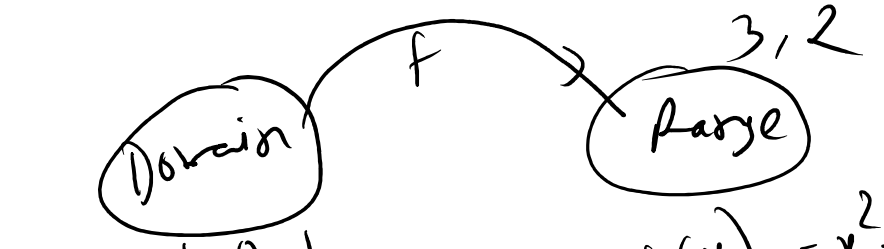
29 October 2020 08:55

Function & Derivatives (F4R)

x — income — indep. var.

y — exp. — dep. var.

$y = f(x)$ → function of x



$x = -1, 0, 1$

$y = f(x) = x^2 + 2$

$y = 2x + 4$ → linear fn.

$y = x^5 + \cdots$ → polynomial fn.

$y = \log x$ → logarithmic "

$y = e^x$ → exponential "

Economics → price, demand, supply, profit, cost

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1 Find the values of x for

$f(x) = f(x+1)$

& $f(x) = x^2 + 3$

Sol: $f(x+1) = f(x)$

$\Rightarrow (x+1)^2 + 3 = x^2 + 3$

$\Rightarrow x^2 + 2x + 1 + 3 = x^2 + 3$

$\Rightarrow 2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$

[$f(x) = x^2 + 3$
$f(2) = 2^2 + 3$
$f(3) = 3^2 + 3$

$f(100) = 100^2 + 3$
$f(x+1) = (x+1)^2 + 3$
$f(x+y) = (x+y)^2 + 3$]

2. $f(x) = x^2 + 3x + 5$
$f(x) = f(x+2)$

Find the value of x

$f(x+2) = f(x)$

$\Rightarrow (x+2)^2 + 3(x+2) + 5 = x^2 + 3x + 5$

$\Rightarrow x^2 + 4x + 4 + 3x + 6 + 5 = x^2 + 3x + 5$

$\Rightarrow 4x + 10 = 0$

$\Rightarrow 4x = -10 \Rightarrow x = -\frac{10}{4}$

$\Rightarrow x = -\frac{5}{2}$

Derivatives

x y

income exp.

$\frac{dy}{dx} = f'(x)$ → rate of change of y w.r.t. x

1. $y = f(x) = c$

$\frac{dy}{dx} = f'(x) = 0$

2. $y = f(x) = x^n$

$\frac{dy}{dx} = f'(x) = nx^{n-1}$

3. $y = f(x) = \log x$

$\frac{dy}{dx} = f'(x) = \frac{1}{x}$

4. $y = f(x) = e^x$

$\frac{dy}{dx} = f'(x) = e^x$

5. $y = f(x) = a^x$

$\frac{dy}{dx} = f'(x) = a^x \log a$

Problems

1. $y = 25 \quad \frac{dy}{dx} = 0$

2. $y = x^5 \quad \frac{dy}{dx} = 5x^4$

3. $y = 6 \log x$

$$\frac{dy}{dx} = \frac{d(6 \log x)}{dx}$$

$$= 6 \frac{d(\log x)}{dx} = \frac{6}{x}$$

★ 4. $y = \sqrt{x} = x^{1/2}$

$$\frac{dy}{dx} = \frac{d(x^{1/2})}{dx} = \frac{1}{2} x^{\frac{1}{2}-1}$$

$$= \frac{1}{2} x^{-1/2}$$

$$= \frac{1}{2 x^{1/2}}$$

x^n

$n x^{n-1}$

$x^n = \frac{1}{x^{-n}}$

$x^{-n} = \frac{1}{x^n}$

★ $\frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$

5. $y = \frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}} = x^{-1/2}$

$$\frac{dy}{dx} = \frac{d(x^{-1/2})}{dx} = -\frac{1}{2} x^{-\frac{1}{2}-1}$$

$$= -\frac{1}{2} x^{-3/2}$$

$\frac{dx}{dx} = -\frac{1}{2}x^{-3/2}$

5.11.20 **Rules of Derivatives**

1. **Addition/Subtraction Rule**

$$y = u \pm v \pm w$$

$$\frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx} \pm \frac{dw}{dx}$$

1. $y = 5^x + x^5 + 5^5 - e^x$

$$\frac{dy}{dx} = \frac{d(5^x)}{dx} + \frac{d(x^5)}{dx} + \frac{d(5^5)}{dx} - \frac{d(e^x)}{dx}$$

$$= 5^x \log 5 + 5x^4 + 0 - e^x$$

$$= 5^x \log 5 + 5x^4 - e^x$$

2. $y = x^{10} + e^x - 5 \log x$

$$\frac{dy}{dx} = \frac{d(x^{10})}{dx} + \frac{d(e^x)}{dx} - \frac{d(5 \log x)}{dx}$$

$$= 10x^9 + e^x - 5 \times \frac{1}{x}$$

$$= 10x^9 + e^x - \frac{5}{x}$$

3 $y = x^{-5} + 5^{x}$

$\frac{dy}{dx} = \frac{d(x^{-5})}{dx} + \frac{d(5^{x})}{dx}$

$\frac{dx^n}{dx} = nx^{n-1}$

$= -5x^{-5-1} + 5^{x}\log 5$

$= -5x^{-6} + 5^{x}\log 5$

4 $y = \sqrt{x} + 2e^{x} + x^{-2}$

$\frac{dy}{dx} = \frac{d(\sqrt{x})}{dx} + \frac{d(2e^{x})}{dx} + \frac{d(x^{-2})}{dx}$

$= \frac{1}{2\sqrt{x}} + 2\frac{d(e^{x})}{dx} + (-2)x^{-2-1}$

$= \frac{1}{2\sqrt{x}} + 2e^{x} - 2x^{-3}$

2 Multiplication Rule

$y = uv$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

1 $y = \underset{u}{x^{4}} \times \underset{v}{e^{x}}$

$\frac{dy}{dx} = x^{4}\frac{d(e^{x})}{dx} + e^{x}\frac{d(x^{4})}{dx}$

4-1

$\frac{dy}{dx} = \dots \quad \frac{dx}{dx}$

$= x^4 \times e^x + e^x \times 4 \times x^{4-1}$

$= x^4 e^x + 4 e^x x^3$

$= x^3 e^x (x + 4)$

2 $y = (\sqrt{x}) \log x$ (u, v)

$\frac{dy}{dx} = \sqrt{x} \frac{d(\log x)}{dx} + \log x \frac{d(\sqrt{x})}{dx}$

$= \sqrt{x} \times \frac{1}{x} + \log x \times \frac{1}{2\sqrt{x}}$

$x = (\sqrt{x})(\sqrt{x})$

$= \frac{1}{\sqrt{x}} + \log x \times \frac{1}{2\sqrt{x}}$

$= \frac{1}{\sqrt{x}} \left(1 + \frac{\log x}{2}\right)$

3 $y = x^{-2} \times 2^x$

$\frac{dy}{dx} = x^{-2} \frac{d(2^x)}{dx} + 2^x \frac{d(x^{-2})}{dx}$

x^n

$= x^{-2} 2^x \log 2 + 2^x (-2) x^{-2-1}$

$= x^{-2} 2^x \log 2 - 2 \cdot 2^x x^{-3}$

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$y = (x+1)(x^2+3)$ | $u \frac{dv}{dx}$

4/ $y = (x+1)(x^2+3)$

$\frac{dy}{dx} = (x+1)\frac{d(x^2+3)}{dx} + (x^2+3)\frac{d(x+1)}{dx}$

$= (x+1)\left[\frac{d(x^2)}{dx} + \frac{d(3)}{dx}\right] + (x^2+3)\left[\frac{d(x)}{dx} + \frac{d(1)}{dx}\right]$

$= (x+1)\left[2x^{2-1} + 0\right] + (x^2+3)\left[1x^{1-1} + 0\right]$

$= (x+1)[2x] + (x^2+3)(1+0)$

$= 2x(x+1) + x^2 + 3$

$= 2x^2 + 2x + x^2 + 3$

$= 3x^2 + 2x + 3$

$\frac{dx^n}{dx} = nx^{n-1}$, $\frac{d(c)}{dx} = 0$, $\frac{dx}{dx} = 1$

$u\frac{dv}{dx} + v\frac{du}{dx}$

5/ $y = (2x^3 + 3x)(x+5)$

$\frac{dy}{dx} = (2x^3+3x)\frac{d(x+5)}{dx} + (x+5)\frac{d(2x^3+3x)}{dx}$

$= (2x^3+3x)\left(\frac{dx}{dx} + \frac{d(5)}{dx}\right) + (x+5)\left[\frac{d(2x^3)}{dx} + \frac{d(3x)}{dx}\right]$

$= (2x^3+3x)(1+0) + (x+5)[2\times 3x^2$

x^n, nx^{n-1}

nx) $= (2x^3+3x)(1+0) + (x+5)[2\times 3x^{-} + 3\times 1]$

$= (2x^3+3x) + (x+5)(6x^2+3)$

6. $y = (e^x + 5^{-x})(x^5 + \log x)$

$$\frac{dy}{dx} = (e^x + 5^{-x})\frac{d(x^5+\log x)}{dx} + (x^5+\log x)\frac{d(e^x+5^{-x})}{dx}$$

$$= (e^x+5^{-x})\left[\frac{d(x^5)}{dx} + \frac{d(\log x)}{dx}\right] + (x^5+\log x)\left[\frac{d(e^x)}{dx} + \frac{d(5^{-x})}{dx}\right]$$

$$= (e^x+5^{-x})\left[5x^4 + \frac{1}{x}\right] + (x^5+\log x)\left[e^x + 5^{-x}\log 5\right]$$

[Let us sum up

1. Basic formulas $c,\ x^n,\ \log x,\ e^x,\ a^x$ ($\sqrt{x}$, x)
2. Addition/ Rule
3. multiplication ")
4. Division Rule

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

1 $$y = \frac{x^2}{e^x}$$

$$\frac{dy}{dx} = \frac{e^x \frac{d(x^2)}{dx} - x^2 \frac{d(e^x)}{dx}}{(e^x)^2}$$

$$= \frac{e^x \times 2x - x^2 \times e^x}{(e^x)^2}$$

$$= \frac{xe^x(2-x)}{(e^x)^2}$$

$$= \frac{x(2-x)}{e^x}$$ ✓

2 $$y = \frac{x+1}{x^2} = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{x^2 \frac{d(x+1)}{dx} - (x+1)\frac{d(x^2)}{dx}}{(x^2)^2}$$

$$= \frac{x^2(1+0) - (x+1) \times 2x}{x^4}$$

$$= \frac{x^2 - 2x(x+1)}{x^4}$$

$$= \frac{x\{x - 2(x+1)\}}{x^4} \quad x^3$$

$$= \frac{x - 2(x+1)}{x^3}$$

3. $y = \dfrac{\log x + 5^x}{x+5}$

$$\frac{dy}{dx} = \frac{(x+5)\dfrac{d(\log x + 5^x)}{dx} - (\log x + 5^x)\dfrac{d(x+5)}{dx}}{(x+5)^2}$$

$$= \frac{(x+5)\left[\dfrac{d(\log x)}{dx} + \dfrac{d(5^x)}{dx}\right] - (\log x + 5^x)\left(\dfrac{dx}{dx} + \dfrac{d5}{dx}\right)}{(x+5)^2}$$

$$= \frac{(x+5)\left(\frac{1}{x} + 5^x \log 5\right) - (\log x + 5^x)}{(x+5)^2}$$

4. $y = \dfrac{(x+2)^2}{x^3}$

$x^2 + 4x + 4$

$(a+b)^2 = a^2 + 2ab + b^2$

$$\Rightarrow y = \frac{x^2+4x+4}{x^3}$$

$$\frac{dy}{dx} = \frac{x^3 \frac{d(x^2+4x+4)}{dx} - (x^2+4x+4)\frac{d(x^3)}{dx}}{(x^3)^2}$$

$$= \frac{x^3\left[\frac{d(x^2)}{dx} + \frac{d(4x)}{dx} + \frac{d(4)}{dx}\right] - (x^2+4x+4)3x^2}{(x^3)^2}$$

$$= \frac{x^3(2x+4+0) - 3x^2(x^2+4x+4)}{x^6}$$

$$= \frac{x^3(2x+4) - 3x^2(x^2+4x+4)}{x^6}$$

5 $y = \frac{5^x}{x^2}$

$$\frac{dy}{dx} = \frac{x^2 \frac{d(5^x)}{dx} - 5^x \frac{d(x^2)}{dx}}{(x^2)^2}$$

$$= \frac{x^2 \times 5^x \log 5 - 5^x \times 2x^{2-1}}{(x^2)^2}$$

$$= \frac{x^2 \times 5^x \log 5 - 2 \times 5^x x}{}$$

$$\left(x^2\right)^4 = x^8$$

$$= \frac{x^2 \times 5^x \log 5 - 2x \times 5^x}{x^4}$$

6/ $$y = \frac{x^2+10}{x}$$

$$\frac{dy}{dx} = \frac{x\frac{d(x^2+10)}{dx} - (x^2+10)\frac{dx}{dx}}{x^2}$$

$$= \frac{x(2x+0) - (x^2+10)\times 1}{x^2}$$

$$= \frac{2x^2 - (x^2+10)}{x^2}$$

$$= \frac{2x^2 - x^2 - 10}{x^2} = \frac{x^2-10}{x^2}$$

7/ (H.W) $$y = \frac{5^x}{x^5} = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{x^5\frac{d(5^x)}{dx} - 5^x\frac{d(x^5)}{dx}}{}$$