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# Annuity

(1) Present Value of Annuity Regular (PVAR)
(2) future value of Annuity Regular (FVAR)

| compounded | $i$ | $n$ |
|---|---|---|
| yearly | $\frac{R}{100}$ | $n = N$ |
| Half yearly | $\frac{R}{200}$ | $n = 2N$ |
| Quarterly | $\frac{R}{400}$ | $n = 4N$ |
| Monthly | $\frac{R}{1200}$ | $n = 12N$ |

N → No. of years

$$F.V.A.R = A\left[\frac{(1+i)^n - 1}{i}\right]$$

A → Periodic Payment

$$PVAR = A\left[\frac{(1+i)^n - 1}{i \times (1+i)^n}\right]$$

$$\text{Sinking fund} = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$\text{Loan Amount} = EMI\left[\frac{(1+i)^n - 1}{i \times (1+i)^n}\right]$$

Q. find the accumulated value at the end of 4 years for a deposit of Rs 1500 made at the end of every year. rate of interest offered by the bank is 8% PA CI.

Soln: A (Periodic payment) = Rs 1500

$i = \frac{8}{100} = 0.08$, $n = 4$ yrs

$$\text{Accumulated value} = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 1500\left[\frac{(1.08)^4 - 1}{0.08}\right]$$

$$= 1500\left[\frac{1.36048896 - 1}{0.08}\right]$$

$$= \text{Rs } 6759.17$$

Q: find the accumulated value after 4 years of an immediate annuity of Rs 40,000 P.A. with interest compounded at 6% P.A.

Soln: $n = 4$, $i = \frac{6}{100} = 0.06$, A (periodic payment) $= 40,000$

$$\text{Accumulated value} = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 40,000\left[\frac{(1+0.06)^4 - 1}{0.06}\right]$$

$$= 40,000 \times \left[\frac{1.26247696 - 1}{0.06}\right]$$

$$= 40,000 \times \frac{0.26247696}{0.06} = \text{Rs } 174984.64$$

Q. Mr. Aditya invested Rs 6,000 at the end of every year at 10% compound interest rate for some years and received Rs 19860 as the accumulated value. For how many years, the investment was made by Mr. Aditya?

Sol: Maturity value = Rs 19860

A (Periodic payment) = Rs 6000

$i = \frac{10}{100} = 0.1$

$MV = A\left[\frac{(1+i)^n - 1}{i}\right]$

$19860 = 6000\left[\frac{(1+0.1)^n - 1}{0.1}\right]$

$\frac{19860}{6000} = \left[\frac{(1.1)^n - 1}{0.1}\right]$

$3.31 \times 0.1 = (1.1)^n - 1$

$0.331 + 1 = (1.1)^n$

$(1.1)^n = (1.331)$

$(1.1)^n = (1.1)^3$

$\therefore$ **n = 3 years**

04 → n = 4 − 1 = 3

1.331

1.1 X = =

Q. Find the number of years for which an annuity of Rs 10,000 is paid at the end of each year, if its accumulated amount works out to be Rs 53680 with interest compounded at 20% PA?

# Sinking fund

Q. What sum should be set aside every year at 6% P.A. CI for 5 years to replace a machine which is expected to cost Rs 1,20,000? The Salvage value of the existing machinery will be Rs 20,000 at the end of 5 years.

Soln: Total Money required at the end of 5 years
= cost of Machine - salvage value of old Machine

$= 1,20,000 - 20,000 = Rs\ 1,00,000$

$\Rightarrow$ Accumulated value = Rs 1,00,000

$i = 6/100 = 0.06, \quad n = 5$ yrs

A (Periodic payment) = ??

$$\text{Accumulated Value} = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$1,00,000 = A\left[\frac{(1+0.06)^5 - 1}{0.06}\right]$$

$$1,00,000 = A\left[\frac{1.3382 - 1}{0.06}\right]$$

$$\frac{1,00,000 \times 0.06}{0.3382} = A$$

$\therefore A = Rs\ 17739.64$

$\Rightarrow$ Rs 17739.64 should be set aside every year.

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Q. Ms. Ananya deposited Rs 45000 at the end of every $\frac{1}{2}$ year for 2 years. The rate of interest is 10% P.A. compounded Half-yearly. What is the accumulated amount at the end of 2 years?

Soln: A (Periodic Payment) = Rs 45000

$n = 2 \times 2 = 4$

$i = \frac{R}{200} = \frac{10}{200} = 0.05$

Maturity value $= A\left[\frac{(1+i)^n - 1}{i}\right]$

$= 45000\left[\frac{(1.05)^4 - 1}{0.05}\right]$

$= 45000\left[\frac{1.21550625 - 1}{0.05}\right]$

$= 45000 \times \frac{0.21550625}{0.05}$

$=$ Rs 193955.625

$=$ Rs 193955.63

Extra

MV (to the nearest ten) = 193960

to the nearest Hundred = 194000

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Q. Ms. Ananya deposited Rs 45000 at the end of every year for 2 years. The rate of interest is 10% P.A. compounded Half-yearly. What is the accumulated amount at the end of 2 years?

Soln: As the amount is deposited at the end of every year, hence the time period of the annuity is a year. However the rate of interest is 10% PA compounded H.Y. Hence we will have to find out the effective ROI per annum.

(find out the CI on Rs 100 for a year at 10% P.A. compounded H.Y)

$P = Rs\ 100,\ i = 0.05,\ n = 2$

$$\text{Amount} = P[1+i]^n = 100[1.05]^2$$
$$= 100 \times 1.1025$$
$$A = Rs\ 110.25$$
$$\therefore CI = A - P = 110.25 - 100$$
$$= Rs\ 10.25$$

$\Rightarrow$ effective ROI = 10.25%

Now we have an annuity of Rs 45000 at the end of each year for 2 years at 10.25% PA

$$i = \frac{10.25}{100} = 0.1025,\ n = 2$$

$$\therefore \text{Maturity value} = 45000\left[\frac{(1+0.1025)^2 - 1}{0.1025}\right]$$

$$= Rs\ 94612.5$$

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# Present value of Annuity

$$PVAR = A\left[\frac{(1+i)^n - 1}{i \times (1+i)^n}\right]$$

**Q:** Find the present value of an annuity at the end of 4 years at 10% p.a CI for a sum of Rs 10,000 deposited every year.

**Soln:**

$$PVAR = 10{,}000\left[\frac{(0.1)^4 - 1}{0.1 \times (1.1)^4}\right] \checkmark$$

$$= 10{,}000\left[\frac{1.4641 - 1}{0.1 \times 1.4641}\right] \checkmark \quad \checkmark$$

$$= \frac{10{,}000 \times 0.4641}{0.1 \times 1.4641} = ₹31698.65$$

1.1 X = = = M+ − 1 × 10,000 ÷ 0.1 ÷ MRC =

$$\frac{18}{2 \times 3} \qquad = \frac{18}{6} = 3$$

$18 \div 2 \times 3 = 27$ | $\frac{(18 \div 2)}{3}$

$18 \div 2 \div 3 = 3$ | $18 \div 2 \div 3$

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Q: How much money should a person invest at 7% P.A. compound interest so that he would get an annuity of Rs 1,00,000 at the end of each year for the next four years after which the principal money will be over?

$$PVAR = A\left[\frac{(1+i)^n - 1}{i \times (1+i)^n}\right]$$

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Q Machine 'X' costs Rs 5000 and has a useful life of 4 years. Machine 'y' costs Rs 4000 and has a useful life of 3 years. Machine 'X' is supposed to generate an annual savings of Rs 2800 while Machine 'y' is supposed to generate an annual savings of Rs 3000. Assuming the time value of money is 10% PA, determine which machine is preferable?

Soln: Machine X:

$A = Rs\ 2800,\ n = 4,\ i = 0.1$

$$PVAR = 2800\left[\frac{(1.1)^4 - 1}{0.1 \times (1.1)^4}\right]$$

$= (1.1)X == M^+ - 1 \times 2800 \div 0.1 \div MRC =$

$PVAR = 8875.62$

Net present value of Machine X

= present value of savings − Present cost

= 8875.62 − 5000 = Rs 3875.62

Machine Y:

$A = Rs\ 3000,\ n = 3,\ i = 0.1$

$$PVAR = 3000\left[\frac{(1.1)^3 - 1}{0.1 \times (1.1)^3}\right]$$

$= Rs\ 7460.56$

Net present value = 7460.56 − 4000

= Rs 3460.56

# EMI

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## EMI-Equated Monthly Installment

$$\text{Loan Amount} = EMI\left[\frac{(1+i)^n - 1}{i \times (1+i)^n}\right]$$

**Two Methods to solve**

1. Reducing Balance Method
2. Flat rate of Interest Method

EMI → Partial repayment toward Loan Amt + Interest of the given time.

→ $\text{Interest} = \frac{P \times N \times R}{100}$ [P → Loan Amt, N → No. of years]

$\text{Amount} = P + I$

$EMI = \frac{\text{Amount}}{\text{No. of months}}$

Q: A loan of Rs 80,000 is to be paid or returned in 3 monthly instalments at the rate of 12% P.A. compounded monthly. Find the EMI using reducing balance method. Also find the interest and principal repayment for each month.
(Amortization Table)

Soln: Loan Amt = Rs 80,000

$n = 3$ , $i = \frac{12}{1200} = 0.01$

$$\text{Loan Amt} = EMI\left[\frac{(1+i)^n - 1}{i \times (1+i)^n}\right]$$

$$80{,}000 = EMI\left[\frac{(1+0.01)^3 - 1}{0.01 \times (1+0.01)^3}\right]$$

$$80{,}000 = EMI\left[2.940985207 23\right]$$

1.01 x = = M+ − 1 ÷ 0.01 ÷ MRC =

$$\therefore EMI = \frac{80{,}000}{2.940985207 23}$$

EMI = Rs 27201.77

PTO

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| S.NO | Loan Amt | EMI | Interest on Outstanding Loan | Repayment Towards loan | Balance Amt |
|---|---|---|---|---|---|
| 1. | 80,000 | 2720.77 | 800 | 26401.77 | 53598.23 |
| 2. | 53598.23 | 27201.77 | 535.98 | 26665.79 | 26932.44 |
| 3. | 26932.44 | 2720.77 | 269.32 | 26932.45 | 0 |

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