MCQ Practice of Unit 2

Practical 2.4

Properties of Cyclic group

1. Suppose G is a cyclic group such that G has exactly three subgroups viz. $G, \{e\}$ and a subgroup pf order 5. Then the order of G is
   (a) 5 (b) 10 √(c) 25 (d) 125

$O(G) = 25$
$\Rightarrow O(H_1) = 1, \; O(H_2) = 5, \; O(H_3) = 25$
(c)

2. The number of elements of order 5 in $\mathbb{Z}_{1000}$ is
   (a) 1 √(b) 4 (c) 5 (d) none of these

$\phi(5) = 5 - 1$
$= 4$
(b)

3. Let G be a cyclic group of infinite order. Then the number of elements of finite order in G is
   √(a) 1 (b) 2 (c) 5 (d) 0

$O(e) = 1$

4. The number of generators of an infinite cyclic group is
   (a) 1 √(b) 2 (c) 0 (d) infinite

No of generators of infinite cyclic group = 2 (b)

5. The number of elements of order 5 in the cyclic group of order 25 is
   (a) 1 (ii) 2 (iii) 5 √(iv) none of these

$G = \langle a \rangle, \; O(G) = 25$
$= \{a, a^2, a^3, \ldots a^{24}, a^{25} = e\}$
(d)
$(a^5)^5 = a^{25} = e \Rightarrow O(a^5) = 5, \; O(a^{20})^5 = e$

(d) $\{a, a, a, \ldots\}$
$(a^5)^5 = a^{25} = e \Rightarrow o(a^5) = 5$
$(a^{10})^5 = a^{50} = e \Rightarrow o(a^{10}) = 5$
$(a^{15})^5 = e \Rightarrow o(a^{15}) = 5$
$o(a^{20})^5 = e$
$\Rightarrow o(a^{20}) = 5$
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6. Let G be a group and $a \in G$. If $0(a) = 24$ then $\langle a^{21} \rangle \cap \langle a^{10} \rangle$ has order
(a) 1 (b) 2 (c) 3 (d) 4

$\langle a^{21} \rangle \cap \langle a^{10} \rangle = \{e\}$
$(21, 10) = 1$

7. The number of subgroups of $(\mathbb{Z}_{20}, +)$ is
(a) 6 (b) 5 (c) 3 (d) 2

$o(\mathbb{Z}_{20}) = 20$
Possible subgroups order can be
1, 2, 4, 5, 10, 20
(a)

8. G is a cyclic group having exactly 3 subgroups namely G itself, $\{e\}$ and a subgroup of order 7. Then order of G is
(a) 14 (b) 49 (c) 7 p where p is any prime (d) cannot say

$H_1 = G$, $o(H_1) = o(G) = 7$
$H_2 = \{e\}$, $o(H_2) = 1$
$o(H_3) = 7$

9. The total number of elements of order 8 in $(\mathbb{Z}_{80,00,000}, +)$ is
(a) 8 (b) 10,00,000 (c) 4 (d) None of the above

$\phi(8) = \phi(2^3) = 8\left(1 - \frac{1}{2}\right) = 8 \times \frac{1}{2} = 4$
(c)

10. Let G be a group and $a \in G$. If $0(a) = 0(a^2)$, then $0(a)$ is
(a) even (b) odd (c) a prime (d) None of the above

let $o(a) = 2n \Rightarrow a^{2n} = e$

let $o(a) = 2n \Rightarrow a^{2n} = e$

$\Rightarrow (a^2)^n = e \Rightarrow o(a^2) = n$

$o(a) = p \Rightarrow a^p = e$

(c) $\Rightarrow (a^p)^2 = e^2 = e \Rightarrow (a^2)^p = e$

$\Rightarrow o(a^2) = p$

11. Let G be an infinite cyclic group generated by a. The generators of $\langle a^3 \rangle$ are
(a) a^3, a^2 ~~(b)~~ a^3, a^{-3} (c) a^3, a^m where $(m, 3) = 1$ (d) a^3, a^p where p is prime

$\langle a^3 \rangle = \langle a^3, a^6, a^9, a^{12}, \ldots \rangle$

$a^3 \cdot a^{-3} = 1$

$\therefore (a^3)^{-1} = a^{-3}$ (b)

12. Consider the subgroup H generated by $\bar{3}$ of the group $(\mathbb{Z}_{15}, +)$, then the number of generators of H is
~~(a) 4~~ (b) 2 (c) 3 (d) 4

$H = \langle \bar{3} \rangle$

(a) $= \{ \bar{3}, \bar{6}, \bar{9}, \overline{12}, \bar{0} \}$

$\bar{3} + \overline{12} \equiv 0 \pmod{15}$, $\overline{12}$ another generator

$\bar{6} = \bar{6}$, $6^2 = \bar{6} + \bar{6} = 12$, $6^3 = \bar{6} + \bar{6} + \bar{6} = \bar{3}$

$6^4 = 24 = \bar{9} \pmod{15}$, $6^5 = 30 \equiv 0 \pmod{15}$

13. Let $G = \{\bar{4}, \bar{8}, \overline{12}, \overline{16}\}$ under multiplication modulo 20. Then
(a) G is cyclic and $G = \langle \bar{4} \rangle$ (b) G is not cyclic
~~(c)~~ G is cyclic and $G = \langle \bar{8} \rangle$ (d) None of the above

$4^1 \equiv 4$ $8 \equiv 8 \pmod{20}$

$4^2 \equiv 16$ $8^2 \equiv 4 \pmod{20}$

$4^3 \equiv 4$ $8^3 \equiv 12 \pmod{20}$

$4^4 \equiv 16$ $8^4 \equiv 16 \pmod{20}$

(c) $\langle \bar{8} \rangle$

14. Let $G = \langle a \rangle$ and order of G be 40. Then the elements of order 10 in G are
(a) a^4, a^8, a^{12}, a^{16} ~~(b)~~ $a^4, a^{12}, a^{28}, a^{36}$ (c) $a^4, a^{16}, a^{24}, a^{32}$ (d) a^4, a^8, a^{24}, a^{28}

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(a) a^4, a^8, a^{12}, a^{16} ✓(b) $a^4, a^{12}, a^{28}, a^{36}$ (c) $a^4, a^{16}, a^{24}, a^{32}$ (d) a^4, a^8, a^{24}, a^{28}

$a^{40} = e$

$(a^4)^{10} = a^{40} = e \Rightarrow o(a^4) = 10$

$(a^{12})^{10} = a^{120} = e \Rightarrow o(a^{12}) = 10$

$o(a^{28}) = 10$

$o(a^{36}) = 10$ (b)

15. The generators of $20\mathbb{Z} \cap 30\mathbb{Z}$ are

✓(a) $60, -60$ (b) $10, -10$ (c) $20, 30$ (d) None of the above

$20\mathbb{Z} = \{ \cdots -60, -40, -20, 0, 20, 40, 60 \cdots \}$

$30\mathbb{Z} = \{ \cdots -90, -60, -30, 0, 30, 60, 90 \cdots \}$

$20\mathbb{Z} \cap 30\mathbb{Z} = \{ \cdots -120, -60, 0, 60, 120 \cdots \}$

(a) $60, -60$

16. Let G be a group and $a \in G$ such that $O(\langle a^5 \rangle) = 12 = O(\langle a^4 \rangle)$. Then possible orders of 'a' are

(a) 20 (b) 12 or 60 (c) 24 ✓(d) None of these

$a^{20} = e \Rightarrow (a^5)^4 = a^{20}$

$\langle a^5 \rangle = \{ a^5, a^{10}, a^{15}, \cdots a^{60} = e \}$

$\langle a^4 \rangle = \{ a^4, a^8, a^{12} \cdots a^{48} = e \}$

$$\begin{array}{r|l} 6 & 60, 48 \\ \hline 2 & 10, 8 \\ \hline & 5, 4 \end{array}$$

Order should be = 2 X 6 X 5 X 4 = 240

17. Let G be a cyclic group of order n generated by 'a' then $\langle a^r \rangle = \langle a^s \rangle$ implies

(a) $(r, s) = 1$ (b) $s = (n, r)$ ✓(c) $(n, r) = (n, s)$ (d) $r/(n, s)$

$(n, r) = (n, s)$

as $\langle a^r \rangle = \langle a^s \rangle$

18. If G is a cyclic group of order 11, then number of generators of G is

(a) 2 (b) 10 (c) 11 (d) None of the above

$o(G) = 11$

$\phi(11) = 11 - 1$

$= 10$

$\Rightarrow$ (b)

19. Let n = Number of elements of order 4 in $(\mathbb{Z}_{4k}, +)$ and m =Number of elements of order 4 in $(\mathbb{Z}_{8k}, +)$ (k is a positive number). Then

(a) $m = 2n$ (b) $m = n^2$ (c) $m < n$ (d) $m = n$

$\phi(4) = \phi(2^2) = 4\left(1 - \frac{1}{2}\right) = 2$

(d) $m = n$

21. Every proper subgroup of S_3 is

(a) cyclic (b) non-abelian (c) of order 3 (d) of order 2

$S_3 = \{I, (12), (13), (23), (123), (132)\}$

$H = \{I, (12)\} = \{I, (13)\} = \{I, (23)\}$

$H = \{I, (123), (132)\}$

(a)

22. The number of elements of order 5 in $\mathbb{Z}_{20}$ is

(a) 1 (b) 2 (c) 4 (d) 5

(c) $\phi(5) = 5 - 1 = 4$

23. The number of elements of order 10 in $\mathbb{Z}_{10}$ is

(a) 4 (b) 10 (c) 5 (d) 0

$\phi(10) = \phi(2 \times 5) = 10\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{5}\right)$

(a)

$= \cancel{10} \times \frac{1}{\cancel{2}} \times \frac{4}{\cancel{5}}$

$= 4$

24. Let G be a group and $a \in G$. If $o(a) = 20$, then $o(a^4)$:

(a) 15 (b) 5 (c) 12 (d) 20

$o(a^4) = 5$, $a^{20} = e$

$(a^4)^5 = e$

(b)

25. If $H_1 = <\overline{20}>$ and $H_2 = <\overline{15}>$ in $\mathbb{Z}_{30}$ under addition modulo 30, then

(a) $|H_1| = 10$ and $|H_2| = 15$
(b) $|H_1| = 3$ and $|H_2| = 2$
(c) $|H_1| = 5$ and $|H_2| = 3$
(d) None of these.

$H_1 = \{\overline{20}, \overline{10}, \overline{0}\}$

$H_2 = \{\overline{15}, \overline{0}\}$

(b)

26. If $H_1 = <\overline{3}>$ and $H_2 = <\overline{15}>$ in $\mathbb{Z}_{18}$ under addition modulo 18, then

(a) $|H_1| = 6$ and $|H_2| = 6$
(b) $|H_1| = 6$ and $|H_2| = 5$
(c) $|H_1| = 15$ and $|H_2| = 3$
(d) None of these.

$H_1 = \langle \overline{3} \rangle = \{\overline{3}, \overline{6}, \overline{12}, \overline{15}, \overline{0}\}$

$H_2 = \langle \overline{15} \rangle = \{\overline{15}, \overline{12}, \overline{9}, \overline{6}, \overline{3}, \overline{0}\}$

(a)

28. If $m, n \in \mathbb{Z}$, then the generator of $<m> \cap <n>$ is

(a) mn (b) $\gcd(m, n)$ (c) $\text{lcm}(m, n)$ (d) None of these.

Generator of $< m > \cap < n >$ is the g.c.d (m , n)
Option (b)

29. Let p be prime. If a group G has more than $p - 1$ elements of order p, then

(a) G is cyclic
(b) G is not cyclic
(c) G has a unique subgroup of order p.
(d) None of these.

Option (a) As if G has more than p-1 elements of order p then it must be cyclic.