Compound Interest

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Sum lent (P) = Rs 10,000
N = 3 years
R = 5% PA

SI for 1 year = 5% of 10000
= Rs 500
Amount for 1st yr = 1050
SI for 3 yrs = 500 × 3 = 1500

for 1st year
P_1 = Rs 10,000
R = 5% PA
N = 1 year
$I_1 = \frac{10000 \times 5 \times 1}{100}$ = Rs 500
$A_1 = P_1 + I_1$
A_1 = Rs 10,500

for 2nd year
$P_2 = A_1$ = Rs 10,500
R = 5%, N = 1 yr
$I_2 = \frac{10500 \times 5 \times 1}{100}$ = Rs 525
$A_2 = P_2 + I_2 = 1050 + 525$
= Rs 11025

for 3rd year
$P_3 = A_2$ = Rs 11025
R = 5%, N = 1 year
$I_3 = \frac{11025 \times 1 \times 5}{100}$ = Rs 551.25
$A_3 = 11025 + 551.25$
A_3 = 11576.25

∴ CI = $A_3 - P_1$ = Rs 1576.25

$$A = P\left[1 + \frac{R}{100}\right]^N$$
$$= 10,000\left[1 + \frac{5}{100}\right]^3$$
$$= 10,000\left[1 + 0.05\right]^3$$
$$= 10,000\left[1.05\right]^3$$
$$= 10,000 \times 1.157625$$
A = Rs 11576.25

Q. Mr Kumar borrowed Rs 20,000 for 4 years at 8% PA CI, compounded annually. Find
(i) Interest earned in 3 years
(ii) Interest earned in 4 years
(iii) Total amount to be repaid at the end of the given time period.
(iv) Interest earned in 4th year.

Soln: (i) $N = 3$ yrs, $R = 8\%$ PA, $P = 20{,}000$

$$A_3 = P\left[1+\frac{R}{100}\right]^N$$

$$= 20{,}000\left[1+\frac{8}{100}\right]^3 = 20{,}000\left[1.08\right]^3$$

$$= 20{,}000 \times 1.259712 = 25194.24$$

$$\therefore CI = A_3 - P = 25194.24 - 20{,}000$$

$$= \text{Rs } 5194.24$$

Q: The simple interest at 20% p.a on a certain sum of money for 4 years is Rs 25600. Find the compound interest on the sum at the same rate for the same period.

Soln: $R = 20\%$ p.a., $N = 4$ yrs, $SI = Rs\ 25600$

$$P = \frac{SI \times 100}{R \times N} = \frac{25600 \times 100}{20 \times 4}$$

$$\therefore \boxed{P = Rs\ 32000}$$

Compound Interest

$P = Rs\ 32000$, $N = 4$, $R = 20\%$

$$A = P\left[1 + \frac{R}{100}\right]^N = 32000\left[1 + \frac{20}{100}\right]^4$$

$$= 32000\left[1.2\right]^4$$

$$= 32000 \times 2.0736$$

$$A = Rs\ 66355.20$$

$$\therefore CI = A - P = 66355.20 - 32000$$

$$= Rs\ 34355.20$$

Compound Interest

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Q: A sum of Rs 50,000 was lent for 3 years at 10% PA CI. find

(i) Sum outstanding at the beginning of 3rd yr.
(ii) Interest Accrued (accumulated) in 2 years
(iii) interest earned during the third year.
(iv) Total interest earned in 3 years.

Soln: (i) find A_2 (Amount at the end of 2nd yr)

$$A_2 = P\left[1+\frac{R}{100}\right]^N = 50,000\left[1+\frac{10}{100}\right]^2$$

$$= 50,000\,[1.1]^2 = 50,000 \times 1.21$$

$$A_2 = 60500$$

(ii) $(I_2 + I_1) = A_2 - P_1 = 60500 - 50,000$
$= Rs\ 10500$

(iii) $A_3 = 50,000\,[1.1]^3 = 50000 \times 1.331$
$= 66550$

$\therefore CI = 66550 - 50000 = Rs\ 16550$
$(I_1+I_2+I_3)$

(iv) $I_3 = (I_1+I_2+I_3) - (I_1+I_2)$
$= 16550 - 10500 = Rs\ 6050$

OR

$I_3 = A_3 - A_2$

$= 50,000\,[1.1]^3 - 50,000\,[1.1]^2$

$= 50,000\,(1.1)^2\,[1.1-1]$

$= 50,000 \times 1.21 \times 0.1 =$

$= Rs\ 6050$

$= Rs\ 6050$

Q: A sum of Rs 40,000 was lent for 4 years at 12% PA CI. find
(i) Sum outstanding at the beginning of 3rd yr.
(ii) Interest Accrued (accumulated) on 3 years
(iii) interest earned during the 4th year.
(iv) Total interest earned in 4 years.
(v) Interest earned in 3rd year.

Soln: $A_2 = P_3 = P\left[1+\frac{R}{100}\right]^2$

$= 40,000\left[1+\frac{12}{100}\right]^2$

$= 40,000 \times 1.12 \times 1.12 = 40000 \times 1.2544$

$= 50176$ [CI = 10176]

(ii) $A_3 = 40,000\left[1+\frac{12}{100}\right]^3 = 40000 \times (1.12)^3$

$A_3 = 40000 \times 1.4049 = 56197.12$

$\therefore CI = 56197.12 - 40000 = Rs\ 16197.12$

(iii) $A_4 = 40000\left[1.12\right]^4$

$= 40,000 \times 1.5735 = Rs\ 62940$

Int. earned in 4th year $= A_4 - A_3 = A_4 - P_4$

$= 62940 - 56197.12$

$Rs = 6742.88$

Interest earned in n^{th} year = Amount in n year − Amount in (n−1) yrs

Interest earned in 3rd year

$= A_3 - A_2$

$= P\left[1+\frac{R}{100}\right]^3 - P\left[1+\frac{R}{100}\right]^2$

$= P\left[1+\frac{R}{100}\right]^2\left[1+\frac{R}{100}-1\right]$

$= P\left[1+\frac{R}{100}\right]^2 \times \frac{R}{100}$

Int. earned in N^{th} year

$= P\left[1+\frac{R}{100}\right]^{N-1} \times \frac{R}{100}$

$= 40,000\left[1.12\right]^2 \times 0.12$

$= 40,000 \times 1.12 \times 1.12 \times 0.12$

$=$

Compounded	i	n
Yearly	$\frac{R}{100}$	$n = N$
Half yearly	$\frac{R}{2\times100}$	$n = 2N$
Quarterly	$\frac{R}{4\times100}$	$n = 4N$
Monthly	$\frac{R}{12\times100}$	$n = 12N$

$N \rightarrow$ No. of years

Q: A sum of Rs 80,000 was lent for 2 years at 10% PA CI. Find the difference in the interest when it is compounded (i) Half yearly & Quarterly.

Soln: Half yearly

$P = Rs\ 80,000$, $i = \frac{10}{200} = 0.05$, $n = 2\times2 = 4$

$\therefore A = P[1+i]^n$

$= 80,000\ [1+0.05]^4 = 80,000 \times (1.05)^4$

$= 80,000 \times 1.2155 = Rs\ 36465$

Quarterly

$i = \frac{10}{400} = 0.025$, $n = 2\times4 = 8$

$A = 80,000\ [1.025]^8 = 80,000 \times 1.2184 = Rs\ 36552$

$\therefore$ Difference in the interest $= 36552 - 36465$

$= Rs\ 87$

Compounded	i	n
Yearly	$\frac{R}{100}$	$n = N$
Half yearly	$\frac{R}{2 \times 100}$	$n = 2N$
Quarterly	$\frac{R}{4 \times 100}$	$n = 4N$
Monthly	$\frac{R}{12 \times 100}$	$n = 12N$

N → No. of years

Q: Find the amount and compound interest on a sum of Rs 10,000 for 2 years at 12% PA when the rate of interest is compounded (i) yearly (ii) Half-yearly (iii) Quarterly (iv) Monthly

Soln: (i) $P = Rs\ 10,000$, $n = 2$, $i = \frac{R}{100} = \frac{12}{100} = 0.12$

$$A = P[1+i]^n$$

$$A = 10,000[1+0.12]^2 = 10,000[1.12]^2$$

$$A = 10,000 \times 1.2544 = 12544$$

$$\therefore CI = A - P = 12544 - 10,000 = Rs\ 2544$$

(ii) $n = 2 \times 2 = 4$ Hy, $i = \frac{R}{200} = \frac{12}{200} = 0.06$

$$A = 10,000[1+0.06]^4 = 10,000[1.06]^4$$

$$= 10,000 \times 1.262476 = Rs\ 12624.76$$

$$CI = 12624.76 - 10,000 = Rs\ 2624.76$$

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(iii) $n = 4 \times 2 = 8$ quarters, $i = \frac{12}{4 \times 100} = 0.03$

$A = 10,000[1+0.03]^8 = 10,000[1.03]^8$

$A = 10,000 \times 1.2667700 = 12667.70$

$CI = Rs\ 2667.70$

(iv) $n = 12 \times 2 = 24$ months, $i = \frac{12}{12 \times 100} = 0.01$

$A = 10,000[1+0.01]^{24} = 10,000[1.01]^{24}$

$= 10,000 \times 1.2697346$

$A = Rs\ 12697.35$

$\therefore CI = Rs\ 2697.35$

Q: A Particular sum of money amounts to Rs. 497778.75 in 3 years and Rs 474075 in 2 years. find the sum and the rate of compound interest.

Soln: $A = P[1+i]^n$

$A_3 = 497778.75 = P[1+i]^3$ ——— (A)

$A_2 = 474075 = P[1+i]^2$ ——— (B)

A ÷ B

$$\frac{497778.75}{474075} = \frac{P[1+i]^3}{P[1+i]^2} = (1+i)$$

$\Rightarrow 1+i = 1.05 \Rightarrow i = 1.05 - 1 = 0.05$

$i = \frac{R}{100} = 0.05 \Rightarrow \boxed{R = 5\%}$

$474075 = P[1+0.05]^2 \Rightarrow P[1.1025]$

$P = \frac{474075}{1.1025} \Rightarrow P = Rs\ 4,30,000/-$

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Q: A sum of Rs 63,000 is invested in a fixed deposit giving 10% p.a. compound interest. find the interest in the 4th year.

Soln: Interest in 4th year

$= A_4 - A_3$

$= P[1+i]^4 - P[1+i]^3$

$P[1+i]^3\ [1+i-1]$

$i = \frac{R}{100} = \frac{10}{100} = 0.1$

$= 63000[1+0.1]^4 - 63000[1+0.1]^3$

$= 63000[1.1]^4 - 63000[1.1]^3$

$= 63000 \times 1.4641 - 63000 \times 1.331$

$= 92238.30 - 83853$

$= Rs\ 838530$

Q A sum of Rs 50,000 is invested in a fixed deposit giving 7% p.a. compound interest. find the interest in the 3rd year.

Q The CI and SI on a sum of money at a certain rate for 2 years is Rs 8200 and Rs 8000 respectively. Find the sum and rate of interest.

Note: When compounded annually, CI for 1st yr = SI for 1st yr

CI for 1st yr = SI of 1st yr $= \frac{8000}{2} = Rs\ 4000$

CI for 2nd yr = CI for 2 yrs − CI for 1st yr

$= 8200 - 4000 = Rs\ 4200$

$I_1 = Rs\ 4000$, $I_2 = Rs\ 4200$

contd →

1 - Rs 4000, 2 = Rs 4200

Contd -

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$$R = \frac{(I_2 - I_1) \times 100}{I_1 \times 1} = \frac{200 \times 100}{4000} = 5\% \text{ PA}$$

SI for 1st yr = Rs 4000, R = 5%, N = 1

$$P = \frac{SI \times 100}{R \times N} = \frac{4000 \times 100}{5 \times 1} = \text{Rs } 80,000$$

Q. A sum amounts to Rs 46305 after 3 years at C.I. What will it amount to if simple interest is used?

Q. The maturity amount of Rs 2433600 at 20% P.A. CI after 2 years is same as the maturity amount of a sum at 30% p.a. CI after 2 years. Find the sum.

Q On what sum of money will the difference between CI and SI for 2 years at 4% PA be Rs 56?

Q. Find the present value of Rs 40,00,000 required 4 years from now if the compound interest rate is 5%.

Q. For how many years must Rs 3,20,00,000 be invested at 8% PA to get Rs 3,46,37,829.12 if the interest is compounded quarterly?

Q. Find the maturity amount of a deposit of Rs 400,000 compounded semi-annually for 1½ years at 7% PA.

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Q. The maturity amount of Rs 2433600 at 20% P.A. CI after 2 years is same as the maturity amount of a sum at 30% p.A. CI after 2 years. Find the sum.

Soln: Case I.

$P = Rs\ 2433600$, $n = 2$ yrs. $i = \frac{20}{100} = 0.2$

$A = P[1+i]^n$

$= 2433600[1+0.2]^2 = 2433600 \times 1.44$

$A = 3504384$ ——— (A)

Case II :-

$A = Rs\ 3504384$, $n = 2$ years, $i = \frac{30}{100} = 0.3$

$P = ?$

$3504384 = P[1+0.3]^2$

$\Rightarrow 3504384 = P \times 1.69$

$\Rightarrow P = \frac{3504384}{1.69} = Rs\ 20,73,600/-$

Q. For how many years must Rs 3,20,00,000 be invested at 8% PA to get Rs 3,46,37,829.12 if the interest is compounded quarterly?

Soln: $P = Rs\ 3,20,00,000$, $A = Rs\ 3,46,37,829.12$

$n = ?$ $i = \frac{R}{400} = \frac{8}{400} = 0.02$

$3,46,37,829.12 = 3,20,00,000[1+0.02]^n$

$\Rightarrow \frac{346378\cancel{29.12}}{32000000} = (1.02)^n \Rightarrow (1.02)^n = 1.08243216$

$(1.02)^n = (1.02)^4 \Rightarrow n = 4$ Quarters

$\therefore$ No. of years $= \frac{4}{4} = 1$ years

$\therefore$ No. of years $= \frac{4}{4} = 1$ years

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Q. The difference between the compound interest, compounded annually and the simple interest on a certain sum for 2 years at 6% per annum is Rs. 18. Find the sum.

Q. A certain sum amounts to Rs. 72900 in 2 years at 8% per annum compound interest, compounded annually. Find the sum.

HW

Soln: Let the principal be Rs P

$R = 6\%$ PA, $n = 2$ yrs

CI − SI = Rs 18

Amount $= P\left[1+\frac{R}{100}\right]^2 \Rightarrow A = P\left[1+\frac{6}{100}\right]^2$

$\Rightarrow A = P[1.06]^2 = 1.1236P$

$CI = A - P = 1.1236P - P = 0.1236P$ ——— (A)

$SI = \frac{PNR}{100} = \frac{P \times 2 \times 6}{100} = 0.12P$

CI − SI = Rs 18

$0.1236P - 0.12P = Rs\ 18$

$0.0036P = Rs\ 18$

$\therefore P = \frac{Rs\ 18}{0.0036} = Rs\ 5000$

The simple interest accrued on an amount of Rs. 20,000 at the end of three years is Rs.7,200. What would be the compound interest accrued on the same amount at the same rate in the same period ?

Soln: P = Rs 20,000. SI = Rs 7200, N = 3 yrs

$R = \frac{SI \times 100}{P \times N}$

$= \frac{7200 \times 100}{20000 \times 3}$

$R = 12\%$ PA

$\Rightarrow i = 0.12$

$CI = P[(1+i)^n - 1]$

$= 20,000[(1+0.12)^3 - 1]$

$= 20,000[1.404928 - 1]$

$= 20,000 \times 0.404928$

$CI = Rs\ 8098.56$

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Mr. Akash invested Rs. 20,000 with rate of interest at 20 % p.a. The interest was compounded half yearly for first year and in the next year it was compounded yearly. What will be the total interest earned at the end of two years?

Soln: Rs 20,000.

i_1 (when compounded Half yearly) $= \frac{20}{200} = 0.10$, $n_1 = 2$ H.Y

i_2 (when compounded yearly) $= \frac{20}{100} = 0.20$, $n_2 = 1$

$$\text{Amount} = P[1+i_1)^{n_1}(1+i_2)^{n_2}$$

$$= 20,000\,[1+0.1]^2\,[1+0.2]^1$$

$$= 20,000 \times 1.21 \times 1.2$$

Amount = Rs 29040

$\therefore$ CI = A − P = Rs 29040 − 20,000

= Rs 9040

Note: When the rate of interest is different for different years, then

$$\text{Amount (A)} = P[1+i_1)^{n_1}(1+i_2)^{n_2} - - - - - -.$$

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Chaitanya borrowed Rs. 8,000 at a compound interest rate of 10% per annum. If he repays Rs. 1,500 at the end of the first year and Rs. 2,000 at the end of the second year, find his outstanding loan at the beginning of the third year.

Soln: $P_1 = Rs\ 8000$

$i = \frac{10}{100} = 0.10$

$n = 1$

$I_1 = P_1 \times i \times n = \frac{P \times N \times R}{100}$

OR

$CI = P[(1+i)^n - 1]$

$= 8000[(1+0.1)^1 - 1]$

$= 8000[0.1]$

$= 800$

$A_1 = P_1 + CI = 8800$

Balance Amount Payable after repayment of Rs 1500 $= 8800 - 1500 = 7300 = P_2$

$A_2 = 7300[(1+0.1)^1$

$= 7300 \times 1.1 = Rs\ 8030$

Balance Amount after repayment of Rs 2000

$= Rs\ 8030 - 2000 = \underline{Rs\ 6030} = P_3.$

∴ Outstanding loan at the beginning of 3rd year = Rs 6030

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A man starts a business with a capital of Rs.1000000. He incurs a loss of 4% during the first year. But he makes a profit of 5% during the second year on his remaining investment. Finally, he makes a profit of 10% on his new capital during the third year. Find his total profit at the end of three years.

$$\text{Amount} = P\left[1-i_1\right]^{n_1}\left[1+i_2\right]^{n_2}\left[1+i_3\right]^{n_3}$$

$$1,00,00,00\left[1-0.04\right]^{1}\times\left[1+0.05\right]^{1}\left[1+0.10\right]^{1}$$

$$= 1,00,00,00 \times 0.96 \times 1.05 \times 1.10$$

$$A = \text{Rs } 11,08800$$

$$\text{Profit} = 1108800 - 1,00,00,00$$

$$= \text{Rs } 108800$$

The population of a village increases by 10% every year. If the present population is 60000, what will be the population of the village after 3 years? Also find the population 3 years ago.

I ⇒ $A = \text{Present population}\left[1+\text{Rate of appreciation}\right]^{n}$

$$\Rightarrow \text{future population} = 60,000\left[1+\frac{10}{100}\right]^{3}$$

$$= 60,000\left[1.1\right]^{3}$$

$$= 60,000 \times 1.331 = 79860$$

II : $60,000 = \text{Present pop}\left[1+\frac{10}{100}\right]^{3}$

$$\frac{60,000}{(1.1)^3} = \text{Population 3 years ago}$$

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