions of the homogeneous linear system of differential equations $\begin{cases} \dfrac{dx}{dt} = 2y - 4x \\ \dfrac{dy}{dt} = 3x - 3y \end{cases}$ is

(a) $\begin{cases} x = 3e^{2t} \\ y = e^{2t} \end{cases}$ (b) $\begin{cases} x = 2e^{-t} \\ y = 3e^{-t} \end{cases}$ (c) $\begin{cases} x = e^{t} \\ y = 3e^{t} \end{cases}$ (d) None of these

Descriptive questions

1. Show that both $\begin{cases} x = e^{3t} \\ y = e^{3t} \end{cases}$ and $\begin{cases} x = e^{2t} \\ y = 2e^{2t} \end{cases}$ are solutions of the system $\begin{cases} \dfrac{dx}{dt} = 4x - y \\ \dfrac{dy}{dt} = 2x + y \end{cases}$.

   Hence or otherwise, write general solution of the system.

2. Show that both $\begin{cases} x = e^{4t} \\ y = e^{4t} \end{cases}$ and $\begin{cases} x = e^{-2t} \\ y = -e^{-2t} \end{cases}$ are solutions of the system $\begin{cases} \dfrac{dx}{dt} = x + 3y \\ \dfrac{dy}{dt} = 3x + y \end{cases}$.

   Show in two ways that these solutions are linearly independent solutions on every closed and bounded interval.

3. Show that $\begin{cases} x = 3t - 2 \\ y = -2t + 3 \end{cases}$ is a particular solution of the non-homogeneous system $\begin{cases} \dfrac{dx}{dt} = x + 2y + t - 1 \\ \dfrac{dy}{dt} = 3x + 2y - 5t - 2 \end{cases}$. Find its general solution, given that $\begin{cases} x = 2e^{4t} \\ y = 3e^{4t} \end{cases}$ and $\begin{cases} x = e^{-t} \\ y = -e^{-t} \end{cases}$ are solutions of the corresponding homogeneous system.

4. By using the method of solving Homogeneous Linear system with constant coefficients, solve the following system: $\begin{cases} \dfrac{dx}{dt} = -3x + 4y \\ \dfrac{dy}{dt} = -2x + 3y \end{cases}$

5. By using the method of solving Homogeneous Linear system with constant coefficients, solve the following system: $\begin{cases} \dfrac{dx}{dt} = 4x - 2y \\ \dfrac{dy}{dt} = 5x + 2y \end{cases}$

6. By using the method of solving Homogeneous Linear system with constant coefficients, solve the following system: $\begin{cases} \dfrac{dx}{dt} = 5x + 4y \\ \dfrac{dy}{dt} = -x + y \end{cases}$

Practical 3.7
Unit 1

Q.1. Explain the method of solving the equation $M\,dx + N\,dy = 0$ when the variables in it are separable.

Q.2. Define an exact differential equation. State and prove the necessary and sufficient condition for the equation $M\,dx + N\,dy = 0$ to be exact, where M and N are functions of x and y.

Q.3. Define an integrating factor (I.F.) of $M\,dx + N\,dy = 0$, where M and N are functions of x and y. Show that if one I.F. of the above equation is known, then it is possible to find an infinite number of I.F.'s of the same equation.

Q.4. Let $M\,dx + N\,dy = 0$ be a homogenous differential equation of the first order and first degree. Show that $\frac{1}{Mx+Ny}$ is an integrating factor of the equation where $Mx + Ny \neq 0$.

Q.5. If the equation $M\,dx + N\,dy = 0$ is of the type $f_1(xy)ydx + f_2(xy)xdy = 0$, show that

(i) $\frac{1}{Mx-Ny}$ is an integrating factor of the equation if $Mx - Ny \neq 0$.

(ii) The variables, in the given equation, can be separated if $Mx - Ny = 0$.

Q.6. If $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ is a function of x alone, say, $f(x)$, then show that $e^{\int f(x)dx}$ is an integrating factor of the equation $M\,dx + N\,dy = 0$.

Q.7. If $\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$ is a function of y alone, say, $f(y)$, then show that $e^{\int f(y)dy}$ is an integrating factor of the equation $M\,dx + N\,dy = 0$.

Q.8. Find an integrating factor of the linear equation $\frac{dy}{dx} + Py = Q$, P and Q being functions of x alone. Hence solve the equation.

OR

Show that the general solution of the differential equation $\frac{dy}{dx} + py = Q$ is
$y = e^{-\int P\,dx}\left[\int Q \cdot e^{\int P\,dx} dx + c\right]$. or $y \cdot e^{\int P\,dx} = \int Q e^{\int P\,dx} dx + c$

Q.9. Discuss the solution of Bernoulli's differential equation $\frac{dy}{dx} + Py = Qy"$ where P and Q are function of x.

Q.10. What are orthogonal trajectories of a system of curves? Explain the method of obtaining orthogonal trajectories of a system of curves when its equation is given in (i) Cartesian co-ordinates, (ii) polar co-ordinates.

Q.11. Suppose that x_0 bacteria are placed in a nutrient solution at time $t = 0$, and that $x = x(t)$ is the population of the colony at a later time t. If food and living space are unlimited, and if as a consequence the population at any moment is increasing at a rate proportional to the population at that moment, find x as a function of t.

Q.12. A tank contains 50 gallons of brine in which 75 pounds of salt are dissolved. Beginning at time $t = 0$, brine containing 3 pounds of salt per gallon flows in at the rate of 2 gallons per minute, and the mixture (which is kept uniform by stirring) flows out at the same rate. When will there be 125 pounds of dissolved salt in the tank? How much dissolved salt is in the tank after a long time?

Q.13. The current i at t in a given circuit containing an inductance L, resistance R and a steady e.m.f. E_0 is given by $L\frac{di}{dt} + Ri = E_0$. If $i = 0$ when $t = 0$, show that $i = \frac{E_0}{R}\left(1 - e^{-Rt/L}\right)$.

Q.14. Determine the current in an RL circuit if the applied EMF is $E(t) = E_0 \cos \omega t$, where E_0 and ω are constants.

Unit 2

1. If y_g is the general solution of the homogeneous equation $y'' + P(x)y' + Q(x)y = 0$ and y_p is any particular solution of the equation $y'' + P(x)y' + Q(x)y = R(x)$, then prove that $y_g + y_p$ is the general solution of $y'' + P(x)y' + Q(x)y = R(x)$.
2. If $y_1(x)$ and $y_2(x)$ are any two solutions of the equation $y'' + P(x)y' + Q(x)y = 0$, then show that $c_1y_1(x) + c_2y_2(x)$ is also a solution, for any constants $c_1$and c_2.
3. Let $y_1(x)$ and $y_2(x)$ be any two solutions of the differential equation $y'' + P(x)y' + Q(x)y = 0$ on $[a, b]$, then prove that their Wronskian$W(y_1, y_2) = y_1y_2' - y_2{y_1}'$ is either identically zero or never zero on $[a, b]$.
4. Let $y_1(x)$ and $y_2(x)$ be any two solutions of the differential equation $y'' + P(x)y' + Q(x)y = 0$ on the interval $[a, b]$. Then prove that their Wronskian $W(y_1, y_2) = y_1y_2' - y_2{y_1}'$ is identically zero if and only if $y_1(x)$ and $y_2(x)$ are linearly dependent on $[a, b]$.
5. Let $y_1(x)$ and $y_2(x)$ be two linearly independent solutions of the equation $y'' + P(x)y' + Q(x)y = 0$ on $[a, b]$, then show that $c_1y_1(x) + c_2y_2(x)$ is the general solution of $y'' + P(x)y' + Q(x)y = 0$ on $[a, b]$.
6. If $y_1(x)$ is a non-zero solution to the differential equation $y'' + P(x)y' + Q(x)y = 0$, show that another linearly independent solution $y_2(x)$ is given by
$$y_2(x) = y_1(x) \int \frac{1}{y_1^2} e^{-\int P dx} dx$$
7. Obtain the auxillary equation of the differential equation $y'' + py' + q = 0$, where p and q are constants.
8. Let $m_1$and m_2 be the roots of the auxiliary equation of the differential equation $y'' + py' + q = 0$, where p and q are constants. Discuss the general solution of the differential equation when
   (a) $m_1$and$m_2$are real and unequal.
   (b) $m_1$and$m_2$are real and equal.
   (c) $m_1$and $m_2$are complex roots.
9. Consider the differential equation $y'' + py' + qy = R(x)$, where p, q are constants and R is a function of x. Let $y_1$and $y_2$be two linearly independent solutions of $y'' + py' + qy = 0$. If $y = v_1y_1 + v_2y_2$ is a solution of $y'' + py' + qy = R(x)$, where$v_1$and$v_2$are functions of x then prove that $v_1 = -\int \frac{y_2 R}{W} dx$ and$v_2 = \int \frac{y_1 R}{W} dx$, where W is the Wronskian determinant of $y_1$and y_2.

Unit 3

1. Define Wronskian $W[t]$ of the two solutions $\begin{cases} x = x_1(t) \\ y = y_1(t) \end{cases}$ and $\begin{cases} x = x_2(t) \\ y = y_2(t) \end{cases}$ of the homogeneous system $\begin{cases} \dfrac{dx}{dt} = a_1(t)x + b_1(t)y \\ \dfrac{dy}{dt} = a_2(t)x + b_2(t)y \end{cases}$. Show that this Wronskian is either identically zero or nowhere zero on $[a,b]$.

2. When do we say that the two solutions $\begin{cases} x = x_1(t) \\ y = y_1(t) \end{cases}$ and $\begin{cases} x = x_2(t) \\ y = y_2(t) \end{cases}$ of the homogeneous system $\begin{cases} \dfrac{dx}{dt} = a_1(t)x + b_1(t)y \\ \dfrac{dy}{dt} = a_2(t)x + b_2(t)y \end{cases}$ are linearly independent on $[a,b]$? Show that the linear independence of the solutions on $[a,b]$ implies $\begin{cases} x = c_1x_1(t) + c_2x_2(t) \\ y = c_1y_1(t) + c_2y_2(t) \end{cases}$ to be the general solution of the homogeneous system on $[a,b]$.

3. If the two solutions $\begin{cases} x = x_1(t) \\ y = y_1(t) \end{cases}$ and $\begin{cases} x = x_2(t) \\ y = y_2(t) \end{cases}$ of the homogeneous system $\begin{cases} \dfrac{dx}{dt} = a_1(t)x + b_1(t)y \\ \dfrac{dy}{dt} = a_2(t)x + b_2(t)y \end{cases}$ are linearly independent on $[a,b]$ and if $\begin{cases} x = x_p(t) \\ y = y_p(t) \end{cases}$ is any particular solution of the non-homogeneous system $\begin{cases} \dfrac{dx}{dt} = a_1(t)x + b_1(t)y + f_1(t) \\ \dfrac{dy}{dt} = a_2(t)x + b_2(t)y + f_2(t) \end{cases}$ on $[a,b]$, then show that $\begin{cases} x = c_1x_1(t) + c_2x_2(t) + x_p(t) \\ y = c_1y_1(t) + c_2y_2(t) + y_p(t) \end{cases}$ is the general solution of the non-homogeneous system on $[a,b]$.

4. Define Homogeneous Linear system (of two equations) with constant coefficients. What is the Auxiliary equation of such a system? State the expressions for the general solution of the system, in case the roots of the auxiliary equation are (i) real and distinct, (ii) distinct complex and (iii) real and equal.

5. If the homogeneous system $\begin{cases} \dfrac{dx}{dt} = a_1(t)x + b_1(t)y \\ \dfrac{dy}{dt} = a_2(t)x + b_2(t)y \end{cases}$ has two solutions $\begin{cases} x = x_1(t) \\ y = y_1(t) \end{cases}$ and $\begin{cases} x = x_2(t) \\ y = y_2(t) \end{cases}$ on $[a,b]$, then show that $\begin{cases} x = c_1x_1(t) + c_2x_2(t) \\ y = c_1y_1(t) + c_2y_2(t) \end{cases}$ is also a solution of the system on $[a,b]$ for any real numbers c_1 and c_2.

6. Obtain the general solution of a Homogeneous Linear system (of two equations) with constant coefficients, when its Auxiliary equation has two real and distinct roots.

7. Obtain the general solution of a Homogeneous Linear system (of two equations) with constant coefficients, when its Auxiliary equation has two distinct complex roots.

8. Obtain the general solution of a Homogeneous Linear system (of two equations) with constant coefficients, when its Auxiliary equation has real and equal roots.

9. Define Homogeneous Linear system and Non-Homogeneous Linear system of differential equations. What do we mean by the general solution of any such system?

10. State two conditions to show that the two solutions $\begin{cases} x = x_1(t) \\ y = y_1(t) \end{cases}$ and $\begin{cases} x = x_2(t) \\ y = y_2(t) \end{cases}$ of the homogeneous system $\begin{cases} \dfrac{dx}{dt} = a_1(t)x + b_1(t)y \\ \dfrac{dy}{dt} = a_2(t)x + b_2(t)y \end{cases}$ are linearly dependent on $[a,b]$. When do we say that the solutions are linearly independent?

11. If $\begin{cases} x = x_1(t) \\ y = y_1(t) \end{cases}$ and $\begin{cases} x = x_2(t) \\ y = y_2(t) \end{cases}$ are linearly independent solutions of the homogeneous system $\begin{cases} \dfrac{dx}{dt} = a_1(t)x + b_1(t)y \\ \dfrac{dy}{dt} = a_2(t)x + b_2(t)y \end{cases}$, then show that $\begin{cases} x = v_1(t)x_1(t) + v_2(t)x_2(t) \\ y = v_1(t)y_1(t) + v_2(t)y_2(t) \end{cases}$ is a particular solution of $\begin{cases} \dfrac{dx}{dt} = a_1(t)x + b_1(t)y + f_1(t) \\ \dfrac{dy}{dt} = a_2(t)x + b_2(t)y + f_2(t) \end{cases}$, provided the functions $v_1(t)$ and $v_2(t)$ satisfy the system $\begin{matrix} v_1'x_1 + v_2'x_2 = f_1 \\ v_1'y_1 + v_2'y_2 = f_2 \end{matrix}$.