RATIO, PROPORTION AND PERCENTAGE

Ratio

The ratio of two quantities 'a' and 'b' of the same kind and in the same units is a fraction abab which shows that how many times one quantity is of the other and is written as a : b and is read as 'a is to b' where b ≠ 0.

Terms of the ratio

In the ratio a : b, the quantities a and b are called terms of the ratio. Here, 'a' is called the first term or the antecedent and `b' is called the second term or consequent.

Example:

In the ratio 5 : 9, 5 is called the antecedent and 9 is called the consequent.

Properties of ratio

If the first term and the second term of a ratio are multiplied/divided by the same non-zero number, the ratio does not change.

● a/b = xa/xb, (x ≠ 0) So, a : b = xa : xb

● a/b = (a/x)/(b/x), (x ≠ 0) So, a : b = a/x : b/x

Ratio in the simplest form

A ratio a : b is said to be in the simplest form if a and b have no common factor other than 1.

Example:

Express 15 : 10 in the simplest form.

Solution:

15/10

= (15 ÷ 5)/(10 ÷ 5)

= 3/2 (In this we cancelled the common factor 5)

Thus, we have expressed the ratio 15/10 in the simplest form, i.e., 3/2 and the terms 3 and 2 have common factor only 1.

Note:

● In ratio, quantities being compared must be of the same kind, otherwise the comparison becomes meaningless.

For example; comparing 20 pens and 10 apples is meaningless.

● They must be expressed in the same units.

● In a ratio, order of the terms is very important. The ratio a: b is different from b : a.

● The ratio has no units.

For example; Dozen = 12, Gross = 144, Score = 20

Decade = 10, Century = 100, Millennium = 1000

Example:

Express the following ratios in the simplest form.

(a) 64 cm to 4.8 m

(b) 36 minutes to 36 seconds

(c) 30 dozen to 2 hundred

Solution:

(a) Required ratio = 64 cm/4.8 m

= 64 cm/(4.8 × 100) cm

= 64 cm/480m

= 64/480

= 2/15

= 2 : 15

(b) Required ratio = 36 minutes/36 seconds

= (36 × 60 seconds)/(36 seconds)

= 60/1

= 60 : 1

(c) Required ratio = (30 dozen)/(2 hundred)

= (30 × 12)/(2 × 100)

= 3/10

= 3 : 10

Simplification of ratio

If the terms of the ratio are expressed in fraction form; then find the Least Common Multiple of the denominators of these fractions. Now, multiply each fraction by the L.C.M. The ratio is simplified.

Example:

Simplify the following ratios.

(a) $^5/_2 : {}^3/_8 : {}^4/_9$

(b) $2^1/_7 : 3^2/_5$

Solution:

(a) The L.C.M. of 2, 8, 9 = 2 × 2 × 2 × 3 × 3

= 8 × 9

= 72

Now, multiplying each fraction by the L.C.M.

5/2 × 72 = 160   3/8 × 72 = 27   4/9 × 72 = 32

So, the ratio becomes 160 : 27 : 32

(b) $2^1/_7 : 3^2/_5$

= 15/7 : 17/5 (Here, we have used (a/b)/(c/d) = abab × dcdc)

= 15/7 × 5/17

= 75/119

So, the ratio becomes 75 : 119

Comparison of ratios

Ratios can be compared as fractions. Convert them into equivalent ratios as we convert the given fractions into equivalent fractions and then compare.

Example:

Which ratio is greater?

$2^1/_3 : 3^1/_2$, 2.5 : 3.5, 4/5 : 3/2

Solution:

Simplifying the given 3 ratios

$2^1/_3 : 3^1/_2 = {}^7/_3 : {}^7/_2 = {}^7/_3 \div {}^7/_2 = {}^7/_3 \times {}^2/_7 = {}^2/_3$

$2.5 : 3.5 = {}^{25}/_{35} = {}^5/_7$

$^4/_5 : {}^3/_2 = {}^4/_5 \times {}^2/_3 = {}^8/_{15}$

$^{2}/_{3}$, $^{5}/_{7}$, $^{8}/_{15}$

L.C.M. of 3, 7, 15 = 105

$^{2}/_{3}$ = (2 × 35)/(3 × 35) = ^{7}<span style='font-size: 50%'>$/_{105}$,

$^{5}/_{7}$ = (5 × 15)/(7 × 15) = $^{45}/_{105}$,

$^{8}/_{15}$ = (8 × 7)/(15 × 7) = $^{56}/_{105}$

7010570105 > 5610556105 > 4510545105

Therefore, $^{2}/_{3}$ > $^{8}/_{15}$ > $^{5}/_{7}$

Therefore, 2$^{1}/_{3}$: 3$^{1}/_{2}$ > 4/5 : 3/2 > 2.5 : 3.5

Dividing the given quantity in the given ratio

If 'p' is the given quantity to be divided in the ratio a : b, then add the terms of the a ratio, i.e., a + b, then the 1st part = {a/(a + b)} × p and 2nd part {b/(a + b)} × p

Example:

Divide Rs.290 among A, B, C in the ratio 1$^{1}/_{2}$, 1$^{1}/_{4}$ and $^{3}/_{8}$.

Solution:

Given ratios = $^{3}/_{2}$: $^{5}/_{4}$: $^{3}/_{8}$.

The L.C.M. of 2, 4, 8 is 8.

So we have $^{3}/_{2}$ × 8 : $^{5}/_{4}$ × 8 : $^{3}/_{8}$ × 8 = 12 : 10 : 3

Therefore, Share of A = 12/29 × 290 = Rs.120

Share of B = 10/29 × 290 = Rs.100

Share of C = 3/29 × 290 = Rs.30

Proportion

We have already learnt that statement of equality of ratios is called proportion, if four quantities a, b, c, d are in proportion, then a : b = c : d or a : b : : c : d (: : is the symbol used to denote proportion).

⇒ ab = cd

⇒ a × d = b × c

⇒ ad = bc

Here *a*, *d* are called the **extreme terms** in which *a* is called the **first term** and *d* is called the **fourth term** and *b*, *c* are called the **mean terms** in which *b* is called the **second term** and *c* is called the **third term**.

Thus, we say, if product of mean terms = the product of extreme terms, then the terms are said to be in proportion.

Also, if **a : b :: c : d**, then d is called the fourth proportional of a, b, c.

Continued Proportion

The three quantities a, b, c are said to be in continued proportion if a : b :: b : c

⇒ ab = bc

⇒ $a \times c = b^2$

⇒ $b^2 = ac$

⇒ $b = \sqrt{ac}$

Here, *b* is called the **mean proportional** of *a* and *c*. The square of **middle term** is equal to the product of **1st term** and **3rd term**.

Also, if **a : b :: b : c**, then c is called the third proportional of a, b.

Example:

Determine if the following are in proportion.

(a) 6, 12, 24

(b) $1\frac{2}{3}$, $6\frac{1}{4}$, $\frac{4}{9}$, $\frac{5}{3}$

Solution:

(a) Here, product of first term and third term = 6 × 24 = 144 and square of middle term = $(12)^2$ = 12 × 12 = 144

(b) $1\frac{2}{3}$, $6\frac{1}{4}$, $\frac{4}{9}$, $\frac{5}{3}$

Here, $a = 1\frac{2}{3}$ $b = 6\frac{1}{4}$ $c = \frac{4}{9}$ $d = \frac{5}{3}$

$a : b = 1\frac{2}{3} : 6\frac{1}{4}$ $c : d = \frac{4}{9} : \frac{5}{3}$

$= \frac{5}{3} : \frac{25}{4}$ $= (4/9)/(5/3)$

$= (5/3)/(25/4)$ $= 4/9 \times 3/5$

$= 5/3 \times 4/25$ $= 4/3 \times 1/5$

$= 4/15$ $= 4/15$

Since, **a : b = c : d**

Therefore, $1\frac{2}{3}$, $6\frac{1}{4}$, $\frac{4}{9}$, $\frac{5}{3}$ are in proportion.

Q1. If 2A = 3B = 4C, find A : B : C

Solution:

Let 2A = 3B = 4C = x

So, A = x/2 B = x/3 C = x/4

The L.C.M of 2, 3 and 4 is 12

Therefore, A : B : C = x/2 × 12 : x/3 × 12 : x/4 = 12

= 6x : 4x : 3x

= 6 : 4 : 3

Therefore, A : B : C = 6 : 4 : 3

Q2. What must be added to each term of the ratio 2 : 3, so that it may become equal to 4 : 5?

Solution:

Let the number to be added be x, then (2 + x) : (3 + x) = 4 : 5

⇒ (2 + x)/(5 + x) = 4/5

5(2 + x) = 4(3 + x)

10 + 5x = 12 + 4x

5x - 4x = 12 - 10

x = 2

Q3. The length of the ribbon was originally 30 cm. It was reduced in the ratio 5 : 3. What is its length now?

Solution:

Length of ribbon originally = 30 cm

Let the original length be 5x and reduced length be 3x.

But 5x = 30 cm

x = 30/5 cm = 6 cm

Therefore, reduced length = 3 cm

= 3 × 6 cm = 18 cm

4. Mother divided the money among Ron, Sam and Maria in the ratio 2 : 3 : 5. If Maria got Rs.150, find the total amount and the money received by Ron and Sam.

Solution:

Let the money received by Ron, Sam and Maria be 2x, 3x, 5x respectively.

Given that Maria has got Rs. 150.

Therefore, 5x = 150

or, x = 150/5

or, x = 30

So, Ron got = 2x

= Rs. 2 × 30 = Rs.60

Sam got = 3x

= 3 × 60 = Rs.90

Therefore, the total amount Rs.(60 + 90 + 150) = Rs.300

5. Divide Rs.370 into three parts such that second part is 1/4 of the third part and the ratio between the first and the third part is 3 : 5. Find each part.

Solution:

Let the first and the third parts be 3x and 5x.

Second part = 1/4 of third part.

= (1/4) × 5x

= 5x/4

Therefore, 3x + (5x/4) + 5x = 370

(12x + 5x + 20x)/4 = 370

37x/4 = 370

x = (370 × 4)/37

x = 10 × 4

x = 40

Therefore, first part = 3x

= 3 × 40

= Rs.120

Second part = 5x/4

= 5 × 40/4

= Rs.50

Third part = 5x

= 5 × 40

= Rs. 200

6. The first, second and third terms of the proportion are 42, 36, 35. Find the fourth term.

Solution:

Let the fourth term be x.

Thus 42, 36, 35, x are in proportion.

Product of extreme terms = 42 ×x

Product of mean terms = 36 X 35

Since, the numbers make up a proportion

Therefore, 42 × x = 36 × 35

or, x = (36 × 35)/42

or, x = 30

Therefore, the fourth term of the proportion is 30.

7. The ratio of number of boys and girls is 4 : 3. If there are 18 girls in a class, find the number of boys in the class and the total number of students in the class.

Solution:

Number of girls in the class = 18

Ratio of boys and girls = 4 : 3

According to the question,

Boys/Girls = 4/5

Boys/18 = 4/5

Boys = (4 × 18)/3 = 24

Therefore, total number of students = 24 + 18 = 42.

8. Find the third proportional of 16 and 20.

Solution:

Let the third proportional of 16 and 20 be x.

Then 16, 20, x are in proportion.

This means 16 : 20 = 20 : x

So, 16 × x = 20 × 20

x = (20 × 20)/16 = 25

Therefore, the third proportional of 16 and 20 is 25.

<u>EXERCISE</u>

1. The ratio of monthly income to the savings in a family is 5 : 4 If the savings be Rs.9000, find the income and the expenses.

2. What should be added to the ratio 5 : 11, so that the ratio becomes 3 : 4?

3. Two numbers are in the ratio 7 : 5. If 2 is subtracted from each of them, the ratio becomes 3 : 2. Find the numbers.

4. Two numbers are in the ratio 3 : 7. If their sum is 710, find the numbers.

5. Find the ratio of A : B: C when

(a) A : B = 3 : 5   A : C = 6 : 7

(b) B : C = 1/2: 1/6   A : B = 1/3 ∶ 1/5

6. A sum of money is divided among Ron and Andy in the ratio 4 : 7. If Andy's share is Rs.616, find the total money.

7. Two numbers are in the ratio 5 : 7. On adding 1 to the first and 3 to the second, their ratio becomes 6/9 . Find the numbers.

8. The difference between two numbers is 33 and the ratio between them is 5 : 2. Find the numbers.

9. The ages of A and B are in the ratio 3 : 5. Four years later, the sum of their ages is 48. Find their present ages.

10. Ramon has notes of Rs.100, Rs.50 and Rs.10 respectively. The ratio of these notes is 2 : 3 : 5 and the total amount is Rs.2,00,000. Find the numbers of notes of each kind.

11. If 4A = 5B = 6C, find the ratio of A : B : C.

12. Divide Rs.430 into 3 parts such that A gets 5/4 of B and the ratio between B and C is 3 : 4.

13. A certain sum of money is divided among A, B, C in the ratio 2 : 3 : 4. If A's share is Rs.200, find the share of B and C.

14. Divide Rs.940 among A, B, C in the ratio 1/3: 1/4 ∶ 1/5

15. The ratio of number of male and female teachers in a school is 3 : 4. If there are 16 female teachers, find the number of male teachers.

16. In a library the ratio of English books to Math books, is the same as the ratio of Math books to Science book. If there are 1200 books on English and 1800 books on Math, find the number of Science books.

17. Set up all the possible proportions from the numbers 12, 15, 8, 10.

18. Find the first term, if second, third and fourth terms are 21, 80, 120.

19. Find the second term, if first, third and fourth terms are 15, 27, 63.

20. Find the mean term, if the other two terms of a continued proportion are 15 and 60.

Answers:

1. Rs.11250, Rs.2250 2. 13 3. 14, 10 4. 213, 497

5. (a) 6 : 10 : 7 (b) 5 : 3 : 1 6. Rs.968 7. 15, 21

8. 55, 22 9. 15 years, 25 years 10. 1000, 1500, 2500

11. 15 : 12 : 10 12. Rs.150, Rs.120, Rs.160 13. Rs.300, Rs.400

14. Rs.400, Rs.300, Rs.240 15. 12 16. 2700

17. (i) 12 : 15 = 8 : 10 (ii) 15 : 12 = 10 : 8 (iii) 12 : 8 = 15 : 10

(iv) 8 : 12 = 10 : 15

18. 14 19. 35 20. 30

Multiple Choice Questions on Ratio and Proportion

1. A ratio equivalent to 3 : 7 is:

(i) 3 : 9; (ii) 6 : 10; (iii) 9 : 21; (iv) 18 : 49

2. The ratio 35 : 84 in simplest form is:

(i) 5 : 7; (ii) 7 : 12; (iii) 5 : 12; (iv) none of these

3. In a class there are 20 boys and 15 girls. The ratio of boys to girls is:

(i) 4 : 3; (ii) 3 : 4; (iii) 4 : 5; (iv) none of these

4. Two numbers are in the ratio 7 : 9. If the sum of the numbers is 112, then the larger number is:

(i) 49; (ii) 72; (iii) 63; (iv) 42

5. The ratio of 1.5 m to 10 cm is:

(i) 1 : 15; (ii) 15 : 10; (iii) 10 : 15; (iv) 15 : 1

6. The ratio of 1 hour to 300 seconds is:

(i) 1 : 12; (ii) 12 : 1; (iii) 1 : 5; (iv) 5 : 1

7. In 4 : 7 : : 16 : 28, 7 and 16 are called

(i) extreme terms; (ii) middle terms; (iii) b middle and c extreme term; (iv) none of these

8. The first, second and fourth terms of a proportion are 16, 24 and 54 respectively. Then the third term is:

(i) 36; (ii) 28; (iii) 48; (iv) 32

9. If 12, 21, 72, 126 are in proportion, then:

(i) 12 × 21 = 72 × 126; (ii) 12 × 72 = 21 × 126; (iii) 12 × 126 = 21 × 72; (iv) none of these

10. If x, y and z are in proportion, then:

(i) x : y : : z : x; (ii) x : y : : y : z; (iii) x : y : : z : y; (iv) x : z : : y : z

11. 7 : 12 is equivalent to:

(i) 28 : 40; (ii) 42 : 71; (iii) 72 : 42; (iv) 42 : 72

12. The length and breadth of a rectangle are in the ratio 3 : 1. If the breadth is 7 cm, then the length of the rectangle is:

(i) 14 cm; (ii) 16 cm; (iii) 18 cm; (iv) 21 cm

13. The value of m, if 3, 18, m, 42 are in proportion is:

(i) 6; (ii) 54; (iii) 7; (iv) none of these

14. Length and width of a field are in the ratio 5 : 3. If the width of the field is 42 m then its length is:

(i) 100 m; (ii) 80 m; (iii) 50 m; (iv) 70 m

Solved Examples on Percentage

1. In an election, candidate A got 75% of the total valid votes. If 15% of the total votes were declared invalid and the total numbers of votes is 560000, find the number of valid vote polled in favour of candidate.

Solution:

Total number of invalid votes = 15 % of 560000

= 15/100 × 560000

= 8400000/100

= 84000

Total number of valid votes 560000 – 84000 = 476000

Percentage of votes polled in favour of candidate A = 75 %

Therefore, the number of valid votes polled in favour of candidate A = 75 % of 476000

= 75/100 × 476000

= 35700000/100

= 357000

2. A shopkeeper bought 600 oranges and 400 bananas. He found 15% of oranges and 8% of bananas were rotten. Find the percentage of fruits in good condition.

Solution:

Total number of fruits shopkeeper bought = 600 + 400 = 1000

Number of rotten oranges = 15% of 600

= 15/100 × 600

= 9000/100 = 90

Number of rotten bananas = 8% of 400

= 8/100 × 400

= 3200/100

= 32

Therefore, total number of rotten fruits = 90 + 32 = 122

Therefore Number of fruits in good condition = 1000 - 122 = 878

Therefore Percentage of fruits in good condition = (878/1000 × 100)%

= (87800/1000)%

= 87.8%

3. Aaron had Rs. 2100 left after spending 30 % of the money he took for shopping. How much money did he take along with him?

Solution:

Let the money he took for shopping be m.

Money he spent = 30 % of m

= 30/100 × m

= 3/10 m

Money left with him = m – 3/10 m = (10m – 3m)/10 = 7m/10

But money left with him = Rs. 2100

Therefore 7m/10 = Rs. 2100

m = Rs. 2100× 10/7

m = Rs. 21000/7

m = Rs. 3000

Therefore, the money he took for shopping is Rs. 3000.

4. In an exam Ashley secured 332 marks. If she secured 83 % makes, find the maximum marks.

Solution:

Let the maximum marks be m.

Ashley's marks = 83% of m

Ashley secured 332 marks

Therefore, 83% of m = 332

⇒ 83/100 × m = 332

⇒ m = (332 × 100)/83

⇒ m =33200/83

⇒ m = 400

Therefore, Ashley got 332 marks out of 400 marks.

5. An alloy contains 26 % of copper. What quantity of alloy is required to get 260 g of copper?

Solution:

Let the quantity of alloy required = m g

Then 26 % of m =260 g

⇒ 26/100 × m = 260 g

⇒ m = (260 × 100)/26 g

⇒ m = 26000/26 g

⇒ m = 1000 g

6. There are 50 students in a class. If 14% are absent on a particular day, find the number of students present in the class.

Solution:

Number of students absent on a particular day = 14 % of 50

i.e., 14/100 × 50 = 7

Therefore, the number of students present = 50 - 7 = 43 students.

7. In a basket of apples, 12% of them are rotten and 66 are in good condition. Find the total number of apples in the basket.

Solution:

Let the total number of apples in the basket be m

12 % of the apples are rotten, and apples in good condition are 66

Therefore, according to the question,

88% of m = 66

⟹ 88/100 × m = 66

⟹ m = (66 × 100)/88

⟹ m = 3 × 25 ⟹ m = 75

Therefore, total number of apples in the basket is 75.

8. In an examination, 300 students appeared. Out of these students; 28 % got first division, 54 % got second division and the remaining just passed. Assuming that no student failed; find the number of students who just passed.

Solution:

The number of students with first division = 28 % of 300

= 28/100 × 300

= 8400/100

= 84

And, the number of students with second division = 54 % of 300

= 54/100 × 300

=16200/100

= 162

Therefore, the number of students who just passed = 300 – (84 + 162)

= 54

Problems on Percentage:

1. In a class 60% of the students are girls. If the total number of students is 30, what is the number of boys?

Answer: 12

2. Emma scores 72 marks out of 80 in her English exam. Convert her marks into percent.

Answer: 90%

3. Mason was able to sell 35% of his vegetables before noon. If Mason had 200 kg of vegetables in the morning, how many grams of vegetables was he able to see by noon?

Answer: 70 kg

4. Alexander was able to cover 25% of 150 km journey in the morning. What percent of journey is still left to be covered?

Answer: 112.5 km

5. A cow gives 24 l milk each day. If the milkman sells 75% of the milk, how many liters of milk is left with him?

Answer: 6 l

Arithmetic Progressions

Introduction:

We see lot of pattern in our daily lives. For example, Rohan salary in his first year is 2000 and he will be getting increment of 500 every year,So his salary for First, second, third, fourth …. Will be of the form 2000,2500,3000,3500
So here we see the pattern in Rohan salary. The difference between two consecutive year salaries is constant. In the similar, world around us present us these pattern in many form. In this chapter, we will learn a special type of pattern called Arithmetic Progression

What is arithmetic progression?

An arithmetic progression is a sequence of numbers such that the difference of any two successive members is a constant

Some Important points about AP

1. The difference is called the common difference of the AP and It is denoted by d
2. The members are called terms. The first member is called first term
3. We can denote common difference by d
4. If a1, a2, a3, a4, a5a1, a2, a3, a4, 5 are the terms in AP then
   D=a2−a1=a3−a2=a4−a3=a5−a4
5. We can represent the general form of AP in the form
   a, a+d, a+2d, a+3d, a+4d,..
   Where aa is first term and dd is the common difference
6. If the AP series has last term then it s finite Arithmetic Progression and if the AP series has infinite then it is called the Inifinite Arithmetic Progression.

Example: Find if the below series is Arithmetic Progression
1,4,7,10,13,16,19,22,25,...
Solution:
D=3=3=3=3 (i.e all the common differences are same)
So it is AP

1. The first and last term of an A.P. are 1 and 11. If the sum of its terms is 36, then the number of terms will be

(a) 5 (b) 6 (c) 7 (d) 8

2. If the sum of three consecutive terms of an increasing A.P. is 51 and the product of the first and third of these terms is 273, then the third term is

(a) 13 (b) 9 (c) 21 (d) 17

3. If 7th and 13th term of an A.P. are 34 and 64 respectively, then its 18th term is

(a) 87 (b) 88 (c) 89 (d) 90

4. If the first, second and last term of an A.P. are *a*, *b* and 2*a* respectively, its sum is

(a) $\frac{ab}{2(b-a)}$ (b) $\frac{ab}{b-a}$

(c) $\frac{3ab}{2(b-a)}$ (d) none of these

5. If the sum of *n* terms of an A.P. is then its *n*th term is

(a) $4n - 3$ (b) $3n - 4$ (c) $4n + 3$ (d) $3n + 4$

6. If 3 times the third term of an A.P. is equal to 5 times the fifth term. Then its 8th term is

(a) 0 (b) 1 (c) 2 (d) 3

7. In an A.P., $a_{m+n} + a_{m-n}$ is equal to

(a) 0 (b) 1 (c) $2a_m$ (d) a_m

8. If *p*th term of an A.P. $\frac{1}{q}$ is and qth term is $\frac{1}{p}$, then the sum of *pq* terms is

(a) $\frac{p-q}{2}$ (b) $\frac{p+q}{2}$

(c) $\frac{pq+1}{2}$ (d) $\frac{pq-1}{2}$

9. If the sum of first *n* even natural number is equal to *k* times the sum of first *n* odd natural number then value of *k* will be

(a) $\frac{1}{n}$ (b) $\frac{n-1}{n}$

(c) $\frac{n+1}{2n}$ (d) $\frac{n+1}{n}$

10. If in an A.P., $S_n = n^2 p$, $S_m = m^2 p$ and S_r denotes the sum of *r* terms of the A.P., then is equal to

(a) $\frac{1}{2}p^3$ (b) mnp

(c) p^3 (d) $(m+n)p^2$

11. The number of terms of an A.P. 3, 7, 11, 15... to be taken so that the sum is 406 is

(a) 5 (b) 10 (c) 12 (d) 14

12. Sum of *n* terms of the series $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}\ldots$ is

(a) 1 (b) $\frac{n(n+1)}{\sqrt{2}}$

(c) $\frac{n(n+1)}{2}$ (d) $2n(n+1)$

13. The 9th term of an A.P. is 449 and 449th term is 9. The term which is equal to zero is

(a) 508^{th} (b) 502^{th} (c) 501^{th} (d) none of these

14. If the sum of *p* terms of an A.P. is *q* and the sum of *q* terms is *p*, then the sum of $(p + q)$ terms will be

(a) $p + q$ (b) $-(p + q)$ (c) $p - q$ (d) 0

15. If are in A.P. then the value of *x* is

(a) 6 (b) 2 (c) 8 (d) 0

16. If sum of *n* terms of an A.P. is then common difference of the A.P. is

(a) 3 (b) 4 (c) 6 (d) 7

17. Sum of first *n* natural number is

(a) $\frac{n(n-1)}{2}$ (b) $\frac{n(n+1)}{2}$

(c) $\frac{n(n+1)(2n+1)}{6}$ (d) $\left[\frac{n(n+1)}{2}\right]^2$

18. If *n*th terms of the APs 63, 65, 67, ... and 3, 10, 17, ... are equal, then *n* is

(a) 27 (b) 23 (c) 15 (d) 13

19. If *p*th term of an A.P. $\frac{3p-1}{6}$, is then sum of first *n* terms of A.P. is

(a) $\frac{n}{12}(3n+1)$ (b) $\frac{n}{12}(3n-1)$

(c) $\frac{n}{6}(3n+1)$ (d) $\frac{n}{6}(3n-1)$

20. Sum of all natural numbers lying between 250 and 1000 which are exactly divisible by 3 is

(a) 157365 (b) 153657 (c) 156375 (d) 155637

ANSWERS

1. (b)	2. (c)	3. (c)	4. (c)	5. (c)
6. (a)	7. (c)	8. (c)	9. (d)	10. (c)
11. (d)	12. (b)	13. (d)	14. (b)	15. (a)
16. (c)	17. (b)	18. (d)	19. (a)	20. (c)

Q.1. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i) The taxi fare after each km when the fare is Rs. 15 for the first km and Rs. 8 for each additional km.

(ii) The amount of air Present in a cylinder when a vacuum pump $\frac{1}{4}$ removes of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every metre of digging, when it costs 150 for the first metre and rises by Rs. 50 for each subsequent metre.

(iv) The amount of money in the account every year, when Rs. 10,000 is deposited at compound interest at 8% per annum.

Sol. (i) Let us consider,

The first term (T_1) = Fare for the first km = Rs. 15 since, the taxi fare beyond the first km is Rs. 8 for each additional km. $\Rightarrow T_1 = 15$

∴ Fare for 2 km = Rs. 15 + 1 x Rs. 8 $\Rightarrow T_2 = a + 8$ [where a = 15]

Fare for 3km = Rs.15 + 2 × Rs. 8 $\Rightarrow T_3 = a + 16$

Fare for 4km = Rs.15 + 3 × Rs.8 $\Rightarrow T_4 = a + 24$

Fare for 5km = Rs.15 + 4 x Rs. 8 $\Rightarrow T_5 = a + 32$

Fare for n km = Rs. 15 + (n – 1)8 $\Rightarrow T_n = a + (n – 1) 8$

We see that above terms form an A.P.

(ii) Let the amount of air in the cylinder = x

∴ Air removed in 1st stroke = $\frac{1}{4}x$

$\Rightarrow$ Air left after 1st stroke = $x - \frac{1}{4}x = \frac{3x}{4}$

Air left after 2nd stoke = $\frac{3x}{4} - \frac{1}{4}\left(\frac{3x}{4}\right) = \frac{3x}{4} - \frac{3x}{16} = \frac{9}{16}x$

Air left after 3nd stoke = $\frac{9}{16}x - \frac{1}{4}\left(\frac{9}{16}x\right) = \frac{9x}{16} - \frac{9x}{64} = \frac{27x}{64}$

Air left after 4nd stoke = $\frac{27}{64}x - \frac{1}{4}\left(\frac{27x}{64}\right) = \frac{27x}{64} - \frac{27}{256} = \frac{91x}{256}$

Thus, the terms are:

$x, \frac{3x}{4}, \frac{9}{16}x, \frac{27}{64}x, \frac{91x}{256}$

Here,

$\frac{3x}{4} - x = \frac{-x}{4}$

$\frac{9}{16}x - \frac{3x}{4} = \frac{-3x}{16}$

Since

$\left(\frac{-x}{4}\right) \neq \left(\frac{-3x}{16}\right)$

The above terms are not in A.P.

(iii) Here, The cost of digging for 1st metre = Rs. 150

The cost of digging for first 2 metres = Rs. 150 + 50 = Rs. 200

The cost of digging for first 3 metres = Rs. 150 + (Rs. 50) × 2 = Rs. 250

The cost of digging for first 4 metres = Rs. 150 + (Rs. 50) × 3 = Rs. 300

∴ The terms are: 150, 200, 250, 300, ...

Since, 200 – 150 = 50

And 250 – 200 = 50

$\Rightarrow$ (200 – 150) = (250 – 200)

∴ The above terms form an A.P.

(iv) ∴ The amount at the end of 1st year = $10000\left(1+\frac{8}{100}\right)^1$

The amount at the end of 2nd year = $10000\left(1+\frac{8}{100}\right)^2$

The amount at the end of 3rd year = $10000\left(1+\frac{8}{100}\right)^3$

The amount at the end of 4th year = $10000\left(1+\frac{8}{100}\right)^4$

∴ The terms are

$$[10000], \left[10000\left(1+\frac{8}{100}\right)\right], \left[10000\left(1+\frac{8}{100}\right)^2\right], \left[10000\left(1+\frac{8}{100}\right)^3\right], \ldots\ldots$$

Obviously,

$$\left[10000\left(1+\frac{8}{100}\right)\right] - [10000] \neq \left[10000\left(1+\frac{8}{100}\right)^2\right] - \left[10000\left(1+\frac{8}{100}\right)\right]$$

∴ The above terms are not in A.P.

Q.2. Write first four terms of the AP, when the first term a and the common difference d are given as follows:

(i) a = 10, d = 10

(ii) a = –2, d = 0

(iii) a = 4, d = – 3

(iv) $a = -1, d = \frac{1}{2}$

(v) a = – 1.25, d = – 0.25

Sol. (i) ∵ $T_n = a + (n – 1)d$

∴ For a = 10 and d = 10, we have:

$T_1 = 10 + (1 – 1) \times 10 = 10 + 10 = 10$

$T_2 = 10 + (2 – 1) \times 10 = 10 + 10 = 20$

$T_3 = 10 + (3 – 1) \times 10 = 10 + 20 = 30$

$T_4 = 10 + (4 – 1) \times 10 = 10 + 30 = 40$

Thus, the first four terms of A.P. are:

10, 20, 30, 40.

(ii) T = a + (n – 1) d

∴ For a = – 2and d = 0, we have:

$T_1 = – 2 + (1 – 1) \times 0 = – 2 + 0 = –2$

$T_2 = – 2 + (2 – 1) \times 0 = – 2 + 0 = –2$

$T_3 = – 2 + (3 – 1) \times 0 = – 2 + 0 = –2$

$T_4 = – 2 + (4 – 1) \times 0 = – 2 + 0 = –2$

∴ The first four terms are:

– 2, – 2, – 2 – 2.

(iii) $T_n = a + (n – 1)d$

∴ For a = 4 and d = – 3, we have:

$T_1 = 4 + (1 – 1) \times (– 3) = 4 + 0 = 4$

$T_2 = 4 + (2 – 1) \times (– 3) = 4 + (–3) = 1$

$T_3 = 4 + (3 – 1) \times (– 3) = 4 + (–6) = –2$

$T_4 = 4 + (4 – 1) \times (– 3) = 4 + (–9) = –5$

Thus, the first four terms are:

4, 1, – 2, – 5.

(iv) $T_n = a + (n – 1)d$

For a = – 1 and d = $\frac{1}{2}$, we get

$T_1 = -1 + (1-1) \times \frac{1}{2} = -1 + 0 = -1$

$T_2 = -1 + (2-1) \times \frac{1}{2} = -1 + \frac{1}{2} = -\frac{1}{2}$

$T_3 = -1 + (3-1) \times \frac{1}{2} = -1 + 1 = 0$

$T_4 = -1 + (4-1) \times \frac{1}{2} = -1 + \frac{3}{2} = \frac{1}{2}$

∴ The first four terms are:

$-1, -\frac{1}{2}, 0, \frac{1}{2}$.

(v) $T_n = a + (n-1)d$

∴ For a = –1.25 and d = –0.25, we get

$T_1 = -1.25 + (1-1) \times (-0.25) = -1.25 + 0 = -1.25$

$T_2 = -1.25 + (2-1) \times (-0.25) = -1.25 + 0 = -1.25$

$T_3 = -1.25 + (3-1) \times (-0.25) = -1.25 + 0 = -1.25$

$T_4 = -1.25 + (4-1) \times (-0.25) = -1.25 + 0 = -1.25$

Thus, the four terms are:

–1.25 , –1.50, –1.75, –2.0

Q.3. For the following APs, write the first term and the common difference:

(i) 3, 1, –1, –3, ...

(ii) –5, –1, 3, 7, ...

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, ...$

(iv) 0.6, 1.7, 2.8, 3.9, ...

Sol. (i) We have : 3, 1, –1, –3, ...

⇒ $T_1 = 3$ ⇒ a = 3

$T_2 = 1$

$T_3 = -1$

$T_4 = -3$

∴ $T_2 - T_1 = 1 - 3 = -2$

$T_4 - T_3 = -3 - (-1) = -3 + 2 = -2$

Thus, a = 3 and d = –2

(ii) We have: –5, –1, 3, 7....

⇒ $T_1 = -5$ ⇒ a = –5

$T_2 = -1$ ⇒ $d = T_2 - T_1 = -1 - (-5) = -1 + 5 = 4$

$T_3 = 3$ ⇒ d = –1 + 5 = 4

$T_4 = 7$ $T_4 - T_3 = 7 - 3 = 4$ ⇒ d = 4

Thus, a = –5 and d = 4

(iii) We have: $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, ...$

$\Rightarrow T_1 = \frac{1}{3} \Rightarrow a = \frac{1}{3}$

$T_2 = \frac{5}{3} \Rightarrow d = T_2 - T_1 = \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

$\left.\begin{array}{l} T_3 = \frac{9}{13} \\ T_4 = \frac{13}{3} \end{array}\right\} \Rightarrow d = T_4 - T_3 = \frac{13}{3} - \frac{9}{3} = \frac{4}{3}$

Thus, $a = \frac{1}{3}$ and $d = \frac{4}{3}$

(iv) We have : 0.6, 1.7, 2.8, 3.9, ...

$\Rightarrow T_1 = 0.6 \Rightarrow a = 0.6$

$T_2 = 1.7 \Rightarrow d = T_2 - T_1 = 1.7 - 0.6 = 1.1$

$T_3 = 2.8$

$T_4 = 3.9 \Rightarrow d = T_4 - T_3 = 3.9 - 2.8 = 1.1$

Thus, a = 0.6 and d = 1.1

Q.4. Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

(i) 2, 4, 8, 16...

(ii) $2, \frac{5}{2}, 3, \frac{7}{2}...$

(iii) –1.2, –3.2, –5.2, –7.2, ...

(iv) –10, –6, –2, 2, ...

(v) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}\, 3 + 3\sqrt{2}, ...$

(vi) 0.2, 0.22, 0.222, 0.2222, ...

(vii) 0, – 4, – 8, – 12,...

(viiii) $-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, ...$

(ix) 1, 3, 9, 27,...

(x) a, 2a, 3a, 4a, ...

(xi) $a, a^2, a^3, a^4, ...$

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, ...$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, ...$

Sol. (i) We have: 2, 4, 8, 16, ...

$\left.\begin{array}{l} T_1 = 2 \\ T_2 = 4 \end{array}\right\} \Rightarrow T_2 - T_1 = 4 - 2 = 2$

$\left.\begin{array}{l} T_3 = 8 \\ T_4 = 16 \end{array}\right\} \Rightarrow T_4 - T_3 = 16 - 8 = 8$

Since $2 \neq 8$

$\therefore T_2 - T_1 \neq T_4 - T_3$

$\therefore$ The given numbers do not form an A.P. 5 7

(ii) We have: $2, \frac{5}{2}, 3, \frac{7}{2}, \ldots$

$\therefore T_1 = 2, T_2 = \frac{5}{2}, T_3 = 3, T_4 = \frac{7}{2}$

$T_2 - T_1 = \frac{5}{2} - 2 = \frac{1}{2}$

$T_3 - T_2 = 3 - \frac{5}{2} = \frac{1}{2}$

$T_4 - T_3 = \frac{7}{2} - 3 = \frac{1}{2}$

$\because T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = \frac{1}{2} \Rightarrow d = \frac{1}{2}$

$\therefore$ The given numbers form an A.P.

$\therefore T_5 = T_4 + \frac{1}{2} = \frac{7}{2} + \frac{1}{2} = 4$

$\therefore T_6 = T_5 + \frac{1}{2} = 4 + \frac{1}{2} = \frac{9}{2}$

$\therefore T_7 = T_6 + \frac{1}{2} = \frac{9}{2} + \frac{1}{2} = 5$

Thus, $d = \frac{1}{2}$ and $T_5 = 4$, $T_6 = \frac{9}{2}$ and $T_7 = 5$

(iii) We have: –1.2, –3.2, –5.2, –7.2, ...

$\therefore T_1 = -1.2, T_2 = -3.2, T_3 = -5.2, T_4 = -7.2$

$T_2 - T_1 = -3.2 + 1.2 = -2$

$T_3 - T_2 = -5.2 + 3.2 = -2$

$T_4 - T_3 = -7.2 + 5.2 = -2$

$\therefore T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = -2 = d = -2$

$\therefore$ The given numbers form an A.P.

Such that $d = -2$.

Now, $T_5 = T_4 + (-2) = -7.2 + (-2) = -9.2$

$T_6 = T_5 + (-2) = -9.2 + (-2) = -11.2$

$T_7 = T_6 + (-2) = -11.2 + (-2) = -13.2$

Thus, $d = -2$ and $T_5 = -9.2$, $T_6 = -11.2$ and $T_7 = -13.2$

(iv) We have: –10, – 6, – 2, 2, ...

$\therefore T_1 = -10, T_2 = -6, T_3 = -2, T_4 = 2$

$T_2 - T_1 = -6 + 10 = 4$

$T_3 - T_2 = -2 + 6 = 4$

$T_4 - T_3 = 2 + 2 = 4$

$T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = 4 \Rightarrow d = 4$

$\therefore$ The given numbers form an A.P.

Now, $T_5 = T_4 + 4 = 2 + 4 = 6$

$T_6 = T_5 + 4 = 6 + 4 = 10$

$T_7 = T_6 + 4 = 10 + 4 = 14$

Thus, $d = 4$ and $T_5 = 6$, $T_6 = 10$, $T_7 = 14$

(v) We have:

$3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$

$\therefore T_1 = 3, T_2 = 3+\sqrt{2}, T_3 = 3+2\sqrt{2}, T_4 = 3+3\sqrt{2}$

$T_2 - T_1 = 3+\sqrt{2}-3 = \sqrt{2}$

$T_3 - T_2 = 3+2\sqrt{2}-3-\sqrt{2} = \sqrt{2}$

$T_4 - T_3 = 3+3\sqrt{2}-3-2\sqrt{2} = \sqrt{2}$

$\because T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = \sqrt{2} \Rightarrow d = \sqrt{2}$

⇒ The given numbers from an A.P.

Now, $T_5 = T_4 + \sqrt{2}$

$= 3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2}$

$T_6 = T_5 + \sqrt{2}$

$= 3 + 4\sqrt{2} + \sqrt{2} = 3 + 5\sqrt{2}$

$T_7 = T_6 + \sqrt{2}$

$= 3 + 5\sqrt{2} + \sqrt{2} = 3 + 6\sqrt{2}$

Thus, $d = \sqrt{2}$ and $T_5 = 3 + 4\sqrt{2}, T_6 = 3 + 5\sqrt{2}, \ T_7 = 3 + 6\sqrt{2}.$

(vi) We have: 0.2, 0.22, 0.222, 0.2222, ...

$$\left.\begin{aligned} \therefore T_1 &= 0.2 \\ T_2 &= 0.22 \end{aligned}\right\} \Rightarrow T_2 - T_1 = 0.22 - 0.2 = 0.02$$

$$\left.\begin{aligned} T_3 &= 0.222 \\ T_4 &= 0.2222 \end{aligned}\right\} \Rightarrow T_4 - T_3 = 0.2222 - 0.222 = 0.0002.$$

Since,

$T_2 - T_1 \neq T_4 - T_3$

∴ The given number do not form an A.P.

(vii) We have: 0, –4, –8, –12, ...

$\therefore T_1 = 0, T_2 = -4, T_3 = -8, T_4 = -12$

$T_2 - T_1 = -4 - 0 = -4$

$T_3 - T_2 = -8 + 4 = -4$

$T_4 - T_3 = -12 + 8 = -4$

$\therefore T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = -4, \Rightarrow d = -12$

∴ The given numbers from an A.P.

Now, $T_5 = T_4 + (-4) = -12 + (-4) = -16$

$T_6 = T_5 + (-4) = -16 + (-4) = -20$

$T_7 = T_6 + (-4) = -12 + (-4) = -24$

Thus, $d = -4$ and $T_5 = -16, T_6 = -20, T_7 = -24$

(viii) We have:

$-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \ldots\ldots$

$\therefore T_1 = T_2 = T_3 = T_4 = -\frac{1}{2}$

$T_2 - T_1 = 0$

$T_3 - T_2 = 0$

$T_4 - T_3 = 0$

$\therefore T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = 0 \Rightarrow d = 0$

∴ The given number form an A.P.
Now,

$T_5 = -\frac{1}{2} + 0 = -\frac{1}{2}$

$T_6 = -\frac{1}{2} + 0 = -\frac{1}{2}$

$T_7 = -\frac{1}{2} + 0 = -\frac{1}{2}$

Thus, d = 0 and $T_5 = -\frac{1}{2}, T_6 = -\frac{1}{2}, T_7 = -\frac{1}{2}$

(ix) We have: 1, 3, 9, 27,....

Here, $\left.\begin{matrix} T_1 = 1 \\ T_2 = 3 \end{matrix}\right\} \Rightarrow T_2 - T_1 = 3 - 1 = 2$

$\left.\begin{matrix} T_3 = 9 \\ T_4 = 27 \end{matrix}\right\} \Rightarrow T_4 - T_3 = 27 - 9 = 18$

∴ $T_2 - T_1 \neq T_4 - T_3$
∴ The given numbers do not form an A.P.

(x) We have: a, 2a, 3a, 4a, ...
∴ $T_1 = a, T_2 = 2a, T_3 = 3a, T_4 = 4a$
$T_2 - T_1 = 2a - a = a$
$T_3 - T_2 = 3a - 2a = a$
$T_4 - T_3 = 4a - 3a = a$
∴ $T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = a$
∴ The numbers form an A.P.
Now, $T_5 = T_4 + a = 4a + a = 5a$
$T_6 = T_5 + a = 5a + a = 6a$
$T_7 = T_6 + a = 6a + a = 7a$
Thus, d = a and $T_5 = 5a, T_6 = 6a, T_7 = 7a$

(xi) We have: a, a_2, a_3, a_4,....

$\therefore \left.\begin{matrix} T_1 = a \\ T_2 = a^2 \end{matrix}\right\} \Rightarrow T_2 - T_1 = a^2 - a = a[a-1]$

$\left.\begin{matrix} T_3 = a^3 \\ T_4 = a^4 \end{matrix}\right\} \Rightarrow T_4 - T_3 = a^4 - a^3 = a^3[a-1]$

Since,
$T_1 - T_1 \neq T_4 - T_3$
∴ The given terms are not in A.P.

(xii) We have: $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, ...$

$\therefore T_1 = \sqrt{2}, T_2 = \sqrt{8}, T_3 = \sqrt{18}, T_4 = \sqrt{32}$

$T_2 - T_1 = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$T_3 - T_2 = \sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

$T_4 - T_3 = \sqrt{32} - \sqrt{18} = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

$\because T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = \sqrt{2} \Rightarrow d = \sqrt{2}$

$\therefore$ The given numbers form an A.P.

Now,

$T_5 = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$

$T_6 = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2} = \sqrt{72}$

$T_7 = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2} = \sqrt{98}$

Thus, $d = \sqrt{2}$ and $T_5 = \sqrt{50}, T_6 = \sqrt{72}, T_7 = \sqrt{98}$

(xiii) We have: $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, ...$

$$\left.\begin{aligned} \because T_1 &= \sqrt{3} \\ T_2 &= \sqrt{6} \end{aligned}\right\} \Rightarrow T_2 - T_1 = \sqrt{6} - \sqrt{3} = \sqrt{3}(\sqrt{2} - 1)$$

and $$\left.\begin{aligned} T_3 &= \sqrt{9} \\ T_4 &= \sqrt{12} \end{aligned}\right\} \Rightarrow T_4 - T_3 = \sqrt{12} - \sqrt{9} = 2\sqrt{3} - 3 = \sqrt{3}(2 - \sqrt{3})$$

$\therefore T_2 - T_1 \neq T_4 - T_3$

$\Rightarrow$ The given terms do not form an A.P.

(xiv) We have: $1^2, 3^2, 5^2, 7^2, ...$

$$\left.\begin{aligned} \therefore T_1 &= 1^2 = 1 \\ T_2 &= 3^2 = 9 \end{aligned}\right\} \Rightarrow T_2 - T_1 = 9 - 1 = 8$$

$$\left.\begin{aligned} T_3 &= 5^2 = 25 \\ T_4 &= 7^2 = 49 \end{aligned}\right\} \Rightarrow T_4 - T_3 = 49 - 25 = 24$$

$\therefore$ The given terms do not form an A.P.

(xv) We have: $1^2, 5^2, 7^2, 73, ...$

$\therefore T_1 = 1^2, T_2 = 5^2, T_3 = 7^2 = 49, T_4 = 73$

$T_2 - T_1 = 25 - 1 = 24$

$T_3 - T_2 = 49 - 25 = 24$

$T_4 - T_3 = 73 - 49 = 24$

$\therefore T_2 - T_1 = T_3 - T_2 = T_4 - T_3 = 24, \Rightarrow d = 24$

$\therefore$ The numbers form an A.P.

Now, $T_5 = T_4 + 24 = 73 + 24 = 97$

$T_6 = T_5 + 24 = 97 + 24 = 121$

$T_7 = T_6 + 24 = 121 + 24 = 145$

Thus, $d = 24$ and $T_5 = 97, T_6 = 121, T_7 = 145$

Q.5. Fill in the blanks in the following table, given that 'a' is the first term, 'd' the common difference and an the nth term of the A.P.:

	a	d	n	a_n
(i)	7	3	8	...
(ii)	– 18	...	10	0
(iii)	...	– 3	18	– 5
(iv)	– 18.9	2.5	...	3.6
(v)	3.5	0	105	...

Sol. (i) $a_n = a + (n - 1) d$

$\Rightarrow a_8 = 7 + (8 - 1) 3$

$= 7 + 7 \times 3$
$= 7 + 21$
$\Rightarrow a_8 = 28$

(ii) $\Rightarrow a_{10} = -18 + (10 - 1)d$
$\Rightarrow 0 = -18 + 9d$
$\Rightarrow 9d = 18 \Rightarrow d = \frac{18}{9} = 2$
$\therefore d - 2$

(iii) $a_n = a + (n - 1) d$
$\Rightarrow -5 = a + (18 - 1) \times (-3)$
$\Rightarrow -5 = a + 17 \times (-3)$
$\Rightarrow -5 = a - 51$
$\Rightarrow a = -5 + 51 = 46$
Thus, a = 46

(iv) $a_n = a + (n - 1)d$
$\Rightarrow 3.6 = -18.9 + (n - 1) \times 2.5$
$\Rightarrow (n - 1) \times 2.5 = 3.6 + 18.9$
$\Rightarrow (n - 1) \times 2.5 = 22.5$
$\Rightarrow n - 1 = \frac{22.5}{2.5} = 9$
$\Rightarrow n = 9 + 1 = 10 = 10$
Thus, n = 10

(v) $a_n = a + (n - 1)d$
$\Rightarrow a_n = 3.5 + (105 - 1) \times 0$
$\Rightarrow a_n = 3.5 + 104 \times 0$
$\Rightarrow a_n = 3.5 + 0 = 3.5$ Thus, $a_n = 3.5$

Q.6. Choose the correct choice in the following and justify:

(i) 30th term of the A.P.: 10, 7, 4, ..., is

(A) 97 (B) 77 (C) –77 (D) –87

(ii) 11th term of the A.P.: $-3, -\frac{1}{2}, 2$, ..., is

(A) 28 (B) 22 (C) –38 (D) $-48\frac{1}{2}$

Sol. (i) Here, a = 10, n = 30
$T_n = a + (n - 1)d$ and $d = 7 - 10 = -3$
$\therefore T_{30} = 10 + (30 - 1) \times (-3)$
$\Rightarrow T_{30} = 10 + 29 \times (-3)$
$\Rightarrow T_{30} = 10 - 87 = -77$
Thus, the correct choice is (C) – 77.

(ii) Here, a = – 3, n = 11 and $d = -\frac{1}{2} - (-3) = -\frac{1}{2} + 3 = \frac{5}{2}$
$\therefore T_n = a + (n - 1) d$
$\Rightarrow T_{11} = -3 + (11 - 1) \times \frac{5}{2}$
$\Rightarrow T_{11} = -3 + 25 = 22$
Thus, the correct choice is (B) 22.

Q.7. In the Miming A.Ps., find the missing terms in the boxes:

(i) 2, ☐ , 26
(ii) ☐, 13, ☐, 3
(iii) 5, ☐, ☐, $9\frac{1}{2}$
(iv) – 4, ☐,☐,☐,☐, 6
(v) ☐, 38, ☐, ☐, ☐, –22

Sol. (i) Here, a = 2, $T_3 - 26$
Let common difference = d

$\therefore T_n = a + (n - 1) d$
$\Rightarrow T_3 = 2 + (3 - 1) d$
$\Rightarrow 26 = 2 + 2d$
$\Rightarrow 2d = 26 - 2 = 24$
$\Rightarrow d = \frac{24}{2} = 12$
$\therefore$ The missing term = a + d
= 2 + 12 = 14

(ii) Let the first term = a and common difference = d
Here, $T_2 = 13$ and $T_4 = 3$
$T_2 = a + d = 13$
$T_4 = a + 3d = 3$
$T_4 - T_2 = (a + 3d) - (a + d) = 3 - 13$
$\Rightarrow 2d = -10$
$\Rightarrow d = \frac{-10}{2} = -5$
Now, $a + d = 13 \Rightarrow a + (-5) = 13$
$\Rightarrow a = 13 + 5 = 18$
Thus, missing terms are a and a + 2d or 18 and 18 + (–10) = 8
i.e., $T_1 = 18$ and $T_3 = 8$

(iii) Here, a = 5 and $T_4 = 9\frac{1}{2}$
since, $T_4 = a + 3d$
$\Rightarrow 9\frac{1}{2} = 5 + 3d$
$\Rightarrow 3d = 9\frac{1}{2} - 5 = 4\frac{1}{2}$
$\Rightarrow d = 4\frac{1}{2} \div 3 = \frac{9}{2} \times \frac{1}{3} = \frac{3}{2}$
$\therefore$ The missing terms are:
$T_2 = a + d = 5 + \frac{3}{2} = \boxed{6\frac{1}{2}}$
$T_3 = a + 2d = 5 + 2\left(\frac{3}{2}\right) = \boxed{8}$

(iv) Here, a = – 4 and $T_6 = 6$
$\because T_n = a + (n - 1) d$
$\therefore T_6 = -4 + (6 - 1)d$
$\Rightarrow 6 = -4 + 5d$
$\Rightarrow 5d = 6 + 4 = 10$
$\Rightarrow d = 10 5=2$
$\therefore T_2 = a + d - 4 + 2 = -2$
$T_3 = a + 2d = -4 + 2(2) = 0$
$T_4 = a + 3d = -4 + 3(2) = 2$
$T_5 = a + 4d = -4 + 4(2) = 4$ $\therefore$ The missing terms are $\boxed{-2}, \boxed{0}, \boxed{2}, \boxed{4}$

(v) Here, $T_2 = 38$ and $T_6 = -22$
$\therefore T_2 = a + d = 38$
$T_6 = a + 5d = -22$
$\Rightarrow T_6 - T_2 = a + 5d - (a + d) = -22 - 38$
$\Rightarrow 4d = -60$
$\Rightarrow d = \frac{-60}{4} = -15$
$\therefore a + d = 38 \Rightarrow a + (-15) = 38$
$\Rightarrow a = 38 + 15 = 53$
Now, $T_3 = a + 2d = 53 + 2(-15) = 53 - 30 = 23$

$T_4 = a + 3d = 53 + 3(-15) = 53 - 45 = 8$
$T_5 = a + 4d = 53 + 4(-15) = 53 - 60 = -7$
Thus, the missing terms are $\boxed{53}, \boxed{23}, \boxed{8}, \boxed{-7}$

Q.8. Which term of the A.P.: 3, 8, 13, 18, ..., is 78?

Sol. Let the nth term = 78
Here, $a = 3, \Rightarrow T_1 = 3$ and $T_2 = 8$
$\therefore d = T_2 - T_1 = 8 - 3 = 5$
Now, $T_n = a + (n - 1) d$
$\Rightarrow 78 = 3 + (n - 1) \times 5$
$\Rightarrow 78 - 3 = (n - 1) \times 5$
$\Rightarrow 75 = (n - 1) \times 5$
$\Rightarrow (n - 1) = 75\ 5 = 15$
$\Rightarrow n = 15 + 1 = 16$
Thus, 78 is the 16th term of the given A.P.

Q.9. Find the number of terms in each of the following A.Ps.:
(i) 7, 13, 19, ..., 205
(ii) $18, 15\frac{1}{2}, 13, \ldots, -47$

Sol. (i) Here, a = 7
$d = 13 - 7 = 6$
Let the number of terms be n
$T_n = 205$
Now, $T_n = a + (n - 1) \times d$
$\Rightarrow 7 + (n - 1) \times 6 = 205$
$\Rightarrow (n - 1) \times 6 = 205 - 7 = 198$
$\Rightarrow n - 1 = \frac{198}{6} = 33$
$\therefore n = 33 + 1 = 34$
Thus, the required number of terms is 34.

(ii) Here, a = 18
$d = 15\frac{1}{2} - 18 = -2\frac{1}{2}$
Let the nth term = –47
$\therefore T_n = a + (n - 1)d$
$\Rightarrow -47 = 18 + (n - 1) \times \left(-2\frac{1}{2}\right)$
$\Rightarrow -47 - 18 = (n - 1) \times \left(-\frac{5}{2}\right)$
$\Rightarrow -65 = (n - 1) \times \left(\frac{-5}{2}\right)$
$\Rightarrow n - 1 = -65 \times \left(\frac{-2}{5}\right)$
$\Rightarrow n - 1 = (13) \times (-2) = 26$
$\Rightarrow n = 26 + 1 = 27$ Thus, the required number of terms is 27.

Q.10. Check whether – 150 is a term of the A.P.: 11, 8, 5, 2 ...

Sol. For the given A.P., we have
$a = 11$
$d = 8 - 11 = -3$
Let – 150 is the nth term of the given A.P.
$\therefore T_n = a + (n - 1)d$
$\Rightarrow -150 = 11 + (n - 1) \times (-3)$
$\Rightarrow -150 - 11 = (n - 1) \times (-3)$
$\Rightarrow -161 = (n - 1) \times (-3)$

$\Rightarrow n-1=\frac{-161}{-3}=\frac{161}{3}$

$\Rightarrow n=\frac{161}{3}+1=\frac{164}{3}=54\frac{2}{3}$

But n should be a positive integer.

Thus, – 150 is not a term of the given A.P.

Q.11. Find the 31st term of an A.P. whose 11th term is 38 and the 16th tern is 73.

Sol. Here, T_{31} = ?

T_{11} = 38

T_{16} = 73

If the first term = a and the common difference = d.

Then,

a + (11 – 1) d = 38

⇒ a + 10 – d = 38

and a + (16 – 1) d = 73

⇒ a + 15d = 73

Subtracting (1) from (2), we get

(a + 15d) – (a + 10 – d) = 73 – 38

⇒ 5d = 35

$\Rightarrow d=\frac{35}{5}=7$

From (1),

a + 10(7) = 38

⇒ a + 70 = 38

⇒ a = 38 – 70 = – 32

∴ T_{31} = – 32 + (31 – 1) × 7

⇒ T_{31} = – 32 + 30 × 7

⇒ T_{31} = – 32 + 210

⇒ T_{31} = 174

Thus, the 31st term is 178.

Q.12. An A.P. consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

Sol. Here, n = 50

T_3 = 12

T_n = 106 ⇒ T_{50} = 106

If first term = a and the common difference = d

∴ T_3 = a + 2d = 12 ...(1)

T_{50} = a + 49d = 106 ...(2)

⇒ T_{50} – T_3 = a + 49d – (a + 2d) = 106 – 12

⇒ 47d = 94

$\Rightarrow d=\frac{94}{47}=2$

From (1), we have

a + 2d = 12 ⇒ a + 2(2) = 12

⇒ a = 12 – 4 = 8

Now, T_{29} = a + (29 – 1) d

= 8 + (28) × 2

= 8 + 56 = 64 . Thus, the 29th term is 64.

Q.13. If the 3rd and the 9th terms of an A.P. are 4 and – 8 respectively, which term of this A.P. is zero?

Sol. Here, T_3 = 4 and T_9 = –8

∴ Using T_n = a + (n – 1) d

⇒ T_3 = a + 2d = 4 ...(1)

T_9 = n + 8d = – 8 ...(2)

Subtracting (1) from (2) we get

(a + 8d) – (a + 2d) = – 8 – 4

⇒ 6d = – 12

$\Rightarrow d=\frac{-12}{6}=-2$

Now, from (1), we have:

$a + 2d = 4$
$\Rightarrow a + 2(-2) = 4$
$\Rightarrow a - 4 = 4$
$\Rightarrow a = 4 + 4 = 8$
Let the nth term of the A.P. be 0.
$\therefore T_n = a + (n - 1)d = 0$
$\Rightarrow 8 + (n - 1) \times (-2) = 0$
$\Rightarrow (n - 1) \times - 2 = - 8$
$$\Rightarrow n - 1 = \frac{-8}{-2} = 4$$
$\Rightarrow n = 4 + 1 = 5$
Thus, the 5th term of the A.P. is 0.

Q.14. The 17th term of an A.P. exceeds its 10th term by 7. Find the common difference.
Sol. Let 'a' be the first term and 'd' be the common difference of the given A.P.
Now, using $T_n = a + (n - 1) d$
$T_{17} = a + 16d$
$T_{10} = a + 9d$
According to the condition,
$T_n + 7 = T_{17}$
$\Rightarrow (a + 9d) + 7 = a + 16d$
$\Rightarrow a + 9d - a - 16d = - 7$
$\Rightarrow - 7d = - 7 \Rightarrow d = 1$
Thus, the common difference is 1.

Q.15. Which term of the A.P.: 3, 15, 27, 39, ... will be 132 more than its 54th term?
Sol. Here, $a = 3$
$d = 15 - 3 = 12$
Using $T_n = a + (n - 1) d$, we get
$T_{54} = a + 53d$
$= 3 + 53 \times 12$
$= 3 + 636 = 639$
Let a_n be 132 more than its 54th term.
$\therefore a_n = T_{54} + 132$
$\Rightarrow a_n = 639 + 132 = 771$
Now $a_n = a + (n - 1)d = 771$
$\Rightarrow 3 + (n - 1) \times 12 = 771$
$\Rightarrow (n - 1) \times 12 = 771 - 3 = 768$
$$\Rightarrow (n - 1) = \frac{768}{12} = 64$$
$\Rightarrow n = 64 + 1 = 65$
Thus, 132 more than 54th term is the 65th term.

Q.16. Two A.Ps. have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?
Sol. Let for the 1st A.P., the first term = a
$\therefore T_{100} = a + 99d$
And for the 2nd A.P., the first term = a'
$\therefore T_{100} = a' + 99d$
According to the condition, we have:
$T_{100} - T'100 = 100$
$\Rightarrow a + 99d - (a' + 99d) = 100$
$\Rightarrow a - a' = 100$
Let, $T_{100} - T' = x$

∴ a + 999d – (a' + 999d) = x
⇒ a – a' = x ⇒ x = 100
∴ The difference between the 1000th terms is 100.

Q.17. How many three–digit numbers are divisible by 7?

Ans. The first three digit number divisible by 7 is 105.
The last such three digit number is 994.
∴ The A.P. is 105, 112, 119, ..., 994
Here, a = 105 and d = 7
Let n be the required number of terms.
∴ $T_n = a + (n - 1)d$
⇒ 994 = 105 + (n – 1) × 7
⇒ (n – 1) × 7 = 994 – 105 = 889
$$\Rightarrow (n-1) = \frac{889}{7} = 127$$
⇒ n = 127 + 1 = 128
Thus, 128 numbers of 3–digit are divisible by 7.

Q.18. How many multiples of 4 lie between 10 and 250?

Sol. ∵ The first multiple of 4 beyond 10 is 12.
The multiple of 4 just below 250 is 248.
∴ The A.P. is given by:
12, 16, 20,.... 248
Here, a = 12 and d = 4
Let the number of terms = n
∴ Using $T_n = a + (n - 1) d$, we get
∴ $T_n = 12 + (n - 1) \times 4$
⇒ 248 = 12 + (n – 1) × 4
⇒ (n – 1) × 4 = 248 – 12 = 236
$$\Rightarrow n - 1 = \frac{236}{4} = 59$$
⇒ n = 59 + 1 = 60
Thus, the required number of terms = 60.

Q.19. For what value of n, are the nth terms of two A.Ps.: 63, 65, 67, ... and 3, 10, 17, ... equal?

Sol. For the 1st A.P.
∵ a = 63 and d = 65 – 63 = 2
∴ $T_n = a + (n - 1)d$
⇒ $T_n = 63 + (n - 1) \times 2$
For the 2nd A.P.
a = 3 and d = 10 – 3 = 7
∴ $T_n = a + (n - 1) d$
⇒ $T_n = 3 + (n - 1) \times 7$
Now, according to the condition,
3 + (n – 1) × 7 = 63 + (n – 1) × 2
⇒ (n – 1) × 7 – (n – 1) × 2 = 63 – 3
⇒ 7n – 7 – 2n + 2 = 60
⇒ 5n – 5 = 60
⇒ 5n = 60 + 5 = 65
$$\Rightarrow n = \frac{65}{5} = 13$$
Thus, the 13th terms of the two given A.Ps. are equal.

Q.20. Determine the A.P. whose third term is 16 and the 7th term exceeds the 5th term by 12.

Sol. Let the first term = a and the common difference = d.
∴ Using $T_n = a + (n - 1) d$, we have:
$T_3 = a + 2d$
⇒ a + 2d = 16 ...(1)

And $T_7 = a + 6d$, $T_5 = a + 4d$
According to the condition,
$T_7 - T_5 = 12$
$\Rightarrow (a + 6d) - (a + 4d) = 12$
$\Rightarrow a + 6d - a - 4d = 12$
$\Rightarrow 2d = 12$
$\Rightarrow d = \frac{12}{2} = 6$ ...(2)
Now, from (1) and (2), we have:
$a + 2(6) = 16$
$\Rightarrow a + 12 = 16$
$\Rightarrow a = 16 - 12 = 4$
∴ The required A.P. is
4, [4 + 61], [4 + 2 (6)], [4 + 3 (6)], ...
or 4,10,16,22, ...

Q.21. Find the 20th term from the last term of the A.P.: 3, 8, 13, ..., 253.

Sol. We have, the last term I = 253
Here, $d = 8 - 3 = 5$
Since, the nth term before the last term is given by 1. – (n – 1) d,
∴ We have
20th term from the end = $l - (20 - 1) \times 5$
$= 253 - 19 \times 5$
$= 253 - 95 = 158$

Q.22. The sum of the 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the A.P.

Sol. Let the first term = a
And the common difference = d
∴ Using $T_n = a + (n - 1) d$,
$T_4 + T_8 = 24$
$\Rightarrow (a + 3d) + (a + 7d) = 24$
$\Rightarrow 2a + 10d = 24$
$\Rightarrow a + 5d = 12$
And $T_6 + T_{10} = 44$
$\Rightarrow (a + 5d) + (a + 9d) = 44$
$\Rightarrow 2a + 14d = 44$
$\Rightarrow a + 7d = 22$
Now, subtracting (1) from (2), we get
$(a + 7d) - (a + 5d) = 22 - 12$
$\Rightarrow 2d = 10$
$\Rightarrow d = \frac{10}{2} = 5$
∴ From (1), $a + 5 \times 5 = 12$
$\Rightarrow a + 25 = 12$
$\Rightarrow a = 12 - 25 = -13$
Now, the first three terms of the A.P. are given by:
a, (a + d), (a + 2d)
or –13, (–13 + 5), [–13 + 2 (5)]
or –13, –8, –3

Q.23. Subhash Rao started work in 1995 at an annual salary of Rs. 5000 and received an increment of Rs. 200 each year. In which year did his income reach Rs. 7000?

Sol. Here, a = Rs. 5000 and d = Rs. 200
Say, in the nth year he gets Rs. 7000.
∴ Using $T_n = a + (n - 1) d$, we get
$7000 = 5000 + (n - 1) \times 200$

$\Rightarrow (n-1) \times 200 = 7000 - 5000 = 2000$

$\Rightarrow n - 1 = \frac{2000}{200} = 10$

$\Rightarrow n = 10 + 1 = 11$

Thus, his income becomes Rs. 7000 in 11 years.

Q.24. Rachana saved Rs. 5 in the first week of a year and then increased weekly savings by Rs. 1.75. If in the nth week, her weekly savings become Rs. 20.75, find n.

Sol. Here, a = Rs. 5 and d = Rs. 1.75

∵ In the nth week her savings become Rs. 20.75.

∴ T_n = Rs. 20.75

∴ Using $T_n = a + (n-1)d$, we have

$20.75 = 5 + (n-1) \times (1.75)$

$\Rightarrow (n-1) \times 1.75 = 20.75 - 5$

$\Rightarrow (n-1) \times 1.75 = 15.75$

$\Rightarrow n - 1 = \frac{15.75}{1.75} = 9$

$\Rightarrow n - 9 + 1 = 10$

Thus, the required number of years = 10.

Q.25. Find the sum of the following A.Ps.:

(i) 2, 7, 12, ..., to 10 terms.

(ii) – 37, – 33, – 29, ..., to 12 terms.

(iii) 0.6, 1.7, 2.8, ..., to 100 terms.

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots$, to 11 terms.

Sol. (i) Here, a = 2

d = 7 – 2 = 5

n = 10

Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_{10} = \frac{10}{2}[2 \times 2 + (10-1) \times 5]$

$\Rightarrow S_{10} = 5[4 + 9 \times 5]$

$\Rightarrow S_{10} = 5[49] = 245$

Thus, the sum of first 10 terms is 245.

(ii) We have:

$a = -37$

$d = -33 - (-37) = 4$

$n = 12$

$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{12} = \frac{12}{2}[2(-37) + (12-1) \times 4]$

$= 6[-74 + 11 \times 4]$

$= 6[-74 + 44]$

$= 6 \times [-30] = -180$

Thus, sum of first 12 terms = – 180.

(iii) Here, a = 0.6

d = 1.7 – 0.6 = 1.1

n = 100

$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{100} = \frac{100}{2}[2(0.6) + (100-1) \times 1.1]$

= 50 [1.2 + 99 × 1.1]
= 50 [1.2 + 108.9]
= 50 [110.1]
= 5505
Thus, the required sum of first 100 terms is 5505.

(iv) Here,

$$a = \frac{1}{15}$$

$$d = \frac{1}{12} - \frac{1}{15} = \frac{1}{60}$$

n = 1

$$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{11} = \frac{11}{2}\left[\left(2 \times \frac{1}{15}\right) + (11-1) \times \frac{1}{60}\right]$$

$$= \frac{11}{2}\left[\frac{2}{15} + \left(10 \times \frac{1}{60}\right)\right]$$

$$= \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right]$$

$$= \frac{11}{2}\left[\frac{4+5}{30}\right]$$

$$= \frac{11}{2} \times \frac{9}{30} = \frac{99}{60} = \frac{33}{20}$$

Thus, the required sum of first 11 terms = $\frac{33}{20}$.

Q.26. In an A.P.:
(i) given a = 5, d = 3, a_n = 50, find n and S_n.
(ii) given a = 7, a_{13} = 35, find d and S_{13}.
(iii) given a_{12} = 37, d = 3, find a and S_{12}.
(iv) given a_3 = 15, S_{10} = 125, find d and a_{10}.
(v) given d = 5, S_9 = 75, find a and a_9.

Sol. (i) Here, a = 5, d = 3 and a_n = 50 = l
$a_n = a + (n - 1)d$
∴ 50 = 5 + (n – 1) × 3
⇒ 50 – 5 = (n – 1) × 3
⇒ (n – 1) × 3 = 45
$$\Rightarrow (n-1) = \frac{45}{3} = 15$$
⇒ n = 15 + 1 = 16

$$\text{Now } S_n = \frac{n}{2}(a-1)$$

$$= \frac{16}{2}(5+50)$$

= 8 (55) = 440
Thus, n = 16 and S_n = 440

(ii) Here, a = 7 and a_{13} = 35 = l
∴ $a_n = a + (n - 1)d$
⇒ 35 = 7 + (13 – 1)d
⇒ 35 – 7 = 12d
⇒ 28 = 12d

$\Rightarrow d = \frac{28}{12} = \frac{7}{3}$

Now, using

$S_n = \frac{n}{2}(a + 1)$

$S_{13} = \frac{13}{2}(7 + 35)$

$= \frac{13}{2} \times 42$

$= 13 \times 21 = 273$

$Sn = 273$ and $d = \frac{7}{3}$

(iii) Here, $a_{12} = 37 = l$ and $d = 3$

Let the first term of the A.P. be 'a'.

Now $a_{12} = a + (12 - 1)\ d$

$\Rightarrow 37 = a + 11d$

$\Rightarrow 37 = a + 11 \times 3$

$\Rightarrow 37 = a + 33$

$\Rightarrow a = 37 - 33 = 4$

Now, $S_n = \frac{n}{2}(a + 1)$

$\Rightarrow S_{12} = \frac{12}{2}(4 + 37)$

(iv) $a_3 = 15 = l$

$S_{10} = 125$

Let first term of the A.P. be 'a' and the common difference = d

$\therefore a_3 = a + 2d$

$\Rightarrow a + 2d = 15$ ...(1)

Again $S_n = \frac{n}{2}[2a + (n - 1)\ d]$

$\Rightarrow S_{10} = \frac{10}{2}[2a + (10 - 1)\ d]$

$\Rightarrow 125 = 5\ [2a + 9d]$

$\Rightarrow 2a + 9d = \frac{125}{5} = 25$

$\Rightarrow 2a + 9d = 25$

Multiplying (1) by 2 and subtracting (2) from it, we get

$2\ [a + 2d = 15] - [2a + 9d = 25]$

$\Rightarrow 2a + 4d - 2a - 9d = 30 - 25$

$\Rightarrow -5d = 5$

$\Rightarrow d = \frac{5}{-5} = -1$

$\therefore$ From (1), $a + 2\ (-1) = 15 \Rightarrow a = 15 + 2 \Rightarrow a = 15 + 2 \Rightarrow a = 17$

Now, $a_{10} = a + (10 - 1)\ d$

$= 17 + 9 \times (-1)$

$= 17 - 9 = 8$

Thus, $d = -1$ and $a_{10} = 8$

(v) Here, $d = 5$, $S_9 = 75$

Let the first term of the A.P. is 'a'.

$\therefore S_9 = \frac{9}{2}[2a + (9-1) \times 5]$

$\Rightarrow 75 = \frac{9}{2}[2a + 40]$

$\Rightarrow 75 \times \frac{2}{9} = 2a + 40$

$\Rightarrow \frac{50}{3} = 2a + 40$

$\Rightarrow 2a = \frac{50}{3} - 40 = \frac{-70}{3}$

$\Rightarrow a = \frac{-70}{3} \times \frac{1}{2} = \frac{-35}{3}$

Now, $a_9 = a + (9-1)d$

$= \frac{-35}{5} + (8 \times 5)$

$= \frac{-35}{3} + 40$

$= \frac{-35+120}{3} = \frac{85}{3}$

Thus, $a = \frac{-35}{3}$ and $a_9 = \frac{85}{3}$

Q.27. The first term of an A.P. is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Sol. Here, a = 5

$l = 45 = T_n$

$S_n = 400$

$\because T_n = a + (n-1)d$

$\therefore 45 = 5 + (n-1)d$

$\Rightarrow (n-1)d = 45 - 5$

$\Rightarrow (n-1)d = 40$ ...(1)

Also $S_n = \frac{n}{2}(a+1)$

$\Rightarrow 400 = \frac{n}{2}(5+45)$

$\Rightarrow 400 \times 2 = n \times 50$

$\Rightarrow n = \frac{400 \times 2}{50} = 16$

From (1), we get

$(16-1)d = 40$

$\Rightarrow 15d = 40$

$\Rightarrow d = \frac{40}{15} = \frac{8}{3}$

Q.28. Find the sum of the first 15 multiples of 8.

Sol. The first 15 multiples of 8 are:

8, (8 × 2), (8 × 3), (8 × 4), ..., (8 × 15)

or 8, 16, 24, 32, ..., 120.

These numbers are in A.P., where

a = 8 and l = 120

$\therefore S_{15} = \frac{15}{2}[a + 1] \quad = \frac{15}{2}[8 + 120]$

$= \frac{15}{2} \times 128$

= 15 × 64 = 960

Thus, the sum of first positive 15 multiples of 8 is 960.

Q.29. The first and the last terms of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Sol. We have,

First term a = 17

Last term l = 350 = T_n

Common difference d = 9

Let the number of terms be 'n'

$T_n = a + (n – 1) d$

∴ 350 = 17 + (n – 1) × 9

⇒ (n – 1) × 9 = 350 – 17 = 333

$\Rightarrow n - 1 = \frac{333}{9} = 37$

⇒ n = 37 + 1 = 38

Since, $Sn = \frac{n}{2}(a + l)$

$\therefore S_{38} = \frac{38}{2}(17 + 350)$

Q.30. The sum of the third and the seventh terms of an A.P. is 6 and their product is 8. Find the sum of first sixteen terms of the A.P..

Sol. Here, $T_3 + T_7 = 6$ and $T_3 \times T_7 = 8$

Let the first term = a and the common difference = d

∴ $T_3 = a + 2d$ and $T_7 = a + 6d$

∵ $T_3 + T_7 = 6$

(a + 2d) + (a + 6d) = 6

⇒ 2a + 8d = 6

⇒ a + 4d = 3 ...(1)

Again $T_3 \times T_7 = 8$

⇒ (a + 2d) × (a + 6d) = 8

⇒ (a + 4d – 2d) × (a + 4d + 2d) = 8

⇒ [(a + 4d) – 2d] × [(a + 4d) + 2d] = 8

⇒ [(3) – 2d] × [(3) + 2d] = 8 [From (1)]

⇒ 32 – (2d)2 = 8

⇒ 9 – 4d2 = 8

⇒ –4d2 = 8 – 9 = –1

$\Rightarrow d^2 = \frac{-1}{-4} = \frac{1}{4}$

$\Rightarrow d = \pm\frac{1}{2}$

When $d = \frac{1}{2}$.

From (1), we have:

$$a + 4\left(\frac{1}{2}\right) = 3$$

$\Rightarrow a + 2 = 3$ or $a = 3 - 2 = 1$

Now, Using $S_n = \frac{n}{2}[2a + (n - 1) d]$, we get

$$S_{16} = \frac{16}{2}[2(1) + (16 - 1) \times \frac{1}{2}]$$

$$= 8\left[2 + \frac{15}{2}\right]$$

$= 16 + 60 = 76$

i.e., the sum of first 16 terms = 76

Men $d = -\frac{1}{2}$.

From (1), we have:

$$a + 4\left(-\frac{1}{2}\right) = 3$$

$\Rightarrow a - 2 = 3 \Rightarrow a = 5$

Again, the sum of first sixteen terms

$$S_{16} = \frac{16}{2}[2(5) + (16 - 1) \times \left(-\frac{1}{2}\right)]$$

$$= 8[10 + \left(\frac{-15}{2}\right)]$$

$= 80 - 60 = 20$

i.e., the sum of first 16 terms = 20

Geometric Progression, Series & Sums

Introduction

A **geometric sequence** is a sequence such that any element after the first is obtained by multiplying the preceding element by a constant called the **common ratio** which is denoted by r. The common ratio (r) is obtained by dividing any term by the preceding term, i.e.,

$$r = \frac{a_2}{a_1} = \frac{a_3}{a_2} = \ldots = \frac{a_n}{a_{n-1}}$$

where	r	common ratio
	a_1	first term
	a_2	second term
	a_3	third term
	a_{n-1}	the term before the *n* th term
	a_n	the *n* th term

The **geometric sequence** is sometimes called the **geometric progression** or **GP**, for short.

For example, the sequence 1, 3, 9, 27, 81 is a geometric sequence. Note that after the first term, the next term is obtained by multiplying the preceding element by 3.

The geometric sequence has its sequence formation: $a_1, a_1r, a_1r^2, \ldots, a_1r^{n-1}, a_1r^n$

To find the nth term of a geometric sequence we use the formula:

$$a_n = a_1r^{n-1}$$

where	r	common ratio
	a_1	first term
	a_{n-1}	the term before the n th term
	n	number of terms

Sum of Terms in a Geometric Progression

Finding the sum of terms in a geometric progression is easily obtained by applying the formulas:

nth partial sum of a geometric sequence

$$S_n = \frac{a_1(1-r^n)}{1-r}, \qquad r \neq 1$$

sum to infinity

$$S_\infty = \sum_{n=1}^{\infty} ar^{n-1} = \frac{a_1}{1-r}, -1 < r < 1$$

where	S_n	sum of GP with n terms
	S_∞	sum of GP with infinitely many terms
	a_1	the first term
	r	common ratio
	n	number of terms

Example

Which of the following sequences form a Geometric Progression?

(i) 7, 14, 21, 28, … (ii) 1/ 2 , 1, 2, 4, ... (ii) 5, 25, 50, 75, …

Solution

To check if a given sequence form a G.P. we have to see if the ratio between successive terms are equal.

(i) 7, 14, 21, 28, …

$$\frac{t_2}{t_1} = \frac{14}{7} = 2; \qquad \frac{t_3}{t_2} = \frac{21}{14} = \frac{3}{2}; \qquad \frac{t_4}{t_3} = \frac{28}{21} = \frac{4}{3}$$

Since the ratios between successive terms are not equal, the sequence 7, 14, 21, 28, … is not a Geometric Progression.

(ii)

$$\frac{1}{2}, 1, 2, 4, \ldots$$

$$\frac{t_2}{t_1} = \frac{1}{\frac{1}{2}} = 2; \qquad \frac{t_3}{t_2} = \frac{2}{1} = 2; \qquad \frac{t_4}{t_3} = \frac{4}{2} = 2$$

Here the ratios between successive terms are equal. Therefore the sequence 1/2 , 1, 2, 4, ... is a Geometric Progression with common ratio $r = 2$.

(iii) 5, 25, 50, 75,...

$$\frac{t_2}{t_1} = \frac{25}{5} = 5; \qquad \frac{t_3}{t_2} = \frac{50}{25} = 2; \qquad \frac{t_4}{t_3} = \frac{75}{50} = \frac{3}{2}$$

Since the ratios between successive terms are not equal , the sequence 5, 25, 50, 75,... is not a Geometric Progression.

Example

Find the geometric progression whose first term and common ratios are given by

(i) $a = -7$, $r = 6$

(ii) $a = 256$, $r = 0.5$

Solution

(i) The general form of Geometric progression is a, ar, ar^2 ,.. .

$a = -7$, $ar = -7 \times 6 = -42$, $ar^2 = -7 \times 6^2 = -252$

(ii) The general form of Geometric progression is a, ar, ar^2 ,...

$a = 256$, $ar = 256 \times 0.5 = 128$, $ar^2 = 256 \times (0.5)^2 = 64$

Therefore the required Geometric progression is 256,128, 64,....

Example

Find the 8th term of the G.P. 9, 3, 1,…

Solution

To find the 8th term we have to use the n^{th} term formula $t_n = ar^{n-1}$

First term $a = 9$, common ratio $r = \frac{t_2}{t_1} = \frac{3}{9} = \frac{1}{3}$

$$t_8 = 9 \times \left(\frac{1}{3}\right)^{8-1} = 9 \times \left(\frac{1}{3}\right)^7 = \frac{1}{243}$$

Therefore the 8th term of the G.P. is 1/243

Example

In a Geometric progression, the 4th term is 8/9 and the 7th term is 64/243. Find the Geometric Progression.

Solution

$$4^{\text{th}} \text{ term, } t_4 = \frac{8}{9} \text{ gives } ar^3 = \frac{8}{9} \quad \ldots(1)$$

$$7^{\text{th}} \text{ term, } t_7 = \frac{64}{243} \text{ gives } ar^6 = \frac{64}{243} \quad \ldots(2)$$

$$\text{Dividing (2) by (1) we get, } \frac{ar^6}{ar^3} = \frac{\frac{64}{243}}{\frac{8}{9}}$$

$$r^3 = \frac{8}{27} \text{ gives } r = \frac{2}{3}$$

Substituting the value of r in (1), we get $a \times [2/3]^3 = 8/9 \Rightarrow a = 3$

Therefore the Geometric Progression is $a, ar, ar^2, \ldots$ That is, 3, 2, 4/3,…..

Question

Write down the 8th term in the Geometric Progression 1, 3, 9, ...

Answer

$a_1 = 1;\ a_2 = 3;\ a_3 = 9; n = 8$	write down key terms
$r = \frac{a_2}{a_1} = \frac{3}{1} = 3$	find the common ratio r using $r = \frac{a_2}{a_1}$
$a_8 = a_1 r^{8-1}$	substitute $n = 8$ to $a_n = a_1 r^{n-1}$
$= (1)(3)^7$	substitute $a_1 = 1$ and $r = 3$
$= (3)^7$	multiply (1) and $(3)^7$
$a_8 = 2187$	simplify (3)7 = 2187

Question Find the number of terms in the geometric progression 6, 12, 24, ..., 1536

Answer

$a_1 = 6;\ a_2 = 12;\ \ a_3 = 24; a_n = 1536$	write down key terms
$r = \dfrac{a_2}{a_1} = \dfrac{12}{6} = 2$	find r using $r = \frac{a_2}{a_1}$
$1536 = (6)(2)^{n-1}$	substitute the values of a_1, a_n and r to $a_n = a_1 r^{n-1}$ to find n
$256 = (2)^{n-1}$	divide both sides by 6
$2^8 = 2^{n-1}$	change 256 to its exponential form form whose base = r
$8 = n - 1$	equate the indices since they both have the same base
$8 + 1 = n$	add 1 to both sides
$9 = n$	add 8 and 1

Hence, 1536 is the **9th** term.