Compound Interest

18 January 2021 08:14

Since the compound interest is the 'interest on interest' over period of time, it depends on the frequency of interest redeemed.

Ex. If the interest released yearly, then the principal will change (increase) at the end of each year,
But if the interest released on monthly basis, then the Principal will change (increase) at the end of each month & further interest will have to be calculated accordingly.

Thus, the compound interest formula depends on the frequency of the interest compounded.

The interest can be compounded yearly, half yearly, quaterly or monthly.

* Formula:

(1) $$A = P\left(1 + \frac{R}{100}\right)^n$$

Where,

A = Accumulated value (Total amount)
P = Principal amount
R = Rate of interest
n = period or time

Compound interest formula is given by

$$A = CI + P$$ (Interest compounded yearly)

$$\therefore CI = A - P$$

$$\therefore CI = P\left(1 + \frac{R}{100}\right)^n - P$$

2) If the interest is to be compounded half-yearly.

$$A = P\left(1 + \frac{R}{2(100)}\right)^{2n}$$

3) If the interest is to be compounded quaterly

$\left(1+\frac{R}{2(100)}\right)$

3) If the interest is to be compounded quately.

$$A = P\left(1+\frac{R}{400}\right)^{4n}$$

4) If the interest is to be compounded monthy.

$$A = P\left(1+\frac{R}{1200}\right)^{12n}$$

* The compound interest for k^{th} period is calculated as.

= interest for k periods − interest for (k-1) periods.

It can also be Calculated as

✓ = Accumulated amount after k periods − Accumulated amount after (k-1) periods

$$= P\left(1+\frac{R}{100}\right)^{k-1}\left(\frac{R}{100}\right)$$

* Examples *

1) Mr. Sinha borrows ₹ 12,000 for 4 years at compound interest rate of 8% p.a. How much will be have to repay at the end of period.

Sol.

Given:

$P = 12000$

$n = 4$

$R = 8\%$ p.a.

$A = ?$

Formula:

$$A = P\left(1+\frac{R}{100}\right)^{n}$$

$$= 12000\left(1+\frac{8}{100}\right)^{4}$$

$$= 12000\left(1+0.08\right)^{4}$$

$$\therefore A = 12000\,(1.08)^{4}$$

$\therefore \quad A = 12000 \ (1.360)$

$\therefore \quad \boxed{A = 16320}$

$\therefore$ Amount he had repay is ₹ 16320 //

Q2 At what compound rate of interest a sum of ₹ 20,000 will amount to ₹ 29,282 in Four years.

Soln Given $P = 20000$

$A = 29282$

$n = 4$

$R = ?$

$$A = P\left(1+\frac{R}{100}\right)^n$$

$$\therefore 29282 = 20000\left(1+\frac{R}{100}\right)^4$$

$$\therefore \frac{29282}{20000} = \left(1+\frac{R}{100}\right)^4$$

$$\therefore (1.46)^{1/4} = 1+\frac{R}{100}$$

$$\therefore \left(1.46^{1/2}\right)^{1/2} = 1+\frac{R}{100}$$

$$\therefore 1.10 = 1+\frac{R}{100}$$

$$\therefore \frac{R}{100} = 1.10 - 1$$

$$\Rightarrow \frac{R}{100} = 0.10$$

$$\Rightarrow \boxed{R = 10\%}$$

$\therefore$ Rate of compound interest is 10 %.

Ex3 Find the total amount, with interest if sum of ₹ 8,000 is kept in a bank for 3 years with 12 % p.a Compounded half yearly.

Soln Given: $P = 8000$

$n = 3$

$R = 12\ \%$

$n = 3$

$R = 12\ \%$

$A = ?$

Since the interest is compounded half yearly

$$A = P\left(1 + \frac{R}{200}\right)^{2n}$$

$$\therefore A = 8000\left(1 + \frac{12}{200}\right)^{2\times 3}$$

$$\therefore A = 8000\left(\frac{200+12}{200}\right)^{6}$$

$$\therefore A = 8000\left(\frac{212}{200}\right)^{6} = 8000\,(1.06)^{6}$$

$$\therefore A = 8000 \times 1.41852$$

$$\therefore \boxed{A = 11348.15290}$$

Ex 4 The difference between Simple & compound interest on certain amount for 4 years at 10% p.a. is ₹ 1,282. Find principal.

Soln

Given :- $n = 4$ years

$R = 10\ \%$

$$SI - CI = 1282$$

$$\because SI = \frac{P \times n \times R}{100} = \frac{P \times 4 \times 10}{100} = \frac{4P}{10} = 0.4P$$

$$CI = P\left(1 + \frac{R}{100}\right)^{n} - P = P\left(1 + \frac{10}{100}\right)^{4} - P = P\left(1 + 0.1\right)^{4} - P = P(1.01)^{4} - P$$

$\because SI - CI = 1282$

$$\therefore 0.4P - [P(1.01)^{4} - P] = 1282$$

$$\therefore 0.4P - P(1.04060) + P = 1282$$

$$\therefore P[0.4 - 1.04060 + 1] = 1282$$

$\therefore P[0.35940] = 1282$

$\therefore P = \frac{1282}{0.35940} = 3567.05620$

$\therefore \boxed{P = 3567.05620}$ //

Ex5 The difference between Simple interest & compound interest on a certain principal for 2 years at 6% rate of interest p.a. is ₹ 13.50. find the principal

Soln :. Given : $CI - SI = 13.50$

$n = 2$

$R = 6$

$CI = P\left(1+\frac{R}{100}\right)^n - P = P\left(1+\frac{6}{100}\right)^2 - P = P(1+0.06)^2 - P$

$\therefore CI = P(1.06)^2 - P$ ✓

$SI = \frac{P \times n \times R}{100} = \frac{2 \times 6 \times P}{100} = \frac{12}{100} P = 0.12P$ ✓

$\because CI - SI = 13.50$

$P(1.06)^2 - P - 0.12P = 13.50$

$\therefore P(1.12360) - P - 0.12P = 13.50$

$\therefore P[1.12360 - 1 - 0.12] = 13.50$

$\therefore P[0.00360] = 13.50$

$\therefore P = \frac{13.50}{0.00360} = 3750$

$\therefore \boxed{P = 3750}$

Ex6 Find the amount recieved when a sum of ₹ 12,000 is invested at 15% per an. for 2 years. If the interest compounded quaterly.

Soln Given $P = 12000$

$R = 15$

Soln Given $P = 12000$

$R = 15\%$

$n = 2$

$\because$ interest is Compounded quaterly.

$$A = P\left(1 + \frac{R}{400}\right)^{4n}$$

$$\therefore A = 12000\left(1 + \frac{15}{400}\right)^{4\times 2} = 12000\left(\frac{415}{400}\right)^{8}$$

$$= 12000\,(1.03750)^{8} = 12000 \times 1.34247 = 16109.64941$$

$\therefore$ **A = 16109.64941** (Monthly) (12)

Ex7 Amit lent his friend ₹ 10,000 at a token interest of 2% p.a. to be compounded half yearly. Calculate the amount due to him at the end of 4 years.

Soln: Given: $P = 10000$

$R = 2\%$

$n = 4$

$\because$ Interest is compounded half yearly.

$$A = P\left(1 + \frac{R}{200}\right)^{2n}$$

$$= 10000\left(1 + \frac{2}{200}\right)^{2\times 4}$$

$$= 10000\left(1 + \frac{1}{100}\right)^{8} = 10000\,(1+0.01)^{8} = 10000\,(1.01)^{8}$$

$$= 10000 \times 1.08286$$

$\therefore$ **A = 10828.6**

Ex8: Ritesh borrowed ₹ 5,000 from a money lender at 30% p.a., to be Compounded monthly. Find amount & the compound interest due to him at the end of two years.

Soln Given: $P = 5000$

$R = 30\%$

$n = 2$

$A = ?$ $CI = ?$

∵ The interest compounded monthly.

$$\therefore A = P\left(1 + \frac{R}{1200}\right)^{12n}$$

$$\therefore A = 5000\left(1 + \frac{30}{1200}\right)^{12 \times 2}$$

$$\therefore A = 5000\left(1 + 0.025\right)^{24} = 5000 \times (1.025)^{24}$$

$$\therefore A = 5000 \times 1.80873$$

$$\therefore \boxed{A = 9043.65}$$

Now,

$$CI = A - P = 9043.65 - 5000$$

$$\therefore \boxed{CI = 4043.65}$$

g) X invested ₹ 5,000 in a bank as short term deposit for 9 months at 5% to be compounded quaterly, Find the amount received by him on maturity.

Soln

Given $P = 5000$

$R = 5$

$n = 9$ months $= \frac{9}{12}$ years $= 0.75$ year.

$A = ?$

Since, the interest compounded quaterly

$$A = P\left(1 + \frac{R}{400}\right)^{4n}$$

$$\therefore A = 5000\left(1 + \frac{5}{400}\right)^{4 \times 0.75}$$

$$\therefore A = 5000\left(1 + 0.01250\right)^{3}$$

$$\therefore A = 5000\left(1 + \frac{}{400}\right)^{3}$$

$$\therefore A = 5000\left(1 + 0.01250\right)^{3}$$

$$\therefore A = 5000\left(1.01250\right)^{3}$$

$$\therefore A = 5000 \times 1.03797$$

$$\therefore \boxed{A = 5189.85}$$

Ex 10 For a four year investment, what is better deal, an investment with 12% interest, to be compounded annually or an investment with 11% interest to be compounded half-yearly.

Soln Given:

$n = 4$ $\quad P = 100$

$R_1 = 12\%$ p.a.

$R_2 = 11\%$

1) Interest compounded annually.

$$A_1 = P\left(1 + \frac{R_1}{100}\right)^{n}$$

$$A_1 = 100\left(1 + \frac{12}{100}\right)^{4}$$

$$= 100\left(1 + 0.12\right)^{4} = 100\left(1.12\right)^{4}$$

$$= 100 \times 1.57352 = 157.352$$

$$\boxed{A_1 = 157.352}$$

2) Interest compounded half yearly

$$A_2 = P\left(1 + \frac{R_2}{200}\right)^{2n}$$

$$\boxed{A_2 = 153.468}$$

$A_2 < A_1 \Rightarrow$ deal is better with 12% p.a