H.C.F & L.C.M

Lowest Common Factor (LCM)

Definition: LCM stands for Lowest or Least Common Multiple. In other words, the LCM of two or more numbers is the smallest positive integer divisible by all the given numbers.

Example: Consider this as an example; the LCM of 12 and 15 is 60.

To find the LCM of numbers, first, you need to mention the multiples of each given number.

Thus, the multiples of 12=12,24,36,48,60,72,84… etc.,

Now, the multiples of the second number 15=15,30,45,60,75,90, 105,…. etc,

Hence, 60 is the smallest number that is a multiple of both 12 and 15

Highest Common Factor (HCF)

Definition: The greatest common factor (GCF or GCD or HCF) of a set of whole numbers is the largest positive integer that divides all the given numbers evenly with zero remainders. HCF stands for Highest Common Factor. Thus, HCF is also known as GCF (Greatest Common Factor) or GCD (Greatest Common Divisor).

Example: The HCF of 12 and 15 is 3.

Now, the prime factorisation of 12=2×2×3

Prime factorisation of 15=3×5

As the number 3 is the only common factor for both the numbers 12 and 15, it is the largest number that divides both numbers.

Hence, 3 is the required answer.

LCM and HCF Formula

Relation Between LCM and HCF of Numbers

The relationship between HCF and LCM for two numbers is defined by the HCF and LCM formula given by:

Product of two number= (HCF of the two number) × (LCM of the two numbers)

If in case you take the two numbers as m and n, then the HCF and LCM formula states that as below:

m×n= HCF(m,n)×LCM(m,n)

Using the given above formulas, we can calculate the HCF or LCM if we know one of them and the two numbers.

HCF of the two number =Product of two numbers LCM of two numbers

LCM of the two number =Product of two numbers HCF of two numbers

How to find HCF and LCM?

The two essential methods to calculate HCF and LCM for a given set of numbers is as follows:

Finding HCF using Prime Factorisation Method

The prime factorisation method is also known as the factor tree method. Now, follow the given points to find out the HCF:

Write each of the numbers as a product of its prime factors.

Then you have to list the common factors for both the numbers.

All the common prime factors are the HCF (use the lower power of each common element).

Example: Finding the highest common factor of the numbers 16 and 24 .

You have to write each number as a product of its prime factors. 16=2424=23×3

You know that the product of all the common prime factors is HCF.

The standard prime factor in this example is 23[∵24 can be written as 23×2] So, HCF=23=8

Finding HCF using the Long Division Method

Following are the steps to get the HCF of the given numbers with the Division method.

You have to write the given numbers horizontally and separate them by a comma. Then, draw a line under the two numbers.

Now, draw a horizontal line on the left side and then write the smallest prime number that divides the given numbers without leaving any remainder.

Next, divide the numbers by the lowest prime number and write the quotients separated by a comma under the horizontal line.

You have to repeat the second and the third step until you reach the stage where no common prime factor is available.

The numbers that are on the left-hand side will be the common prime factors of the given numbers.

Thus, to find the HCF, multiply all the numbers that you get on the left side.

Finding LCM using the Prime Factorisation Method Example

To get the LCM, you need to use the maximum number of prime factors between the two or more given numbers.

Example: Find the LCM of 8 and 20 .

First, you need to write the prime factors of each of the given numbers.

So,

8=2×2×2

20=2×2×5

Now, you have to find the number of times the factors have appeared.

The factor 2=3 times

The factor 5=1 time

To get the LCM, you have to multiply each factor by the number of times it occurs in any of the numbers. Thus, you can see that the LCM of the numbers 8 and 20=2×2×2×5=40.

Finding LCM using the Long Division Method Example

To get the LCM of a given set of numbers using the long division method, you use the same technique which is used to find the HCF using the division method. You will see the difference in the final step.

Instead of multiplying all the common factors present on the left-hand side, you have to multiply all numbers obtained at the end of the division.

Example: LCM of the numbers 12 and 18.

So, after performing the long division method, you get the LCM of the number 12 and 18=2×3×2×3=36

LCM and HCF with Co-prime Numbers

LCM of the Co–Prime numbers (m,n) = product of two numbers (m,n)

Given example below will be used to verify the relation.

Example: The numbers 11 and 31 are the two co-prime numbers. Now, verify the LCM of given co-prime numbers is equal to the product of the given numbers.

So, the factors of 11 and 31 are,

11=1×11

31=1×31

HCF of the number 11 and 31=1

LCM of the number 11 and 31=341

Product of the numbers 11 and 31=11×31=341

Hence proved, LCM of the Co–Prime numbers (m,n)=product of two numbers (m,n).

LCM and HCF of Fractions

To get the HCF and LCM of the fractions like mn,pq,uv, etc., you can use the below-given formulas:

LCM of fractions=LCM of the Numerator ÷ HCF of the Denominators

HCF of fractions =HCF of the Numerator ÷LCM of the Denominators

Example: Finding out the LCM of the given fractions 1/4, 3/10, 2/5.

You have to use the given formula and get the HCF and LCM.

LCM of fractions=LCM of the Numerator ÷ HCF of the Denominators

LCM of fractions =LCM(1,3,2)÷HCF(4,10,5)

=6÷1=6

Solved Examples on Relationship between LCM and HCF

Q.1. Identify and write the HCF of the fractions 4/5,5/2,6/7.

Ans: Given, 4/5,5/2,6/7.

To find the HCF of the given fractions, so you have to use the below formula.

HCF of fractions=HCF of Numerators ÷ LCM of Denominators

Now, HCF of fractions=HCF(4,5,6)÷LCM(5,2,7)

Hence, =1/70

Q.2. The highest common factor and lowest common multiple of the two numbers are 18 and 1782, respectively. One number is 162 . Find the other.

Ans: Given to find the other number,

So you have to use the formula that is given below:

HCF×LCM=First number × Second number

We get,

18×1782=162×Second number

18×1782/162=Second number

Hence, the second number =19.

Q.3. If the HCF of two numbers is 3, and their LCM is 54. The first number is 27. Find the other number.

Ans: Given to find the second number,

Use the formula that is given below:

HCF×LCM=Product of two numbers

$3 \times 54 = 27 \times \text{second number}$

Second number = $(3 \times 54)/27 = 162/27 = 6$

Hence, the second number =6.

Q.4. When divided by 5,7,9 and 12, find the least number that leaves the same remainder 3 in each case.

Ans: We know that we have to find out the LCM of the divisors and then add the standard remainder with the number 3.

So, the LCM (5,7,9,12) =1260

Now, add the number 3 to 1260→3+1260=1263

Hence, the required answer is 1263.

Q.5. The two numbers ratio is 5:11. HCF is 7 , so find the numbers.

Ans: We will take numbers as 5 m and 11 m.

Since 5:11 is already the reduced ratio.

So, m has to be the HCF.

Here, the numbers we get is $5 \times 7 = 35$ and $11 \times 7 = 77$

Hence, the numbers are 35 and 77.

Q.6. Highest common factor and lowest common multiple of two numbers are 18 and 1782 respectively. One number is 162, find the other.

Solution:

We know, H.C.F. × L.C.M. = First number × Second number then we get,

18 × 1782 = 162 × Second number

(18×1782)/162 = Second number

Therefore, the second number = 198

Q.7. The HCF of two numbers is 3 and their LCM is 54. If one of the numbers is 27, find the other number.

Solution:

HCF × LCM = Product of two numbers

3 × 54 = 27 × second number

Second number = (3×54)/27

Second number = 6

Q.8. The highest common factor and the lowest common multiple of two numbers are 825 and 25 respectively. If one of the two numbers is 275, find the other number.

Solution:

We know, H.C.F. × L.C.M. = First number × Second number then we get,

825 × 25 = 275 × Second number

(825×25)/275 = Second number

Therefore, the second number = 75

Q.9. Two numbers are in the ratio of 5:11. If their HCF is 7, find the numbers.

Solution :

Let the numbers be 5m and 11m. Since 5:11 is already the reduced ratio, 'm' has to be the HCF. So, the numbers are 5 x 7 = 35 and 11 x 7 = 77.

Q.10. **Find the length of the plank which can be used to measure exactly the lengths 4 m 50 cm, 9 m 90 cm and 16 m 20 cm in the least time.**

Solution :

Let us first convert each length to cm. So, the lengths are 450 cm, 990 cm and 1620 cm. Now, we need to find the length of the largest plank that can be used to measure these lengths as the largest plank will take the least time. For this, we need to take the HCF of 450, 990 and 1620.

$450 = 2 \times 3 \times 3 \times 5 \times 5 = 2 \times 3^2 \times 5^2$

$990 = 2 \times 3 \times 3 \times 5 \times 11 = 2 \times 3^2 \times 5 \times 11$

$1620 = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 = 2^2 \times 3^4 \times 5$

Therefore, HCF (450, 990, 1620) = 2 x 3 x 3 x 5 = 90

Thus, we need a plank of length 90 cm to measure the given lengths in the least time.

Q.11. **Find the greatest number which on dividing 70 and 50 leaves remainders 1 and 4 respectively.**

Solution : The required number leaves remainders 1 and 4 on dividing 70 and 50 respectively. This means that the number exactly divides 69 and 46.

So, we need to find the HCF of 69 (3 x 23) and 46 (2 x 23).

HCF (69, 46) = 23

Thus, 23 is the required number.

Q.12 : **Find the largest number which divides 64, 136 and 238 to leave the same remainder in each case.**

Solution : To find the required number, we need to find the HCF of (136-64), (238-136) and (238-64), i.e., HCF (72, 102, 174).

$72 = 2^3 \times 3^2$

102 = 2 x 3 x 17

174 = 2 x 3 x 29

Therefore, HCF (72, 102, 174) = 2 x 3 = 6

hence, 6 is the required number.

Q13. : **Find the least number which when divided by 5,7,9 and 12, leaves the same remainder 3 in each case**

Solution : In these type of questions, we need to find the LCM of the divisors and add the common remainder (3) to it.

So, LCM (5, 7, 9, 12) = 1260

Therefore, required number = 1260 + 3 = 1263

Q.14 : **Find the largest four digit number exactly divisible by 15,21 and 28.**

Solution : The largest four digit number is 9999.

Now, LCM (15, 21, 28) = 420

On dividing 9999 by 420, we get 339 as the remainder.

Therefore, the required number is 9999-339 = 9660

Q.15 : The policemen at three different places on a ground blow a whistle after every 42 sec, 60 sec and 78 sec respectively. If they all blow the whistle simultaneously at 9:30:00 hours, then at what time do they whistle again together ?

Solution : They all will whistle again at the same time after an interval that is equal to the LCM of their individual whistle blowing cycles.

So, LCM (42, 60, 78) = 2 x 3 x 7 x 10 x 13 = 5460

Therefore, they will blow the whistle again simultaneously after 5460 sec, i.e., after 1 hour 31 minutes, i.e., at 11:01:00 hours.

Q.16. : **Find the least number which when divided by 6,7,8 leaves a remainder 3, but when divided by 9 leaves no remainder .**

Solution : LCM (6, 7, 8) = 168

So, the number is of the form 168m + 3.

Now, 168m + 3 should be divisible by 9.

We know that a number is divisible by 9 if the sum of its digits is a multiple of 9.

For m = 1, the number is 168 + 3 = 171, the sum of whose digits is 9.

Therefore, the required number is 171.

Q.17 : **Two numbers are in the ratio 2:3. If the product of their LCM and HCF is 294, find the numbers.**

Solution : Let the common ratio be 'm'. So, the numbers are 2m and 3m.

Now, we know that Product of numbers = Product of LCM and HCF.

=> 2m x 3m = 294

=> m^2 = 49

=> m = 7

Therefore, the numbers are 14 and 21.

Q.18 : **A rectangular field of dimension 180m x 105m is to be paved by identical square tiles. Find the size of each tile and the number of tiles required.**

Solution : We need to find the size of a square tile such that a number of tiles cover the field exactly, leaving no area unpaved.

For this, we find the HCF of the length and breadth of the field.

HCF (180, 105) = 15

Therefore, size of each tile = 15m x 15m

Also, number of tiles = area of field / area of each tile

=> Number of tiles = (180 x 105) / (15 x 15)

=> Number of tiles = 84

Hence, we need 84 tiles, each of size 15m x 15m.

Q.19 : **Three rectangular fields having area 60 m^2, 84 m^2 and 108 m^2 are to be divided into identical rectangular flower beds, each having length 6 m. Find the breadth of each flower bed.**

Solution : We need to divide each large field into smaller flower beds such that the area of each bed is same.

So, we find the HCF of the larger fields that gives us the area of the smaller field.

HCF (60, 84, 108) = 12

Now, this HCF is the area (in m^2) of each flower bed.

Also, area of a rectangular field = Length x Breadth

=> 12 = 6 x Breadth

=> Breadth = 2 m

Hence, each flower bed would be 2 m wide.

Q.20 : **Find the maximum number of students among whom 182 chocolates and 247 candies can be distributed such that each student gets same number of each. Also, find the number of chocolates and candies each student will get.**

Solution : We need to find the HCF of the number of chocolates and candies available, which would give us the number of students.

HCF (182, 247) = 13

So, there can be 13 students.

Also, Number of chocolates for each student = 182 / 13 = 14

Number of toffees for each student = 247 / 13 = 19

Q.21: Find the greatest number of four digits which when divided by 11, 21, 15, and 28 leaves 5, 15, 9 and 22 as remainders, respectively?

Sol: The LCM of 11, 21, 15 and 28 is 4620. The greatest 4 digit multiple of 4620 is 9240. Now the difference between the number and the respective remainder is same here. So to find the greatest 4 digit number which satisfies the given condition, subtract the common difference from 9240. Hence the required number is 9240 – 6 = 9234.

Q.22: Find the greatest number, which when divides 41, 71 and 91, leaves the same remainder in each case.

Sol: Take the difference between all the three pairs of numbers and their HCF will be the answer i.e. 91 – 41 = 50, 71 – 41 = 30 and 91 – 71 = 20. Now, the HCF of 50, 30 and 20 is 10.