

LINEAR EQUATIONS

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What is an equation?

A statement of equality of two algebraic expressions which involves one or more literals (variables) is called an equation.

$3 + x = 7$ is an equation.

The set of values of variables which makes the open sentence true is called the solution set.

Note:

Every equation has two sides — L.H.S. (left-hand side) and R.H.S. (right-hand side).

Literals involved in the equation are called variables. These are usually denoted by letters of English alphabet.

An equation may contain any number of variables.

Linear equation in one variable

(i) $2x + 5 = 15$

$$2x = 15 - 5$$

$$2x = 10$$

$$x = \frac{10}{2} = 5$$

$$\boxed{x = 5}$$

(ii) $2x + 5 > 15$

$$x > 5$$

(iii) $2x + 5 < 15 \rightarrow x \in \mathbb{I}$

$$x < 5$$

$$\text{Sol}^n \text{ set} = \{4, 3, 2, 1, 0, -1, \dots\}$$

$$x \in \mathbb{W} \rightarrow \{4, 3, 2, 1, 0\}$$

Linear
inequality

Signs of inequalities

- $> \rightarrow$ Greater than
- $\geq \rightarrow$ Greater than or equal to
- $< \rightarrow$ Less than
- $\leq \rightarrow$ Less than or equal to

Solve the given equations

(A) $3(x + 4) = 21$

(B) $2x/3 - 3x/5 = 8$

(C) $3x - 9 = 5x - 3$

(D) $3(x - 3) = 4(2x + 1)$

Solⁿ: (A) $3x + 3 \times 4 = 21$
 $3x + 12 = 21$
 $3x = 21 - 12 = 9$
 $x = \frac{9}{3} = 3$
 $\boxed{x = 3}$

(B) $\frac{2x}{3} - \frac{3x}{5} = 8$ LCM of 3 & 5 = 15

$$\frac{2x \times 5}{3 \times 5} - \frac{3x \times 3}{5 \times 3} = 8$$

$$\frac{10x - 9x}{15} = 8 \Rightarrow x = 8 \times 15$$

$$\therefore \boxed{x = 120}$$

(iii) $3x - 9 = 5x - 3$
 $3x - 5x = -3 + 9$
 $-2x = 6$
 $x = 6 / -2 = -3$
 $\boxed{x = -3}$

(iv) $3(x - 3) = 4(2x + 1)$
 $3x - 9 = 8x + 4$
 $3x - 8x = 4 + 9$
 $-5x = 13$
 $x = 13 / -5 = -\frac{13}{5}$
 $\therefore \boxed{x = -\frac{13}{5}}$

$$(Q) (x-2)/(x-3) = (x-1)/(x+1)$$

$$\frac{x-2}{x-3} = \frac{x-1}{x+1}$$

$$(x-2)(x+1) = (x-1)(x-3)$$

$$x(x+1) - 2(x+1) = x(x-3) - 1(x-3)$$

$$x^2 + x - 2x - 2 = x^2 - 3x - x + 3$$

$$x^2 - x - 2 = x^2 - 4x + 3$$

$$x^2 - x^2 - x + 4x = 3 + 2$$

$$3x = 5$$

$$\boxed{x = 5/3}$$

$$(j) (5-7x)/(2+4x) = -8/7$$

$$\frac{5-7x}{2+4x} = \frac{-8}{7}$$

$$7(5-7x) = -8(2+4x)$$

$$35 - 49x = -16 - 32x$$

$$-49x + 32x = -16 - 35$$

$$-17x = -51$$

$$x = \frac{51}{17}$$

$$\boxed{x = 3}$$

$$(i) (8x-5)/(7x+1) = -4/5$$

$$\frac{8x-5}{7x+1} = \frac{-4}{5}$$

$$5(8x-5) = -4(7x+1)$$

$$40x - 25 = -28x - 4$$

$$40x + 28x = -4 + 25$$

$$68x = 21$$

$$x = \frac{21}{68}$$

$$(Q) 2(x - 2) - 5(x - 5) = 4(x - 8) - 2(x - 2)$$

$$2x - 4 - 5x + 25 = 4x - 32 - 2x + 4$$

$$-3x + 21 = 2x - 28$$

$$-3x - 2x = -28 - 21$$

$$-5x = -49$$

$$\boxed{x = \frac{49}{5}}$$

$$(Q) \frac{2}{3x-1} + \frac{3}{3x+1} = \frac{5}{3x}$$

$$\frac{2}{3x-1} + \frac{3}{3x+1} = \frac{5}{3x}$$

LHS:-

$$\frac{2(3x+1) + 3(3x-1)}{(3x-1)(3x+1)} = \frac{6x+2+9x-3}{9x^2-1}$$

$$= \frac{15x-1}{9x^2-1}$$

$$\text{Now: } \frac{15x-1}{9x^2-1} = \frac{5}{3x}$$

$$3x(15x-1) = 5(9x^2-1)$$

$$45x^2 - 3x = 45x^2 - 5$$

$$+3x = +5$$

$$\boxed{x = \frac{5}{3}}$$

Q: Solve: $(2x + 5)/(x + 4) = 1$

$$\frac{2x+5}{x+4} = 1$$

$$2x+5 = 1(x+4)$$

$$\Rightarrow 2x+5 = x+4$$

$$2x - x = 4 - 5$$

$$\boxed{x = -1}$$

Solve: $5 - 2(x - 1) = 4(3 - x) - 2x$.

$$5 - 2x + 2 = 12 - 4x - 2x$$

$$7 - 2x = 12 - 6x$$

$$-2x + 6x = 12 - 7 = 5$$

$$4x = 5$$

$$x = 5/4$$

Solve: $2/(x + 3) = 3/(5 - x)$

$$\frac{2}{x+3} = \frac{3}{5-x}$$

$$2(5-x) = 3(x+3)$$

$$\Rightarrow 10 - 2x = 3x + 9$$

$$10 - 9 = 3x + 2x$$

$$1 = 5x$$

$$x = 1/5$$

Solve: $2(x - 2) - 5(x - 5) = 4(x - 8) - 2(x - 2)$

$$2x - 4 - 5x + 25 = 4x - 32 - 2x + 4$$

$$-3x + 21 = 2x - 28$$

$$-3x - 2x = -28 - 21$$

$$-5x = -49 \quad \therefore x = 49/5$$

QUES: $(2x - 5)/(3x - 1) = (2x - 1)/(3x + 2)$

$$\frac{2x-5}{3x-1} = \frac{2x-1}{3x+2}$$

$$\begin{aligned} (2x-5)(3x+2) &= (2x-1)(3x-1) \\ 6x^2 + 4x - 15x - 10 &= 6x^2 - 2x - 3x + 1 \\ -11x - 10 &= -5x + 1 \\ -11x + 5x &= 1 + 10 = 11 \\ -6x &= 11 \\ x &= -11/6 \end{aligned}$$

Solve each of the following equations and check your solution by substituting in the equation.

(a) $3x/2 - 10 = 1/2$

(b) $x/2 - x/3 = 8$

(c) $6x - 9 - 2(1 - x) = x + 9$

(d) $0.25(4y - 3) = 0.5y - 9$

(e) $(2 - y)/(y + 7) = 3/5$

(f) $(3x + 2)/(2x - 3) = -3/2$

(g) $(x - 8)/3 = (x - 3)/5$

(h) $(3 - 7x)/(15 + 2x) = 0$

(i) $(0.4y - 3)/(1.5y + 9) = -7/5$

(j) $2/(3x - 1) + 3/(3x + 1) = 5/3x$

(k) $2/(x - 3) + 1/(x - 1) = 5/(x - 1) - 2/(x - 2)$

(l) $15(x - y) - 3(x - 9) + 5(x + 6) = 0$

(m) $y/2 - 1/2 = y/3 + 1/4$

(n) $(0.5y - 9)/0.25 = 4y - 3$

(o) $[17(2 - y) - 5(y + 12)]/(1 - 7y) = 8$

$$(h) 2(x - 2) - 5(x - 5) = 4(x - 8) - 2(x - 2)$$

$$2x - 4 - 5x + 25 = 4x - 32 - 2x + 4$$

$$-3x + 21 = 2x - 28$$

$$-3x - 2x = -28 - 21$$

$$-5x = -49$$

$$\boxed{x = \frac{49}{5}}$$

$$2/(3x - 1) + 3/(3x + 1) = 5/3x$$

$$\frac{2}{3x-1} + \frac{3}{3x+1} = \frac{5}{3x}$$

LHS:-

$$\frac{2(3x+1) + 3(3x-1)}{(3x-1)(3x+1)} = \frac{6x+2+9x-3}{9x^2-1}$$

$$= \frac{15x-1}{9x^2-1}$$

$$\text{Now: } \frac{15x-1}{9x^2-1} = \frac{5}{3x}$$

$$3x(15x-1) = 5(9x^2-1)$$

$$45x^2 - 3x = 45x^2 - 5$$

$$+3x = +5$$

$$\boxed{x = \frac{5}{3}}$$

Q: Solve: $(2x + 5)/(x + 4) = 1$

$$\frac{2x+5}{x+4} = 1$$

$$2x+5 = 1(x+4)$$

$$\Rightarrow 2x+5 = x+4$$

$$2x - x = 4 - 5$$

$$\boxed{x = -1}$$

Solve: $5 - 2(x - 1) = 4(3 - x) - 2x$.

$$5 - 2x + 2 = 12 - 4x - 2x$$

$$7 - 2x = 12 - 6x$$

$$-2x + 6x = 12 - 7 = 5$$

$$4x = 5$$

$$x = 5/4$$

Solve: $2/(x + 3) = 3/(5 - x)$

$$\frac{2}{x+3} = \frac{3}{5-x}$$

$$2(5-x) = 3(x+3)$$

$$\Rightarrow 10 - 2x = 3x + 9$$

$$10 - 9 = 3x + 2x$$

$$1 = 5x$$

$$x = 1/5$$

Solve: $2(x - 2) - 5(x - 5) = 4(x - 8) - 2(x - 2)$

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$$-5x = -49 \quad \therefore x = 49/5$$

QUES: $(2x - 5)/(3x - 1) = (2x - 1)/(3x + 2)$

$$\frac{2x-5}{3x-1} = \frac{2x-1}{3x+2}$$

$$\begin{aligned}(2x-5)(3x+2) &= (2x-1)(3x-1) \\ 6x^2 + 4x - 15x - 10 &= 6x^2 - 2x - 3x + 1 \\ -11x - 10 &= -5x + 1 \\ -11x + 5x &= 1 + 10 = 11 \\ -6x &= 11 \\ x &= -11/6\end{aligned}$$

(QUES) $(3 - 7x)/(15 + 2x) = 0$

(QUES) $(0.4y - 3)/(1.5y + 9) = -7/5$

(QUES) $2/(3x - 1) + 3/(3x + 1) = 5/3x$

(QUES) $2/(x - 3) + 1/(x - 1) = 5/(x - 1) - 2/(x - 2)$

(QUES) $15(x - y) - 3(x - 9) + 5(x + 6) = 0$

(QUES) $y/2 - 1/2 = y/3 + 1/4$

(QUES) $(0.5y - 9)/0.25 = 4y - 3$

(QUES) $[17(2 - y) - 5(y + 12)]/(1 - 7y) = 8$

word problem

27 September 2021 07:55

Q. A boat travels 20 km **upstream** in a river in the same period of time as it takes to travel 30 km **downstream**. If the rate of the stream is 4 km/hr, find the speed of boat in still water.

Ans Let the speed of boat in still water = x km/hr
Relative speed
(i) Downstream $\rightarrow (x+4)$ km/hr
(ii) Upstream $\rightarrow (x-4)$ km/hr

Distance Upstream = 20 km

1, Downstream = 30 km

$$\text{Time} = \frac{\text{Distance}}{\text{speed}}$$

$$\frac{20}{x-4} = \frac{30}{x+4}$$

$$20(x+4) = 30(x-4)$$
$$20x + 80 = 30x - 120$$

$$80 + 120 = 30x - 20x$$

$$200 = 10x$$

$$\therefore x = 200/10 \Rightarrow x = 20$$

$\therefore$ Speed of boat in still water = 20 km/hr.

The length of the rectangle is 4 times its width. If the perimeter of the rectangle is 80 m, find the length and breadth of rectangle.

Soln: Width = x m

$$\text{length} = 4 \times x = 4x$$

$$\therefore P = 2(l + b)$$

$$\Rightarrow 80 = 2(x + 4x)$$

$$\frac{80}{2} = 5x$$

$$\therefore 5x = 40$$

$$x = \frac{40}{5} = 8 \text{ m}$$

$$\therefore \text{length} = 4 \times 8 = 32 \text{ m}$$

$$\text{width} = 8 \text{ m}$$

The present ages of Andy and Mike are in the ratio 5 : 3. If Andy had been 7 years older and Mike 7 years younger, the age of Andy would have been 3 times the age of Mike. Find their present ages.

Soln: Let present age of
 Andy = $5x$ years
 Mike = $3x$ year

given

$$(5x + 7) = 3(3x - 7)$$

$$5x + 7 = 9x - 21$$

$$7 + 21 = 9x - 5x$$

$$4x = 28$$

$$x = 28/4 = 7$$

$\therefore$ Present age of

$$\text{Andy} = 5 \times 7 = 35 \text{ years}$$

$$\text{Mike} = 3 \times 7 = 21 \text{ years.}$$

- A. Three numbers are in the ratio 2 : 3 : 4 and their sum is 54. Find the number.
- B. The sum of three consecutive even numbers is 54. Find the numbers.
- C. Divide 22 into two parts in such a way that $\frac{2}{3}$ of one part is equal to $\frac{1}{4}$ of the other.

Solⁿ A:- Let the numbers be $2x, 3x, 4x$

$$2x + 3x + 4x = 54$$

$$9x = 54 \quad \therefore x = \frac{54}{9} = 6$$

$\therefore$ Numbers are

$$2 \times 6 = 12$$

$$3 \times 6 = 18$$

$$4 \times 6 = 24$$

Solⁿ B: Let the first even no be x

$\therefore$ other two consecutive even nos are $x+2, x+4$

Now, $x + x+2 + x+4 = 54$

$$3x + 6 = 54 \Rightarrow 3x = 54 - 6 = 48$$

$$\therefore x = \frac{48}{3} = 16$$

$\therefore$ Numbers are 16, 18, 20

Solⁿ C: Let the first part be x

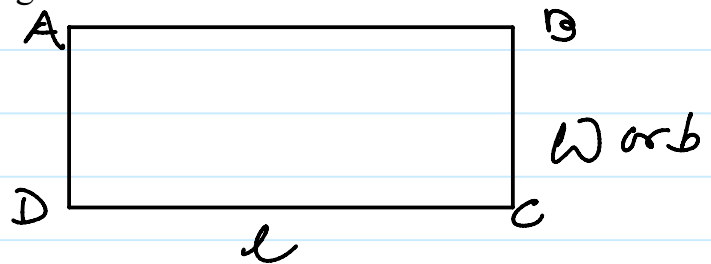
$\therefore$ Second part $(22-x)$

$$\frac{2}{3}(x) = \frac{1}{4}(22-x) \Rightarrow 4 \times 2x = 3(22-x)$$

$$\Rightarrow 8x = 66 - 3x \Rightarrow 8x + 3x = 66 \Rightarrow 11x = 66$$

$$\Rightarrow \boxed{x = 6} \therefore \text{Nos are } \underline{6 \text{ \& } 16}$$

Q. Length of rectangle is 4 m less than 3 times its breadth. If the perimeter of rectangle is 32 m. Find its length and breadth



Let breadth = x m

$\therefore$ length = $3x - 4$

Perimeter of rectangle = $2(l + b)$

$$32 = 2(x + 3x - 4)$$

$$\frac{32}{2} = 4x - 4 \Rightarrow 4x - 4 = 16$$

$$\therefore 4x = 16 + 4 = 20 \quad \therefore x = \frac{20}{4} = 5$$

$\therefore$ Breadth = 5 m

$$\text{Length} = 3 \times 5 - 4 = 11 \text{ m}$$

Two numbers are such that the ratio between them is 5 : 2. If each is increased by 1 the ratio between the new numbers so formed is 7 : 3. Find the original number.

Soln: Let the numbers be $5x$ & $2x$

$$\frac{5x+1}{2x+1} = \frac{7}{3}$$

$$3(5x+1) = 7(2x+1)$$

$$15x + 3 = 14x + 7$$

$$15x - 14x = 7 - 3$$

$$[x=4]$$

$\therefore$ Numbers are $5 \times 4 = 20$ and $2 \times 4 = 8$

A number consists of two digits whose sum is 5. If 9 is subtracted from the number the digits are reversed. Find the number.

Soln: Let the digit at unit's place be x
 " $"$ " $"$ ten's place be $(5-x)$

$$\text{No. formed} = 10(5-x) + x$$

$$= 50 - 10x + x = 50 - 9x \text{ --- (4)}$$

No. formed on reversing the digits

$$\log x + 15 - x = 0 \quad 9x + 5$$

$$(50 - 9x) - (9x + 5) = 9$$

$$50 - 9x - 9x - 5 = 9$$

$$45 - 9 = 18x$$

$$18n = 36$$

$$\therefore \boxed{n=2}$$

Cheek

$$34 - 23 = 9$$

$$\therefore \text{NO}_2 \text{ formed} = 50 - 9 \times 2$$

$$= 50 - 18 = \underline{\underline{32}}$$

Q. The sum of two consecutive multiples of 5 is 55. Find these multiples.

Solution:

Let the first multiple of 5 be x .

Then the other multiple of 5 will be $x + 5$ and their sum = 55

Therefore, $x + x + 5 = 55$

$$\Rightarrow 2x + 5 = 55 \quad \Rightarrow 2x = 55 - 5$$

$$\Rightarrow 2x = 50 \quad \Rightarrow x = 50/2$$

$$\Rightarrow x = 25$$

Therefore, the multiples of 5, i.e., $x + 5 = 25 + 5 = 30$

Therefore, the two consecutive multiples of 5 whose sum is 55 are 25 and 30.

Q. If $3/5^{\text{th}}$ of a number is 4 more than $1/2$ the number, then what is the number?

Solution:

Let the number be x , then $3/5^{\text{th}}$ of the number = $3x/5$

Also, $1/2$ of the number = $x/2$

According to the question,

$3/5^{\text{th}}$ of the number is 4 more than $1/2$ of the number.

$$\Rightarrow 3x/5 - x/2 = 4$$

$$\Rightarrow (6x - 5x)/10 = 4$$

$$\Rightarrow x/10 = 4$$

$$\Rightarrow x = 40$$

The required number is 40.

Q. The difference in the measures of two complementary angles is 12° . Find the measure of the angles.

Solution:

Let the angle be x .

Complement of $x = 90 - x$

Given their difference = 12°

Therefore, $(90 - x) - x = 12^\circ$

$$\Rightarrow 90 - 2x = 12 \quad \Rightarrow -2x = 12 - 90$$

$$\Rightarrow -2x = -78 \quad \Rightarrow 2x/2 = 78/2$$

$$\Rightarrow x = 39 \quad \text{Therefore, } 90 - x = 90 - 39 = 51$$

Therefore, the two complementary angles are 39° and 51°

The difference between the two numbers is 48. The ratio of the two numbers is 7:3. What are the two numbers?

Solution:

Let the common ratio be x .

Let the common ratio be x .

Their difference = 48

According to the question,

$$7x - 3x = 48 \quad \Rightarrow 4x = 48$$

$$\Rightarrow x = 48/4 \quad \Rightarrow x = 12$$

$$\text{Therefore, } 7x = 7 \times 12 = 84$$

$$3x = 3 \times 12 = 36$$

Therefore, the two numbers are 84 and 36.

Q. The cost of two tables and three chairs is Rs.705. If the table costs Rs.40 more than the chair, find the cost of the table and the chair.

Solution:

The table cost Rs. 40 more than the chair.

Let us assume the cost of the chair to be x .

Then the cost of the table = Rs. $40 + x$

The cost of 3 chairs = $3 \times x = 3x$ and the cost of 2 tables $2(40 + x)$

Total cost of 2 tables and 3 chairs = Rs.705

Therefore, $2(40 + x) + 3x = 705$

$$80 + 2x + 3x = 705$$

$$80 + 5x = 705$$

$$5x = 705 - 80$$

$$5x = 625/5$$

$$x = 125 \text{ and } 40 + x = 40 + 125 = 165$$

Therefore, the cost of each chair is Rs.125 and that of each table is Rs.165.

The difference between the two numbers is 48. The ratio of the two numbers is 7:3. What are the two numbers?

Solution:

Let the common ratio be x .

Let the common ratio be x .

Their difference = 48

According to the question,

$$7x - 3x = 48 \Rightarrow 4x = 48$$

$$\Rightarrow x = 48/4 \Rightarrow x = 12$$

$$\text{Therefore, } 7x = 7 \times 12 = 84$$

$$3x = 3 \times 12 = 36$$

Therefore, the two numbers are 84 and 36.

Follow the steps to solve the system of linear equations by using the elimination method:

- (i) Multiply the given equation by suitable constant so as to make the coefficients of the variable to be eliminated equal.
- (ii) Add the new equations obtained if the terms having the same coefficient are opposite signs and subtract if they are of the same sign.
- (iii) Solve the equation thus obtained.
- (iv) Substitute the value found in any one the given equations.
- (v) Solve it to get the value of the other variable.

Solve the system of equation $2x + y = -4$ and $5x - 3y = 1$ by the method of elimination.

$$\begin{array}{l} 2x + y = -4 \\ 5x - 3y = 1 \end{array} \quad \left. \begin{array}{l} \text{Multiply eqn I by 5} \\ \text{" " II by 2} \end{array} \right\} \text{&}$$

$$\begin{array}{r} 10x + 5y = -20 \\ -10x + 6y = -2 \\ \hline 11y = -22 \\ \therefore y = \frac{-22}{11} = -2 \end{array}$$

Substitute $y = -2$ in eqn I

$$\begin{aligned} 2x - 2 &= -4 \\ 2x &= -4 + 2 = -2 \\ x &= \frac{-2}{2} = -1 \end{aligned}$$

$\therefore$ Soln set $(-1, -2)$

Steps to solve by Substitution Method

- (i) Find the value of one variable in terms of the other from one of the given equations.
- (ii) Substitute the value of this variable in the other equation.
- (iii) Solve the equation and get the value of one of the variables.
- (iv) Substitute the value of this variable in any of the equation to get the value of other variable.

Solve the system of equation $2x + y = -4$ and $5x - 3y = 1$ by the method of substitution.

$$\begin{aligned} 2x + y &= -4 \quad \text{--- (i)} \\ 5x - 3y &= 1 \quad \text{--- (ii)} \end{aligned}$$

from eqn (i)

$$y = -4 - 2x \quad \text{--- (4)}$$

Substitute $y = -4 - 2x$ in eqn (ii)

$$5x - 3(-4 - 2x) = 1$$

$$5x + 12 + 6x = 1$$

$$11x = 1 - 12 = -11$$

$$\therefore x = \frac{-11}{11} \Rightarrow \boxed{x = -1}$$

Substitute $x = -1$ in eqn (i)

$$\begin{aligned} y &= -4 - 2(-1) \\ &= -4 + 2 = -2 \end{aligned}$$

$$\therefore \boxed{x = -1, y = -2}$$

Cross multiplication method

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

$$\begin{array}{ccc} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{array}$$

$$\frac{x}{\begin{array}{cc} b_1 & c_1 \\ b_2 & c_2 \end{array}} = \frac{y}{\begin{array}{cc} c_1 & a_1 \\ c_2 & a_2 \end{array}} = \frac{-1}{\begin{array}{cc} a_1 & b_1 \\ a_2 & b_2 \end{array}}$$

$$\frac{x}{(b_1 \times c_2) - (b_2 \times c_1)} = \frac{y}{(c_1 \times a_2) - (a_1 \times c_2)} = \frac{-1}{(a_1 \times b_2) - (a_2 \times b_1)}$$

Solve the system of equation $2x + y = -4$ and $5x - 3y = 1$ by the method of substitution.

$$2x + y = -4$$

$$5x - 3y = 1$$

$$\begin{array}{ccc} 2 & 1 & -4 \\ 5 & -3 & 1 \end{array}$$

$$\frac{x}{\begin{array}{cc} 1 & -4 \\ -3 & 1 \end{array}} = \frac{y}{\begin{array}{cc} -4 & 2 \\ 1 & 5 \end{array}} = \frac{-1}{\begin{array}{cc} 2 & 1 \\ 5 & -3 \end{array}}$$

$$\frac{x}{(1 \times 1) - (-4 \times -3)} = \frac{y}{(-4 \times 5) - (1 \times 2)} = \frac{-1}{(2 \times -3) - (5 \times 1)}$$

$$\frac{x}{1-12} = \frac{y}{-20-2} = \frac{-1}{-6-5} \Rightarrow \frac{1}{11}$$

$$\frac{x}{-11} = \frac{y}{-22} = \frac{1}{11}$$

$$\frac{x}{-11} = \frac{1}{11}$$

$$\& \frac{y}{-22} = \frac{1}{11}$$

$$x = \frac{-11}{11} = -1$$

$$y = \frac{-22}{11} = -2$$

$$\therefore \boxed{x = -1, y = -2}$$

Q. Solve the given equations for x & y

$$\begin{array}{rcl} 2x + 3y & = & 13 \text{ --- (i)} \\ 5x - 2y & = & 4 \text{ --- (ii)} \\ \hline \end{array}$$

LCM of 3 & 2 = 6
 Multiplying eqn (i) by 2
 eqn (ii) by 3

$$\begin{array}{rcl} 4x + 6y & = & 26 \\ 15x - 6y & = & 12 \\ \hline 19x & = & 38 \\ x & = & \frac{38}{19} = 2 \end{array} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{Add} \quad \therefore \boxed{x=2}$$

Substituting value of $x=2$ in eqn (i)

$$\begin{array}{l} 5 \times 2 - 2y = 4 \\ 10 - 4 = +2y \end{array} \quad \Rightarrow 2y = 6 \quad \therefore \boxed{y=3}$$

∴ Solution set = (2, 3)

Q. Solve the given equations for x & y by substitution method

$$\begin{aligned} 2x + 3y &= 13 \quad \text{--- (i)} \\ 5x - 2y &= 4 \quad \text{--- (ii)} \end{aligned}$$

Soln: From eqⁿ (I)

$$2x = 13 - 3y$$

$$\therefore x = \frac{13 - 3y}{2} \quad \text{--- (A)}$$

Substitute $x = \frac{13 - 3y}{2}$ in eqⁿ (II)

$$5\left(\frac{13 - 3y}{2}\right) - 2y = 4$$

$$\frac{65 - 15y - 4y}{2} = 4$$

$$65 - 19y = 4 \times 2$$

$$65 - 8 = 19y \Rightarrow 19y = 57$$

$$\therefore y = \frac{57}{19} \Rightarrow \boxed{y = 3}$$

Substitute $y = 3$ in eqⁿ (A)

$$x = \frac{13 - 3 \times 3}{2} = \frac{4}{2} = 2$$

$$\therefore \text{soln set} = (2, 3)$$

Q. Solve the given equations for x & y
by **Cross-Multiplication Method**

$$2x + 3y = 13 \quad \text{--- (i)}$$

$$5x - 2y = 4 \quad \text{--- (ii)}$$

$$\begin{array}{ccc} x & y & c \\ 2 & 3 & 13 \\ 5 & -2 & 4 \end{array} \quad \begin{array}{ccc} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{array}$$

$$\frac{x}{\frac{b_1 c_2 - (b_2 c_1)}{(a_1 b_2 - (a_2 b_1))}} = \frac{y}{\frac{c_1 a_2 - (c_2 a_1)}{(a_1 b_2 - (a_2 b_1))}} = \frac{-1}{\frac{a_1 b_2 - (a_2 b_1)}{(a_1 b_2 - (a_2 b_1))}}$$

$$\frac{x}{\frac{3 \cdot 4 - (-2 \cdot 13)}{2 \cdot (-2) - (5 \cdot 3)}} = \frac{y}{\frac{13 \cdot 5 - (4 \cdot 2)}{2 \cdot (-2) - (5 \cdot 3)}} = \frac{-1}{\frac{2 \cdot (-2) - (5 \cdot 3)}{2 \cdot (-2) - (5 \cdot 3)}}$$

$$\frac{x}{12 - (-26)} = \frac{y}{65 - 8} = \frac{-1}{-4 - 15}$$

$$\frac{x}{12 + 26} = \frac{y}{57} = \frac{1}{19} \Rightarrow \frac{x}{38} = \frac{y}{57} = \frac{1}{19}$$

$$\frac{x}{38} = \frac{1}{19}$$

$$x = \frac{38}{19} = 2$$

$$\frac{y}{57} = \frac{1}{19}$$

$$y = \frac{57}{19} = 3$$

$$\therefore \boxed{x=2, y=3}$$

1. A fraction is such that if numerator is multiply by 3 and the denominator is reduce by 2 we get $\frac{3}{5}$ but if the numerator is increased by 4 and the denominator is doubled we get $\frac{5}{14}$. Find the fraction.
2. The sum of the numerator and denominator of a fraction is 12. If the denominator is increased by 1, the fraction becomes $\frac{7}{6}$. Find the fraction.
3. A number consists of two digits whose sum if 5. When the digits are reversed, the number becomes greater by 9. Find the number.
4. If twice the age of son is added to age of father, the sum is 56. But if twice the age of the father is added to the age of son, the sum is 82. Find the ages of father and son.

Solⁿ: 1: Numerator $\rightarrow x$ } fraction = $\frac{x}{y}$
 Denominator $\rightarrow y$ } formed

$$\frac{x \times 3}{y - 2} = \frac{3}{5} \Rightarrow 5 \times 3x = 3(y - 2)$$

$$15x = 3y - 6 \Rightarrow 15x - 3y = -6 \quad \text{--- (i)}$$

$$\frac{x + 4}{2y} = \frac{5}{14} \Rightarrow 14(x + 4) = 5(2y)$$

$$\Rightarrow 14x + 56 = 10y \Rightarrow 14x - 10y = -56 \quad \text{--- (ii)}$$

$$\begin{array}{r} 15x - 3y = -6 \\ 14x - 10y = -56 \end{array}$$

$$\begin{array}{r} 150x - 30y = -60 \\ -42x - 30y = -168 \\ \hline 108x = 108 \end{array} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Subtract eq (ii) from (i)}$$

$$108x = 108$$

$$x = \frac{108}{108} = 1 \Rightarrow \boxed{x = 1}$$

Substitute x in eq (i)

$$15 \times 1 - 3y = -6 \Rightarrow 15 + 6 = 3y \Rightarrow 3y = 21$$

$$\Rightarrow \boxed{y = 7}$$

$$\therefore \boxed{\text{fraction} = \frac{1}{7}}$$

$$\therefore \text{fraction} = \frac{1}{7}$$

The sum of a two digit number and the number obtained by reversing the digit is 110.
The difference between the digits is 4. Find the number.

Solⁿ: Let the digit at ten's place be y
unit's place be x .

$$\therefore \text{Number formed} = 10y + x$$

$$\text{Number formed by reversing the digits} = 10x + y$$

$$(10y + x) + (10x + y) = 110$$

$$11x + 11y = 110$$

$$x + y = 10 \quad \text{--- (A)}$$

$$x - y = 4 \quad \text{--- (B)}$$

$$\begin{array}{r} x + y = 10 \\ x - y = 4 \\ \hline 2x = 14 \end{array} \quad \left. \begin{array}{l} \text{(A)} \\ \text{(B)} \end{array} \right\} \text{Add}$$

$$2x = 14$$

$$\boxed{x = 7}$$

$$\therefore y = 10 - x = 10 - 7 = 3 \Rightarrow \boxed{y = 3}$$

$$\therefore \boxed{\text{Number formed} = 37}$$

2. The sum of the numerator and denominator of a fraction is 12. If the denominator is increased by 1, the fraction becomes $\frac{7}{6}$. Find the fraction.

Solⁿ: Numerator = x

Denominator = y

$$\therefore \text{Fraction} = \frac{x}{y}$$

$$x + y = 12 \quad \text{--- (i)}$$

$$\frac{x}{y+1} = \frac{7}{6}$$

$$\Rightarrow 6x = 7(y+1)$$

$$6x - 7y = 7 \quad \text{--- (ii)}$$

$$x + y = 12 \Rightarrow y = 12 - x$$

$$6x - 7(12 - x) = 7$$

$$6x - 84 + 7x = 7 \Rightarrow 13x = 7 + 84 = 91$$

$$\therefore x = \frac{91}{13}$$

$$\Rightarrow \boxed{x = 7} \quad \therefore \boxed{y = 5} \quad \left[\frac{7}{5} \right]$$

Ten years ago, mother was 12 times as old as her daughter and ten years hence she will be twice as old as her daughter will be. Find the present ages.

Soln: Let the present age of
Mother be x yrs
Daughter " y yrs

- 10 years ago:

Mother $\rightarrow x-10$

Daughter $\rightarrow y-10$

$$(x-10) = 12(y-10)$$

$$\therefore x-10 = 12y-120 \Rightarrow x = 12y-120+10$$

$$\therefore \boxed{x = 12y-110} \text{ ————— (A)}$$

10 years Hence

Mother $\rightarrow x+10$

Daughter $\rightarrow y+10$

$$(x+10) = 2(y+10)$$

$$x+10 = 2y+20$$

$$x = 2y+20-10$$

$$\boxed{x = 2y+10} \text{ ————— (B)}$$

from eqn (A) and (B)

$$12y-110 = 2y+10$$

$$12y-2y = 10+110 = 120$$

$$10y = 120 \quad \therefore \boxed{y = 12}$$

$$x = 2 \times 12 + 10 \Rightarrow \boxed{x = 34}$$

$\therefore$ Age of Mother = 34 yrs

" " Daughter = 12 yrs

A boat goes 6 km upstream and 57 km downstream in 5 hours. In 9 hours it can go 21 km upstream and 38 km downstream. Determine the speed of stream and that of boat in still water.

Solⁿ: Let the speed of stream be x km/hr
 " " boat be y km/hr

$$\begin{aligned} \text{Relative speed upstream} &= y - x \\ \text{" " downstream} &= y + x \end{aligned}$$

Case I $T_D = 57 / y + x$
 $T_U = 6 / y - x$

$$\begin{aligned} \frac{57}{y+x} + \frac{6}{y-x} &= 5 \quad \text{--- (A)} \quad \times 7 \\ \frac{38}{y+x} + \frac{21}{y-x} &= 9 \quad \text{--- (B)} \quad \times 2 \end{aligned}$$

$$\begin{aligned} \frac{399}{y+x} + \frac{42}{y-x} &= 35 \\ \frac{76}{y+x} + \frac{42}{y-x} &= 18 \end{aligned}$$

$$\frac{323}{y+x} = 17 \Rightarrow y+x = \frac{323}{17} = 19 \quad \text{--- (1)}$$

Substitute $y+x=19$ in eqⁿ (A)

$$3 \frac{57}{19} + \frac{6}{y-x} = 5 \Rightarrow \frac{6}{y-x} = 5 - 3 = 2$$

$$\therefore y-x = \frac{6}{2} = 3 \quad \text{--- (ii)}$$

$$\begin{aligned} y+x &= 19 \\ y-x &= 3 \\ \hline 2y &= 22 \\ \therefore y &= 11 \end{aligned}$$

$$\therefore x = 19 - 11 = 8$$

$\therefore$ speed of stream = 8 km/hr
 speed of boat = 11 km/hr.

HOMEWORK

1. A fraction is such that if numerator is multiply by 3 and the denominator is reduce by 2 we get $\frac{3}{5}$ but if the numerator is increased by 4 and the denominator is doubled we get $\frac{5}{14}$. Find the fraction.
2. The sum of the numerator and denominator of a fraction is 12. If the denominator is increased by 1, the fraction becomes $\frac{7}{6}$. Find the fraction.
3. A number consists of two digits whose sum if 5. When the digits are reversed, the number becomes greater by 9. Find the number.
4. The sum of a two digit number and the number obtained by reversing the digit is 110. The difference between the digits is 4. Find the number.
5. Seven times a 2 – digit number is equal to 4 times the number obtained by reversing the digits. The difference between the digits is 1. Find the number.
6. point A and B are 50 km part on a highway. A car starts from A and another car start from B at the same time. If they travelled in the same direction, they meet in 5 hours but if they move towards each other they meet in 1 hour. Find their speeds.
7. A boat goes 6 km upstream and 57 km downstream in 5 hours. In 9 hours it can go 21 km upstream and 38 km downstream. Determine the speed of stream and that of boat in still water.
8. The distance between two stations is 340 km. two trains starts simultaneously from these stations on parallel tracks to cross each other. If the speed on one of them is greater than the other by 5 km/hr and the distance between the two trains after 2 hours of their start is 30 km, find the speed of each train.
9. The area of a rectangle gets reduced by 10 square units if its length is reduced by 4 units and breadth is increase by 2 units. If we increased the length by 3 units and breadth by 4 units, the area is increased by 96 square units. Find the length and breadth of the rectangle.

10. Raga travel 150 km to his home partly by bus and partly by auto-rickshaw. He takes 2 hours if travels 30 km by bus and rest by auto-rickshaw. He takes 5 minutes longer if he travel 50 km by bus and the rest by auto-rickshaw. Find the speed of bus and the auto-rickshaw.

Answers

1. $\frac{1}{7}$ 2. $\frac{7}{5}$ 3. 23 4. 37
5. 12 6. 30kmph, 20 kmph
7. Speed of boat 11 kmph, speed of steam 8 kmph
8. 80 kmph, 75 kmph 9. $L = 15$, $b = 8$
10. Speed of bus = 60 kmph, speed of auto-rickshaw 80 kmph